

Physics 3 Summer 1989

Lab 7 - Elasticity

Theory

All materials deform to some extent when subjected to a stress (a force per unit area). Elastic materials have internal forces which restore the size and shape of the object when the stress is removed. If the deformation, or strain (the ratio of the change in length to the initial length), is directly proportional to the applied stress and if it is completely reversible so that the deformation disappears when the stress does, then the material is said to be perfectly elastic. An elastic material exerts a restoring force when stretched and this can lead to oscillation when the elastic material is attached to a mass.

The basic law of elasticity is Hooke's law. If a force \mathbf{F} is applied to an elastic object (such as a spring), the object will undergo a change in length \mathbf{x} which will be directly proportional to \mathbf{F} . Mathematically, this can be expressed as

$$\vec{F} = k(\vec{x})$$

where k is a proportionality constant called the spring constant. The spring constant has units of dynes per centimeter or newtons per meter. The restoring force \mathbf{F}_R , the force exerted by the object on the agent responsible for the applied force, is equal in magnitude to F and oppositely directed to \mathbf{x} ,

$$\vec{F}_R = -k(\vec{x}) \quad . \quad (1)$$

Equation (1) is the mathematical form of Hooke's law.

This laboratory will be a study of the elasticity in a spring, a copper wire and a rubber tube.

The Spring Constant of Springs

Consider a spring of length x_0 which is suspended from a support as shown in figure 1. If a mass is attached to the free end of the spring, the spring will stretch and come to rest with a new length x_1 . Since the system is in equilibrium, the applied force, the weight of the mass, must equal the restoring force. Applying Hooke's law yields

$$mg = k(x_1 - x_0) \quad .$$

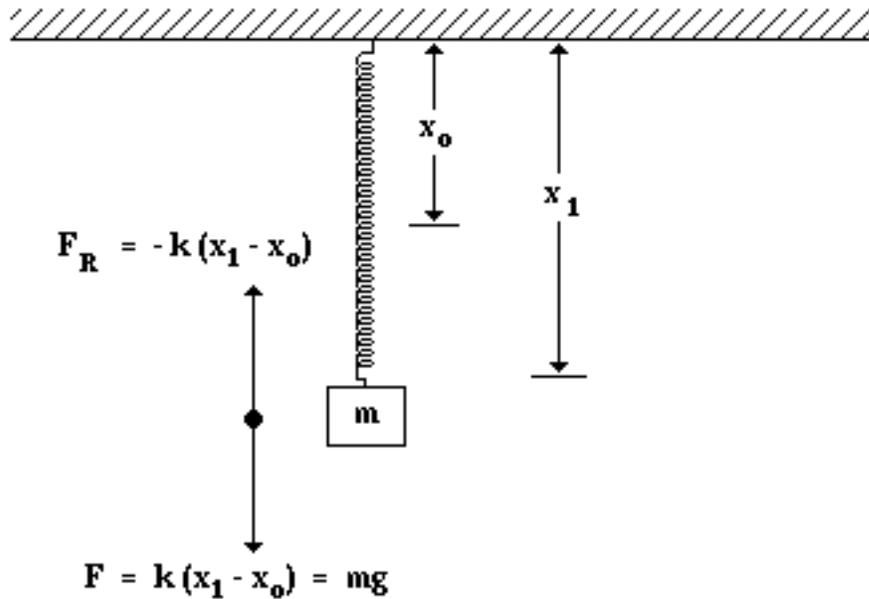


Figure 1

We could measure k for the spring by hanging a known mass from the spring and measuring x_1 . But this would mean that our value of k would depend on only one measurement (x_1) in which there might be a lot of error. A little more sophisticated method is as follows. We notice that $mg = k(x_1 - x_0)$ can be written as

$$x_1 = \left(\frac{g}{k}\right)m + x_0 \quad (2)$$

which is of the form

$$y = mx + b$$

where $y = x_1$, $x = m$. So if we hang lots of different masses on the spring and measure the extension x_1 for each, the slope of the graph is g/k . We can find k , then, by calculating the slope. This makes the value of k depend on many measurements of x_1 and makes it likely that random errors in measurement will cancel (if one x_1 measurement is a little too high, another will be a little too low, etc).

