Web Appendix to Accompany

“Economic Perspectives on the Advance Market Commitment for Pneumococcal Vaccines”

Health Affairs, August 2011.

Christopher M. Snyder
Dartmouth College Department of Economics and National Bureau of Economic Research

Wills Begor
Dartmouth College Department of Economics

Ernst R. Berndt
MIT Sloan School of Management and National Bureau of Economic Research

1. Introduction
This web appendix provides technical details on the theoretical economic model used in the article “Economic Perspectives on the Advance Market Commitment for Pneumococcal Vaccines.” This theoretical model was used to generate the numerical examples in Exhibit 3 of the paper.

In Section 2 of this web appendix, we define the variables used in the model. In Section 3 we provide equations for supplier profits (used to compute equilibrium supply in the model) and for net social benefits (used as the criterion for judging the performance of the program and to compare alternative designs). In Section 4, we illustrate how to use the model to generate the numerical examples provided in Exhibit 3 of the paper. In the final section we discuss a variety of extensions of the basic model, allowing for competition, demand uncertainty, cost uncertainty, and so forth.
2. Definition of terms

We will analyze two different designs for the advance market commitment: the design as actually implemented, with a cap on the subsidy a firm can earn, and the design as initially proposed, without a subsidy cap.

We begin by defining the terms of the design of the advance market commitment as actually implemented. The cap on the subsidy associated with this design was tied to the fraction of the supply target that the firm commits to supply. We will fill in additional terms for the other design of the program later when we turn to analyzing that design.

Let $Q$ be the supply target, the maximum number of doses needed annually to vaccinate all children in all GAVI countries according to an appropriate schedule. Let $q_i$ be the quantity of the vaccine supplied by firm $i$ annually under the commitment. Let $A$ be the dollar amount of the advance market commitment fund used to pay the subsidy. (All monetary values herein are stated in nominal US dollars.) Let $s$ be the dollar amount of the “top up” subsidy per dose paid out of the fund. This is paid for all the initial doses sold during the initial subsidy period, which lasts until the supplier has exhausted its due portion of the advance market commitment fund. Let $T_s$ be the length of time (in years) of this initial subsidy period. Let $t$ be the “tail price.” This is the baseline price to which the subsidy $s$ is added for the initial subsidized doses sold; it is also the agreed upon price for units sold under the commitment after the initial subsidy period. Let $T$ be the length of time in years of the whole supply commitment period, including both the initial period ($T_s$) during which the supplier receives subsidized price $s + t$ and the subsequent period during which supplier receives just the tail price $t$.

Regarding the supplier’s costs, let $c$ be the unit production cost per dose. Let $k$ be the per-dose cost, paid up front, of installing a unit of capacity. As discussed in the paper, this
includes the cost of constructing a vaccine manufacturing plant as well as a subtle bargaining cost explained there. In the case of multiple suppliers, we assume they face the same costs, but it would be straightforward to extend the model to allow for heterogeneous firms. Let $L$ be the lifespan of the facility built to manufacture the vaccine.

Let $b$ be the value to the donors of the health benefits provided by the vaccine; this social value is expressed in monetary terms per dose. Assume all parties in the model (both donors and firms) face the same common interest rate—the market interest rate $r$—and use this to discount profit and social benefit streams earned over a period of time back to present values. It is straightforward to extend the model to allow for private and social discount rates to differ, but we will save on notation by having a single rate.

3. Profits and social benefits

It turns out to be easiest to analyze the program using a continuous-time model. Firms maximize the present-discounted value of the stream of profit (revenues minus unit production cost) from the program net of the up-front cost of installing capacity. Most of the analysis focuses on the simple case of a single supplier. In Section 5.1 we show how to extend the model to allow for competition.

Donors (or the program designer on the donors’ behalf) maximize the present discounted value of social benefits net of program expenditures. By subtracting off program expenditures we are implicitly assuming that donors place no weight on suppliers’ profits under the program. We are not advocating that firm profits receive no weight in social welfare. Rather, we have two other reasons for making this assumption. First, we are trying to reflect the objectives of donors who have a focus on global health rather than some broad utilitarian social planner who takes into account the surplus of all members of society including producers. For example, the donor
may be the finance ministry of country 1. Even if this finance ministry places positive weight on profits accruing to firms located within its own borders, if the supplier participating in the advance market commitment happens to be located in country 2, country 1’s finance ministry may not weight that firm’s profit in its objectives. Another reason for assuming that donors place no weight on supplier profits is that we want to base any recommendation for enhancing the program’s generosity toward firms (if such recommendations happen to emerge from the analysis) on the most conservative assumptions possible. That is, any such recommendations will be based purely on the health benefits provided and not on some positive weight on firm profits.

