Introduction to Probability

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To our wives
and in memory of
Reese T. Prosser

Contents

Preface

Probability theory began in seventeenth century France when the two great French mathematicians, Blaise Pascal and Pierre de Fermat, corresponded over two problems from games of chance. Problems like those Pascal and Fermat solved continued to influence such early researchers as Huygens, Bernoulli, and DeMoivre in establishing a mathematical theory of probability. Today, probability theory is a well-established branch of mathematics that finds applications in every area of scholarly activity from music to physics, and in daily experience from weather prediction to predicting the risks of new medical treatments.

This text is designed for an introductory probability course taken by sophomores, juniors, and seniors in mathematics, the physical and social sciences, engineering, and computer science. It presents a thorough treatment of probability ideas and techniques necessary for a firm understanding of the subject. The text can be used in a variety of course lengths, levels, and areas of emphasis.

For use in a standard one-term course, in which both discrete and continuous probability is covered, students should have taken as a prerequisite two terms of calculus, including an introduction to multiple integrals. In order to cover Chapter 11, which contains material on Markov chains, some knowledge of matrix theory is necessary.

The text can also be used in a discrete probability course. The material has been organized in such a way that the discrete and continuous probability discussions are presented in a separate, but parallel, manner. This organization dispels an overly rigorous or formal view of probability and offers some strong pedagogical value in that the discrete discussions can sometimes serve to motivate the more abstract continuous probability discussions. For use in a discrete probability course, students should have taken one term of calculus as a prerequisite.

Very little computing background is assumed or necessary in order to obtain full benefits from the use of the computing material and examples in the text. All of the programs that are used in the text have been written in each of the languages TrueBASIC, Maple, and Mathematica.

This book is on the Web at http://www.dartmouth.edu/~chance, and is part of the Chance project, which is devoted to providing materials for beginning courses in probability and statistics. The computer programs, solutions to the odd-numbered exercises, and current errata are also available at this site. Instructors may obtain all of the solutions by writing to either of the authors, at jlsnell@dartmouth.edu and cgrinst1@swarthmore.edu. It is our intention to place items related to this book at this site, and we invite our readers to submit their contributions.

FEATURES

Level of rigor and emphasis: Probability is a wonderfully intuitive and applicable field of mathematics. We have tried not to spoil its beauty by presenting too much formal mathematics. Rather, we have tried to develop the key ideas in a somewhat leisurely style, to provide a variety of interesting applications to probability, and to show some of the nonintuitive examples that make probability such a lively subject.

Exercises: There are over 600 exercises in the text providing plenty of opportunity for practicing skills and developing a sound understanding of the ideas. In the exercise sets are routine exercises to be done with and without the use of a computer and more theoretical exercises to improve the understanding of basic concepts. More difficult exercises are indicated by an asterisk. A solution manual for all of the exercises is available to instructors.

Historical remarks: Introductory probability is a subject in which the fundamental ideas are still closely tied to those of the founders of the subject. For this reason, there are numerous historical comments in the text, especially as they deal with the development of discrete probability.

Pedagogical use of computer programs: Probability theory makes predictions about experiments whose outcomes depend upon chance. Consequently, it lends itself beautifully to the use of computers as a mathematical tool to simulate and analyze chance experiments.

In the text the computer is utilized in several ways. First, it provides a laboratory where chance experiments can be simulated and the students can get a feeling for the variety of such experiments. This use of the computer in probability has been already beautifully illustrated by William Feller in the second edition of his famous text *An Introduction to Probability Theory and Its Applications* (New York: Wiley, 1950). In the preface, Feller wrote about his treatment of fluctuation in coin tossing: "The results are so amazing and so at variance with common intuition that even sophisticated colleagues doubted that coins actually misbehave as theory predicts. The record of a simulated experiment is therefore included."

In addition to providing a laboratory for the student, the computer is a powerful aid in understanding basic results of probability theory. For example, the graphical illustration of the approximation of the standardized binomial distributions to the normal curve is a more convincing demonstration of the Central Limit Theorem than many of the formal proofs of this fundamental result.

Finally, the computer allows the student to solve problems that do not lend themselves to closed-form formulas such as waiting times in queues. Indeed, the introduction of the computer changes the way in which we look at many problems in probability. For example, being able to calculate exact binomial probabilities for experiments up to 1000 trials changes the way we view the normal and Poisson approximations.

ACKNOWLEDGMENTS FOR FIRST EDITION

Anyone writing a probability text today owes a great debt to William Feller, who taught us all how to make probability come alive as a subject matter. If you find an example, an application, or an exercise that you really like, it probably had its origin in Feller's classic text, *An Introduction to Probability Theory and Its Applications*.