In part I of this lab, you will experimentally measure the spring constants of several single springs and several spring systems using the method described above.

Elasticity in Metals - The Copper Wire

As it turns out the value of k for a spring made of a given material is not constant at all, but depends on the dimensions of the spring - its length, cross-sectional area, shape etc. Two springs made of the same material but with different lengths will have different values of k . The spring

constant, therefore, is a property of the sample under consideration and does not give a measure of the elasticity of the material itself. In most applications, however, we are not interested in the properties of a particular sample, but rather in the fundamental properties of the material out of which a sample is made. Hooke's law may be written so that the proportionality constant depends only on the material of which the object is made and not on its dimensions.

If we consider a copper wire as our "spring", experiment shows that the amount it stretches for a given force doubles if the cross-sectional area A of the wire is halved. But, if the same force per unit area is used, all copper wires do the same thing - they stretch a certain proportion of their length.

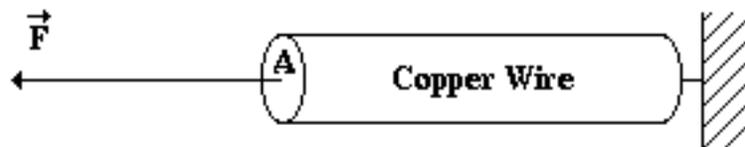


Figure 2

Mathematically,

$$\frac{\Delta L}{L} = \frac{1}{Y} \frac{F}{A} \quad (3)$$

where L is the length of the wire, Y is a constant, $\Delta L/L$ is the proportional change in length of the wire and F/A is the force per unit area. F/A is called the stress and $\Delta L/L$ is called the strain. We can rewrite equation (3) as

$$\frac{F}{A} = Y \frac{\Delta L}{L} \quad (4)$$

which shows that the stress is directly proportional to the strain. Since all copper wires stretch this way, Y is a constant measuring the elasticity of copper. Y is called the Young's modulus. The Young's moduli for many materials have been experimentally measured and can be found in tables. Equation (4) can be written as

$$F = \frac{YA}{L} \Delta L \quad (5)$$

From this we see that the spring constant is related to Young's modulus of the material forming the spring by

$$k = Y \left(\frac{A}{L} \right) \quad (6)$$

According to equation (4), for elastic materials a plot of stress vs strain will be a straight line with a slope equal to Y .