3.1. Analysis of program as implemented (with cap on subsidy)

The length of the subsidy period $T_s$ solves the following equation:

$$sq_i T_s = \left(\frac{q_i}{Q}\right) A.$$  \hspace{1cm} (1)

The left-hand side is the total subsidy earned under the program. The right-hand side is the cap on the subsidy, constraining the supplier to extract no more from the fund $A$ than the proportion $q_i/Q$ of the total target it supplies. Solving equation (1) yields

$$T_s = \frac{A}{sQ}.$$  \hspace{1cm} (2)

Supplier $i$’s profit equals

$$\int_0^\tau (t-c)q_i e^{-rx} \, dx + \int_{T_s}^\tau sq_i e^{-rx} \, dx - kq_i.$$  \hspace{1cm} (3)

The first term is the subsidy earned during the initial period until the supplier hits the cap. The second term is the margin over costs provided by the tail price for all units sold during the program. The second term adds the revenue from the subsidy earned during the initial period.
until it hits its cap in year $T_s$. The last term is the up-front cost of installing capacity for the $q_i$ units.

The net social benefit from supplier $i$’s participation in the program is

$$
\int_0^T (b - t) q_i e^{-rx} dx + \int_{T}^{L} (b - c) q_i e^{-rx} dx - \frac{q_i A}{Q} + \int_0^{T_s} r \left( \frac{Aq_i}{Q} - sxq_i \right) e^{-rx} dx. \tag{4}
$$

The first term is the net of social benefit over the tail price over the duration of the program. The second term accounts for the social benefit coming after the program while the production facility is still in operation and remains able to produce doses for sale to GAVI-eligible countries. We assume that these doses are sold at marginal cost; this assumption is discussed in the next paragraph. The last two terms account for the expense of the advance market commitment subsidy fund. The accounting convention is that the whole amount up to the cap is set aside in a separate account (hence the subtraction of this proportion in the third term). The account earns interest on the declining portion of the advance market commitment subsidy fund as it is paid out to the supplier. This interest, represented by the last term, goes back to the donors.

Several assumptions embedded in equation (4) require further discussion. First, consider the assumption that the firm continues to supply the market after the advance market commitment runs its course but while the manufacturing facility is still operational. One might worry that the supplier may raise the price above cost to GAVI-eligible countries when the tail period expires or may divert capacity toward middle-income countries when its supply commitment expires. The historical experience of UNICEF and the World Health Organization procuring medicines at or below production cost from suppliers’ excess capacity argues against a price increase by the supplier. Indeed, the whole logic of the advance market commitment is to
provide a subsidy because of the problem that the price typically paid for medicines procured for
developing countries at best covers production cost but does not cover earlier investment
expenditures.

The diversion of supply to middle-income countries is perhaps a more significant worry. The
model can be adjusted to allow the firm to supply nothing to GAVI-eligible countries after
the program expires. The qualitative implications of the analysis should continue to hold
because the length of time in question—the $L - T$ years between the expiration of the advance
market commitment and the lifespan of the plant—is not very long in the baseline numerical
example, and the social benefit earned during this period is heavily discounted because it comes
at the end of the game.

Equation (4) also combines all program expenses—whether borne by the donors to the
advance market commitment fund, by GAVI, or by the aid-recipient countries—and subtracts
them off from the social benefit to get a net social benefit value. Our perspective is that all these
funds ultimately are sourced from donors (whether the donors to GAVI and other aid donors to
the countries themselves). This expansive definition of expenses will prevent us from
concluding that one program design is better than another merely because it shifted expenses
from one form (say a tail price) to another (say an advance market commitment subsidy or a
purchase guarantee). Holding total expenditures constant, the specific terms of the program are
more or less fungible from the donors’ perspective.

### 3.2. Analysis of program as initially designed (without cap on subsidy)

The advance market commitment was initially proposed without a cap on subsidy.

Mathematically, this means that instead of the subsidy period $T_s$ solving equation (1), it solves
\[ sq_i T_s = A, \]  \hspace{1cm} (5)

implying

\[ T_s = \frac{A}{sq_i}. \]  \hspace{1cm} (6)

Also, the period of this form of advance market commitment is assumed to stretch the full \( T \) years beyond the initial \( T_s \) subsidy period. That is, this form of advance market commitment lasts a total of \( T_s + T \) years.

The assumption on the length of the program without a subsidy cap deserves some discussion. Comparing equations (2) and (6) shows that the length of the subsidy period \( T_s \) will generally differ between the design with a subsidy cap and the design without. This leaves some ambiguity over how to set the overall length of the program to facilitate the most informative comparison between the programs. Among other options, one could equalize the overall length of the commitment, equalize the length of the tail period, or equalize the ratio of the tail to subsidy period. Our assumption that the program without a subsidy cap lasts for a full \( T \) years after the subsidy fund runs out is easier to operationalize than these other options; if as in the benchmark example, \( t = c \), the length of the program is immaterial because the firm supplies its committed quantity over the lifespan of the facility and earns a zero margin on sales outside the subsidy period in any event.