This book had its start with a course given jointly at Dartmouth College with Professor John Kemeny. I am indebted to Professor Kemeny for convincing me that it is both useful and fun to use the computer in the study of probability. He has continuously and generously shared his ideas on probability and computing with me. No less impressive has been the help of John Finn in making the computing an integral part of the text and in writing the programs so that they not only can be easily used, but they also can be understood and modified by the student to explore further problems. Some of the programs in the text were developed through collaborative efforts with John Kemeny and Thomas Kurtz on a Sloan Foundation project and with John Finn on a Keck Foundation project. I am grateful to both foundations for their support.

I am indebted to many other colleagues, students, and friends for valuable comments and suggestions. A few whose names stand out are: Eric and Jim Baumgartner, Tom Bickel, Bob Beck, Ed Brown, Christine Burnley, Richard Crowell, David Griffeath, John Lamperti, Beverly Nickerson, Reese Prosser, Cathy Smith, and Chris Thron.

The following individuals were kind enough to review various drafts of the manuscript. Their encouragement, criticisms, and suggestions were very helpful.

Ron Barnes	*University of Houston, Downtown College*
Thomas Fischer	*Texas A & M University*
Richard Groeneveld	*Iowa State University*
James Kuelbs	*University of Wisconsin, Madison*
Greg Lawler	*Duke University*
Sidney Resnick	*Colorado State University*
Malcom Sherman	*SUNY Albany*
Olaf Stackelberg	*Kent State University*
Murad Taqqu	*Boston University*
Abraham Wender	*University of North Carolina*

In addition, I would especially like to thank James Kuelbs, Sidney Resnick, and their students for using the manuscript in their courses and sharing their experience and invaluable suggestions with me.

The versatility of Dartmouth's mathematical word processor PREPPY, written by Professor James Baumgartner, has made it much easier to make revisions, but has made the job of typist extraordinaire Marie Slack correspondingly more challenging. Her high standards and willingness always to try the next more difficult task have made it all possible.

Finally, I must thank all the people at Random House who helped during the development and production of this project. First, among these was my editor Wayne Yuhasz, whose continued encouragement and commitment were very helpful during the development of the manuscript. The entire production team provided efficient and professional support: Margaret Pinette, project manager; Michael Weinstein, production manager; and Kate Bradfor of Editing, Design, and Production, Inc.

ACKNOWLEDGMENTS FOR SECOND EDITION

The debt to William Feller has not diminished in the years between the two editions of this book. His book on probability is likely to remain the classic book in this field for many years.

The process of revising the first edition of this book began with some high-level discussions involving the two present co-authors together with Reese Prosser and John Finn. It was during these discussions that, among other things, the first co-author was made aware of the concept of "negative royalties" by Professor Prosser.

We are indebted to many people for their help in this undertaking. First and foremost, we thank Mark Kernighan for his almost 40 pages of single-spaced comments on the first edition. Many of these comments were very thought-provoking; in addition, they provided a student's perspective on the book. Most of the major changes in the second edition have their genesis in these notes.

We would also like to thank Fuxing Hou, who provided extensive help with the typesetting and the figures. Her incessant good humor in the face of many trials, both big ("we need to change the entire book from Lamstex to Latex") and small ("could you please move this subscript down just a bit?"), was truly remarkable.

We would also like to thank Lee Nave, who typed the entire first edition of the book into the computer. Lee corrected most of the typographical errors in the first edition during this process, making our job easier.

Karl Knaub and Jessica Sklar are responsible for the implementations of the computer programs in Mathematica and Maple, and we thank them for their efforts. We also thank Jessica for her work on the solution manual for the exercises, building on the work done by Gang Wang for the first edition.

Tom Shemanske and Dana Williams provided much TeX-nical assistance. Their patience and willingness to help, even to the extent of writing intricate TeX macros, are very much appreciated.

The following people used various versions of the second edition in their probability courses, and provided valuable comments and criticisms.

Marty Arkowitz	*Dartmouth College*
Aimee Johnson	*Swarthmore College*
Bill Peterson	*Middlebury College*
Dan Rockmore	*Dartmouth College*
Shunhui Zhu	*Dartmouth College*

Reese Prosser and John Finn provided much in the way of moral support and camaraderie throughout this project. Certainly, one of the high points of this entire

endeavour was Professor Prosser's telephone call to a casino in Monte Carlo, in an attempt to find out the rules involving the "prison" in roulette.

Peter Doyle motivated us to make this book part of a larger project on the Web, to which others can contribute. He also spent many hours actually carrying out the operation of putting the book on the Web.

Finally, we thank Sergei Gelfand and the American Mathematical Society for their interest in our book, their help in its production, and their willingness to let us put the book on the Web.