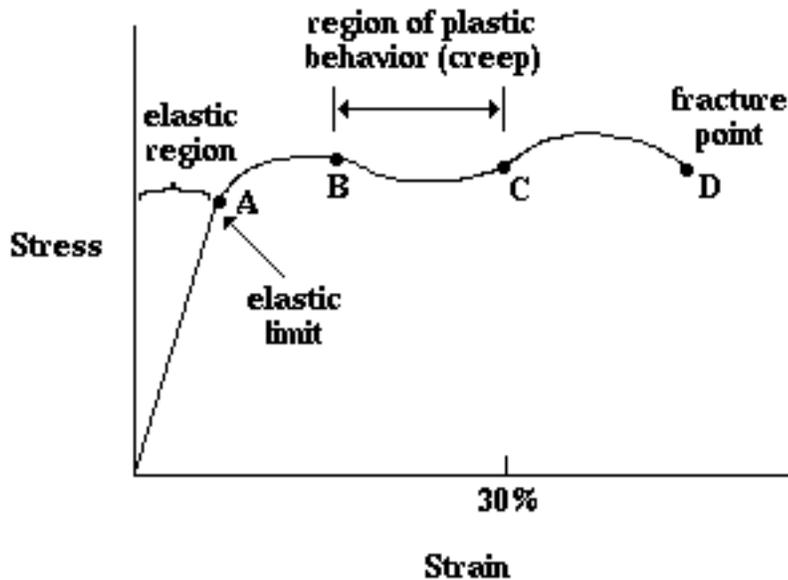


Figure 3

If a piece of copper wire is stretched by hanging weights on it, a plot of stress vs strain such as shown in figure 3 is obtained. Note that only over a portion of the graph does the copper wire exhibit elastic behavior. Initially the strain is proportional to the applied stress; if the weights are removed, the wire shrinks back to its original length. On the graph, this is the region between points O and A. In this region, Hooke's law is obeyed. If too much stress is applied, the wire will experience a permanent strain; the wire is still elastic, but has been permanently elongated. The maximum stress which the wire can experience and still return to its original length is called the elastic limit (point A on the graph). If greater than critical stress is applied, the wire will creep; that is, it will continue to stretch even if no additional stress is applied. This is the region B to C in the figure 3. Finally, if enough stress is applied, the wire will sag and break (point D on the graph). Point D is called the fracture point.

In part II of this lab, you will hang weights on a copper wire and measure the stress and strain. From that data you will compute Young's modulus for copper and plot a stress vs strain graph similar to the one in figure 3.

Rubber Elasticity

A rubber band and a copper wire both stretch under tension, but there is a fundamental difference between the underlying mechanisms of the two cases. In the case of the copper wire, the stretching changes the interatomic distances slightly. The tension in the wire is the sum total effect of all the atoms in a cross-section pulling on one another. Rubber is made up of very long chain polymer molecules. They are tangled and intertwined in a very disordered state. When stretched, they become slightly more ordered and aligned. Thermal agitation disorders the rubber molecules and the greater the agitation the greater the tension. When rubber is heated the vibrations of the molecules become more intense, the molecules become more tangled and the rubber

contracts. When copper is heated the atomic vibrations increases, the interatomic distances open up and the copper expands.

The Young's modulus for copper and other "hard" materials (e.g. metals, glass fibers, minerals) is about 10^{11} in MKS units while the modulus for rubber and other "soft" elastic materials is about 10^6 . Rubber is used to make "elastic" bands, but rubber and rubber-like substances have very poor elastic properties. If a weight is hung on a sample of rubber, it stretches almost immediately to a new length, but then continues to stretch slowly and exponentially in time to a final length. This is called creep. It means that the Young's modulus is not an exact quantity, but is time dependent. If weights are added one at a time

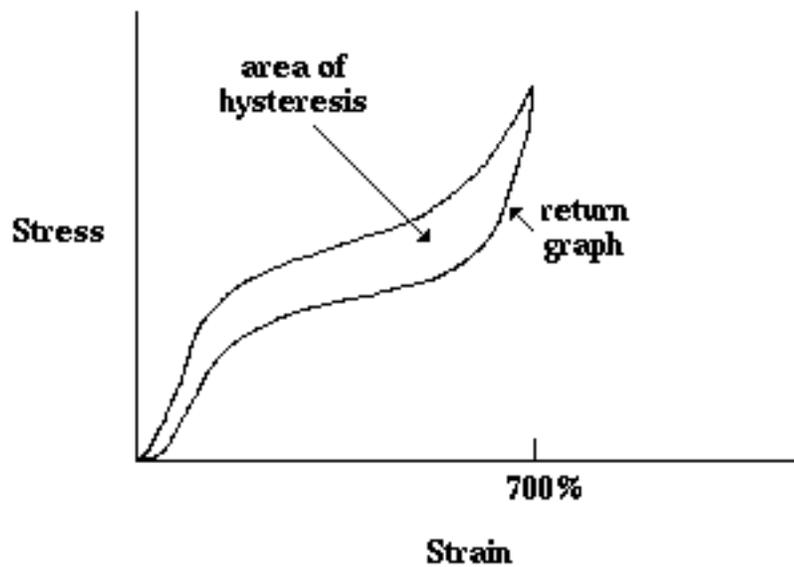


Figure 4

and the change in length is measured, the graph of weight vs change in length is the stress vs strain diagram. If the weights are now removed one at a time and the new contracted lengths plotted, a "return" stress-strain diagram is obtained. The two curves are not the same, as shown in figure 4. This effect is called hysteresis. Copper wire does not creep and does not show hysteresis as long as the stresses are moderate, but rubber-like substances always do.

