ABSTRACT
Synthetic Red Giant Branch Bump (RGBB) magnitudes are generated with the most recent theoretical stellar evolution models computed with the Dartmouth Stellar Evolution Program (DSEP) code. They are compared to the observational work of Natali et al., who present RGBB magnitudes for 72 globular clusters. A DSEP model using a chemical composition with enhanced α capture (α/Fe) = 0.4 and an age of 12 Gyr shows agreement with observations over metallicities ranging from [Fe/H] = 0 to [Fe/H] ≈ −1.5, with discrepancy emerging at lower metallicities.

QUANTIFYING CONSISTENCY: 13 Gyr, α-enhanced DSEP model
We implement a χ² minimization routine to assess the goodness of fit of our best model to N2013’s data and subsets thereof. The reduced χ² score for the models fits the entire sample is 1.38, corresponding to a p-score of 0.0175, or a ~2% chance of recreating this observational spread with our model. The score is computed via

\[ \chi^2 = \sum \left( \frac{(M_{\text{mod}} - M_{\text{obs}})}{\delta M_{\text{mod}}^2} + \frac{(M_{\text{obs}} - M_{\text{true}})}{\delta M_{\text{obs}}^2} \right)^2 \]

with \( r_i = \sqrt{12 (\text{obs} - \text{true}) + 2 (\text{obs} + \text{true})} \).

IDENTIFYING OUTLIERS
A GC may be classified as an outlier in two ways: (1) if its contribution to the total reduced χ² score, or its p-score, is sufficiently large, and (2) if its p-score computed using the 4D Local Outlying Factor (LOF) algorithm is sufficiently large. The impact on the model-observation goodness of fit as outliers are removed from the sample is shown in these tables.

\[ \chi^2 \text{ Analysis} \]

<table>
<thead>
<tr>
<th>Sample</th>
<th>Reduced χ² score</th>
<th>p-value</th>
<th>GC</th>
<th>4D Rank</th>
</tr>
</thead>
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<tr>
<td>NZ2013</td>
<td>1.12</td>
<td>0.28</td>
<td>3.26</td>
<td>NGC 6093</td>
</tr>
<tr>
<td>1.38</td>
<td>0.0175</td>
<td>0.018</td>
<td>10.01</td>
<td>NGC 6254</td>
</tr>
<tr>
<td>1.38</td>
<td>0.0175</td>
<td>0.018</td>
<td>10.01</td>
<td>NGC 6254</td>
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<td>10.01</td>
<td>NGC 6254</td>
</tr>
</tbody>
</table>

Members of the LOF-tagged anomalous cluster group are removed from the sample beginning with the most discrepant and working outward. The degree of discrepancy is determined by the individual contribution a data point makes to the χ² score (\( r_i^2 \)).

LOF Routine

The 4D LOF routine identifies the most anomalous clusters based on density using a given point’s distance from the bulk distribution. Results are model-independent. The \( r_i^2 \) rank indicates how discrepant that cluster is with respect to the χ² metric. The sample indicates the observational population (defined by NZ2013) to which the GC belongs.

χ² Analysis using Kraft & Ivans Metallicity Scale
To test our results’ dependence on uncertainty in metallicity, the reduced χ² scores are computed adopting the cluster metallicities reported in Kraft & Ivans. Clusters for which Kraft & Ivans metallicities were not available are removed from the NZ2013 sample, leaving 40 clusters total.

IDENTIFYING ANOMALOUS CLUSTERS

We compare DSEP’s RGBB magnitude predictions to predictions given by other stellar evolution codes, including the Yonsei-Yale (YY) models, the Victoria-Regina (V) models, the PARSEC models, the BaSTI models, and tuned MESA tracks. TOP: Models with scaled solar compositions.

LEFT: Models using similar α-enhancements. The DSEP curve has α = −0.4, the YY curve has α = −0.3, and the BaSTI curve has α = −0.4. All of the models excluding PARSEC agree within a span of −0.2 magnitudes. Errors of this order may be due to differences in microphysical considerations.

OBSERVATIONAL SAMPLE (Natali et al., 2013)
Our analysis uses the observational sample of Natali et al. (2013) (NZ2013), comprising 72 GCs. The NZ2013 sample contains data from ACS, the current HST CCD, for 55 GCs and WFC2 for 17 GCs. NZ2013 separate the data into “silver” and “gold” samples, where the gold data are regarded with higher confidence. Motivated by disagreement between NZ2013’s magnitudes and the magnitudes reported in a previous NGC survey (Zoccali et al., 1999), we examined the raw data and found that NZ2013 may have underestimated their magnitude uncertainties, as demonstrated here. LEFT: Color–magnitude diagram centered on the RGBB region of NGC 6254. RIGHT: Cumulative luminosity function.

RGB MAGNITUDE PREDICTIONS FROM VARIOUS STELLAR MODELS

We compare DSEP’s RGBB magnitudes to predictions made with other stellar evolution models, including the Yonsei-Yale (YY) models, the Victoria-Regina (V) models, the PARSEC models, the BaSTI models, and tuned MESA tracks.

TOp: Models with scaled solar compositions. LEFT: Models using similar α-enhancements. The DSEP curve has α = −0.4, the YY curve has α = −0.3, and the BaSTI curve has α = −0.4. All of the models excluding PARSEC agree within a span of −0.2 magnitudes. Errors of this order may be due to differences in microphysical considerations.

QUANTIFYING THE TRENDS

We quantify the trend by fitting a cubic polynomial (red) to the magnitude differences. We exclude the four anomalous clusters (grey) from this calculation. Theoretical uncertainties from Bjork and Chaboyer (2000) are shown. Our model agrees with NZ2013’s data over the metallicity range [Fe/H]=0 to [Fe/H]≈−1.5, but disagreement amplifies in the most metal-poor regime.

AGE ANALYSIS

We can remove the uncertainty imparted by using NZ2013’s distance moduli by examining instead the difference in magnitude between the RGBB and subgiant branch (SGB). We use the MISTO colors reported by NZ2013 (available for 48 of 72 GCs) to estimate RGBB magnitudes. We may then superimpose 4Gyr-old predicted ΔVs on a grid of DSEP isochrones ranging from 9 to 15 Gyr to estimate the cluster ages.

ACKNOWLEDGEMENTS

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