Supplier \( i \)'s profit under this form of advance market commitment is

\[ \int_0^{T_s+T} (t - c)q_i e^{-rx} dx + \int_0^{T_s} sq_i e^{-rx} dx - kq_i. \]  \hspace{1cm} (7)

Net social benefit is
\[
\int_0^{T_s + T} (b - t) q_i e^{-rx} dx + \int_{T_s + T}^{L} (b - c) q_i e^{-rx} dx - A + \int_0^{T_s} r(A - sxq_i) e^{-rx} dx. \tag{8}
\]

Notice that the full advance market commitment fund \( A \) is now subtracted as an expenditure regardless of how much the supplier produces reflecting the lack of cap on the subsidy. The last term for the interest on the fund also needs to be adjusted from equation (4).

### 3.3. Benchmark values

As noted in the paper, in the benchmark case we set the model variables as follows.

\[
\bar{Q} = 200 \text{ million doses annually} \tag{9}
\]

\[
A = $1.5 \text{ billion} \tag{10}
\]

\[
s = t = c = $3.50 \tag{11}
\]

\[
k = $4.00 \tag{12}
\]

\[
b = $8.50 \tag{13}
\]

\[
T = 10 \text{ years} \tag{14}
\]

\[
L = 15 \text{ years} \tag{15}
\]

\[
r = 5\% \tag{16}
\]

### 4. Solving for equilibrium: an illustration

In this section we demonstrate how to solve for the equilibrium of the model. Appendix Exhibit A1 graphs profit under the program with a cap from equation (3) over the range of quantities between 0 and \( \bar{Q} \).
As the exhibit shows, profit continues to increase over the range of quantities, so the optimal supply for the firm is a corner solution involving the entire target $\bar{Q} = 200$ million annual doses. Net social benefit can be read off a graph of equation (4) over the range of quantities between 0 and $\bar{Q}$, shown in Appendix Exhibit A2.

At an output of $\bar{Q} = 200$ million annual doses, the present discounted value of the stream of net social benefits is $9.1$ billion.
Solving for equilibrium under the advance market commitment without a subsidy cap is similar. Appendix Exhibit A3 graphs supplier profit over the feasible range of outputs.

The interior optimum is reached at 55 million doses annually. The kink in the profit function arises because at low annual output levels the manufacturing plant expires before the advance market commitment terminates.

At an output of 55 million annual doses, Appendix Exhibit A4 shows that the present discounted value of the stream of net social benefits is $1.7 billion.
5. Extensions

Each of the next subsections treats a different extension of the benchmark model. These extensions fill in the lower rows of the table in Exhibit 3 in the paper.

5.1. Modeling competition

We model competition among $N$ symmetric suppliers as a game in which they choose quantities simultaneously. This model of competition is known in the Industrial Organization literature as Cournot competition (see Nicholson and Snyder 2008, Chapter 15, for a textbook treatment).

Consider the advance market commitment without the subsidy cap. Profits for firm $i$ under this form of program are still given by equation (7). The difference is that the length of the subsidy period is shorter: $T_s$ now satisfies

$$s[q_i + (N - 1)q_{-i}]T_s = A,$$  \hspace{1cm} (17)

where $q_{-i}$ is the supply of one of the other firms besides $i$. Solving for $T_s$, we obtain
The Nash equilibrium of this game is a fixed point $q^*$ such that, fixing $q_{-i} = q^*$, the profit-maximizing quantity for supplier $i$ (which can be read off a graph such as in Appendix Exhibit A3) also equals $q^*$.

**5.2. Modeling private cost information**

The paper provides a numerical example in which suppliers have private information about their costs. In the particular example, we model this by assuming that $c$ is a uniform random variable within the interval [$3.00, \ 4.00$]. The designer only knows the distribution of $c$; the supplier knows the specific draw of $c$ from this distribution. We analyze this case by letting $c$ take on values for each 10-cent increment: $3.00, 3.10, 3.20, \ldots, 4.00$. For each of these values of $c$ we solve for the equilibrium individually as described above in Section 4. Then we take the average over the outcomes to produce expected values reported in Exhibit 3 in the paper. Because $c$ is uniformly distributed in the numerical example, it is appropriate to take the simple average; other distributions would require weighting by the relative likelihoods of the cost observations.

**5.3. Modeling demand uncertainty**

Our simple model of demand uncertainty involves a probability $d$ that the full demand for $\bar{Q} = 200$ million annual doses materializes and the complementary probability $1 - d$ that no demand materializes. In the case of the version of advance market commitment with a cap on a firm’s subsidy, supplier $i$’s profit is

$$T_s = \frac{A}{s[q_t + (N-1)q_{-i}]}.$$
\[ d \left[ \int_0^T (t - c)q_i e^{-rx} \, dx + \int_0^{T_s} s q_i e^{-rx} \, dx \right] - k q_i. \]  

(19)

This is the same as in equation (3) except the terms reflecting the present discounted value of the flow of the supplier’s income are scaled down by the probability \( d \). The up-front cost of capacity investment \( k q_i \) is not scaled down because it is sunk before demand is realized.

**References**