In part III of this lab, you will hang weights on a rubber tube and measure weight vs l . From this data you will plot an F vs l graph similar to figure 4 and from the graph compute an approximate value for Young's modulus for rubber.

References

Halliday and Resnick very briefly touch on the topic of Hooke's Law and elasticity in section 3-6 (pages 293-296) of your text. Therefore, the following two books have been put on reserve in Kresge library as supplemental reading. The readings in one of these books should be done before coming to lab.

1. Introduction to Physics for Scientists and Engineers, Bueche -- chapter 6 section 9 and chapter 13 section 3
2. University Physics, Sears and Zemansky -- chapter 10 and chapter 11 section 2

Experimental Purpose

Part I: The Spring Constant of Springs

In this part, you will use the method described in the introduction for experimentally measuring the spring constant of a spring, or a system of springs, to measure the effective spring constant for: (1) three single springs, (2) for two springs hung in parallel and (3) for two springs hung in series.

For the springs hung in parallel, you will confirm that the effective spring constant for the system is given by the expression

$$k = k_1 + k_2$$

where k is the effective spring constant for the system and k_1 and k_2 are the spring constants for the two individual springs.

For the springs hung in series, you will confirm that the effective spring constant for the system is given by the expression

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

where k , k_1 and k_2 are defined as above.

Part II: Elasticity in Metals - The Copper Wire

In this part you will: (1) experimentally measure Young's modulus for copper and (2) take data from which you will plot a stress vs strain graph. From the graph you will determine the elastic limit and the fracture point and identify the elastic region and the region of creep.

Part III: Rubber Elasticity

In this part you will: (1) experimentally measure Young's modulus for rubber and (2) take data from which you will plot an F vs L graph. On the graph you will identify the area of hysteresis.

Procedure

Part I: The Spring Constant of Springs

Note: In part III, you must wait at least 5 minutes between measurements to allow the creep to become negligible. During those time intervals, it is possible to take the data for part I and thereby reduce the overall time needed to take data for the lab. For this reason, the apparatus for parts I and III are located at the same experimental stations. Start your data taking for part III and then return to this section going back to part III at the appropriate times.

The apparatus for this part consists of five springs matched in weight and length but having different spring constants, a frame for suspending the springs, metersticks to measure the extensions of the springs when weights are hung on them and the weights.

1. Adjust the position of the hook on which the spring will hang so that the spring will hang in front of the meterstick. Suspend any of the five springs from the hook and hang 1000 grams on the bottom of the spring. Adjust the height of the meterstick so that the bottom of the spring does not extend below the bottom of the meterstick. Remove the 1000 grams and record the initial position of the bottom of the spring. (Note: The term "bottom of the spring" is ambiguous. For example, it could mean the bottom of the last coil on the main body of the spring or it could mean the bottom of the coil which has been bent at a right angle to the coils for the purposes of hanging weights on the spring. It does not matter which definition you use as long as you use the same definition throughout the data taking.)
2. Hang a two hundred gram mass on the bottom of the spring and record the new position of the bottom of the spring. Attach a second two hundred gram mass to the bottom of the first and again record the position of the bottom of the spring. Continue this process until a total of one thousand grams has been hung on the spring.
3. Remove one of the masses from the spring and record the position of the bottom of the spring. Continue this process until all of the weights have been removed. Does the spring exhibit elastic behavior? Explain.
4. Repeat steps 1 and 2 for two other single springs.
5. Suspend any two of the springs that you used in steps 1 through 4 so that they are hanging side by side from the same hook. You now have two springs hung in parallel. Repeat step 2 for this system. Be sure that when hanging the first mass it is attached to the bottom of both of the springs. To do this, bend open a paper clip and use one end to connect the bottoms of both of the springs together. Hang the first mass from the hook formed by the other end of the open paper clip.
6. Suspend any two springs that you used in steps 1 through 4 so that the bottom of one is attached to the top of the other. You now have two springs hung in series. Readjust the

position of the meterstick to account for the longer initial length of this system. Repeat step 2.

7. (optional): Suspend a spring-mass-spring system from the frame and readjust the position of the meterstick to account for the longer initial length of the system. Repeat step 2. Reverse the order of the springs and repeat the process. Does the order of the springs make a difference?

Part II: Elasticity in Metals - The Copper Wire

The apparatus for this part consists of a sample of copper wire about 2 meters long and approximately 0.08 cm in diameter. It is hung from the ceiling with a hanger for weights attached to its base. Kilogram and half kilogram weights are available - enough to stretch the wire to its breaking point. The small initial elongations of the wire, up to 1 inch, are measured with a dial micrometer. After that the extensions are measured with a meterstick. A micrometer caliper is provided to measure the diameter of the wire, which changes during elongation (the density of the copper stays nearly constant, so the wire shrinks in area as it stretches in length).

1. Remove the hanger from the bottom of the wire and measure its mass. Replace the hanger and place on it two kilogram masses. This is enough to straighten the wire and take out the kinks. Adjust the position of the one holed rubber stopper through which the wire hangs so that the wire does not touch the sides of the stopper. Measure the initial length and initial diameter of the wire. Remove the two kilogram masses. Adjust the position of the dial micrometer so that its piston just touches the bottom of the aluminum cylinder which is glued to bottom of the hanger. (Note: In all that follows, the "bottom" of the hanger is defined as the bottom of the aluminum cylinder.) Do this so that the reading on the dial is as close to zero as possible and then record the initial value. Position the meterstick so that extensions over 1 inch can be measured. Record the initial position of the bottom of the hanger with respect to the meterstick.
2. Carefully place a half kilogram mass on the hanger. (Note: Whenever adding or removing masses from the hanger, do so in such a way that the hanger and the wire are moved as little as possible. Why is this important?) Record the reading on the dial micrometer and measure the diameter of the wire. Add another half kilogram mass and repeat the measurements. Continue this process until a total of two kilograms of mass has been placed on the hanger.
3. Remove one kilogram of mass from the hanger and record the reading on the dial micrometer. Is the copper wire exhibiting elastic behavior? Explain. Replace the kilogram of mass that you removed.
4. Continue to add masses at increments of 0.5 kilograms until a total of 5.0 kilograms of mass have been placed on the hanger. After each mass is added, record the position of the bottom of the hanger and the diameter of the wire. Note that around 5.0 kilograms the wire will begin to show some creep. Therefore, after each mass is added you should look to see if there is

evidence of creep. (What do you look for?) If there is, then wait until the creep stops before making any measurements.

5. Once a total of 5.0 kilograms is reached, continue to add masses in increments of 1.0 kilogram and to record the position of the bottom of the hanger and the diameter of the wire after each mass is added. Do this until the wire sags and breaks which will occur after 12-14 kilograms of mass have been placed on the hanger. (Note: While the elongation of the wire is less than one inch, use the dial micrometer to determine the position of the bottom of the hanger. When the elongation exceeds one inch, remove the dial micrometer and thereafter use the meterstick to determine the position of the bottom of the hanger.) Above about 6.0 kilograms, the creep will continue for a half hour or more and all measurements become approximate. When that happens, wait about 5 minutes between placing the mass on the hanger and taking the measurements.
6. Replace the broken wire with a new one and hang approximately 2 kg of weight on it. Remove the s-shaped metal piece from the ends of the broken wire and put them in the box indicated by your TA. Wire cutters are available in the lab for this purpose. Put the remains of the broken wire into a waste basket.

Part III: Rubber Elasticity

The apparatus in this part is the same as in part I except that the springs are replaced by a sample of rubber tubing with hooks fitted into the ends. The cross-sectional dimensions of the tubing are about 1/8" O.D. x 1/32" wall, but they should be checked using loose pieces and measured with a calibrated eye-piece.

The measurements in this part of the lab can not be made with great precision because the properties of rubber are quite variable and change with time. When you hang a weight on the tube, it takes a very long time, longer than is available in the lab, for the rubber to come to equilibrium. The object is to make some observations, as precisely as possible, to gain an understanding of the difference between metal elasticity and rubber elasticity.

1. Adjust the position of the hook on which the rubber tubing will hang so that the rubber tubing will hang in front of the meterstick. Hang 500 grams on the bottom of the rubber tubing. Adjust the height of the meterstick so that the bottom of the rubber tube does not extend below the bottom of the meterstick. Remove the 500 grams and record the initial position of the bottom of the tubing. Use the calibrated eyepiece to measure the inside and outside diameter of the tubing. There is a cut piece of tubing in the metal drawer for this purpose.
2. Hang a 100 g mass on the rubber tubing. Wait approximately five minutes for the creep to stop (nearly) and record the new position of the bottom of the tubing. Add another 100 g mass and repeat the measurement. Continue doing this until a total mass of 500 g has been hung on the

tube.

3. Reverse the process - removing 100 g at a time, waiting approximately 5 minutes for the rubber to come to equilibrium and recording the extension of the tube - until you get back to 100 g.
4. optional: Hang 800 g on the rubber tube. Measure the extension every ten minutes for one hour. This data can be plotted to show that the creep does in fact die off approximately exponentially.

When you are done with parts I and III, put the springs back in the box from which you got them and put the hooked masses, the paper clips, the calibrated eyepiece and the rubber tubing back into the metal tray.

Lab Report

Follow the usual lab notebook format. Your lab report should include the answers to all of the questions asked in the introduction or procedure, all raw and derived data, and an estimate of the magnitude and sources of error in any data recorded. When answering any question or when giving any comparison or explanation, always refer to specific data to support your statements. Many of the calculations and graphs can be done using the computer program ELAS . ELAS can be called up by typing OLD PHYSLIB***:P3:ELAS . This program is compiled and cannot be listed. A documented version of the source code can be accessed by typing LIST PHYSLIB***:P3:ELAS.DOC . A listing of the first 340 lines will explain what the program does and what information you will need to supply. For this lab, also include the following:

Part I: The Spring Constant of Springs

1. a plot of extension x_1 vs m for the three single springs and for each of the multiple spring systems with a discussion of how well each graph correlates with the theoretically expected graph;
2. the computations of k for each of the single springs and the multiple spring systems; and
3. theoretical derivations showing that:
 - a. for two springs in parallel, the effective spring constant for the system is given by

$$k = k_1 + k_2$$

and

- b. for two springs in series, the effective spring constant for the system is given by

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

- optional: a derivation of the theoretical expression for the effective spring constant for the spring-mass-spring system. (Would this expression change if the spring-mass-spring system was horizontal rather than vertical? Explain.)

Part II: The Elasticity of Metals - The Copper Wire

- a table containing a calculation of the stress, the strain, and Young's modulus for each data pair; show one calculation of the stress, the strain, and Young's modulus starting from the raw data;
- a computation of the average value of Young's modulus for copper with a comparison with the accepted value; and
- a plot of stress vs strain (plotted on 4 x 2 log-log paper) with the following identified - the elastic region, the elastic limit, the plastic region and the fracture point.

Part III: Rubber Elasticity

- a plot of F vs L with an explanation of its shape and what causes the observed hysteresis;
- a computation of an average value of Young's modulus for rubber;

Note: The slope of the F vs L curve (dF/dx) is the effective value of the force constant at any point. Note how far this is from constant. When calculating the Young's modulus for the tube, you must take into account the shrinking of the cross-section of the rubber as the tube stretches. This last factor can be accounted for in the following way. Let A_0 be the unstretched cross sectional area and L_0 be the unstretched length of the rubber tube.

Assuming that $AL = \text{constant}$, we have

$$A_0 L_0 = A (L_0 + L) \quad .$$

Therefore,

$$Y = \frac{\frac{F}{A}}{\frac{L}{L_0}} = \frac{F L_0}{A L} = \frac{F L_0}{A_0 L} \left(1 + \frac{L}{L_0} \right) \quad .$$

The initial value of A_0 can be taken from the nominal dimensions of the unstretched rubber. Do this for at least two widely spaced value of F and L .

3. (optional): a plot of length vs time showing the decay of the creep.

In all parts be sure your units agree throughout your calculations.