How Much Did Manipulation Distort the Libor?*

Job Market Paper

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Abstract

The Libor is a vital benchmark interest rate formed by an unregulated survey of sixteen large banks. These banks own many Libor-referencing contracts and therefore have strong incentives to submit misleading quotes to the survey. In this paper, I measure how much they distorted the Libor between 2005 and 2009. To do so, I specify a model of strategic manipulation which captures the key features of the Libor aggregation mechanism. I use a two-step estimator to back out banks’ unobserved portfolio exposures through their revealed preferences. I find the Libor was accurate prior to the 2007 financial crisis, but was subsequently distorted downward by eight basis points, which cost U.S. municipalities $455 million over the sample period. I propose an alternative aggregation mechanism and find it removes virtually all the systematic bias due to manipulation.

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1 Introduction

Trader: [Would] be nice if you could put 0.90% for 1mth cheers.

Quote Submitter: Sure no prob. I’ll probably get a few phone calls but no worries mate!

Trader: If you may get a few phone calls then put 0.88% then.

Quote Submitter: Don’t worry mate – there’s bigger crooks in the market than us guys!

—Rabobank, internal discussion

The Libor was recently subjected to one of the largest instances of market manipulation in history. Hundreds of trillions of dollars worth of financial contracts were manipulated by large banks, who have since been fined billions of dollars by regulators from four countries.\(^1\) Investigations are ongoing and many class action lawsuits are underway due to the huge volume of contracts adversely affected.\(^2\)

The banks under investigation had simple incentives: they owned contracts whose payouts were functions of the Libor. While recent regulatory investigations have revealed that these portfolio incentives did in fact lead to manipulation, it is not known how this affected the overall Libor rate. It is possible the Libor remained largely unchanged throughout this episode, either due to an inability of banks to consistently execute manipulative intent, or thanks to varying portfolio incentives across banks. On the other hand, manipulation may have caused a systemic distortion, which would undermine the continued value of the Libor as a benchmark. Any persistent distortion may have affected the allocation of funding in this period, as an estimated \$10 trillion of syndicated loans use the Libor as the variable interest rate (Wheatley (2012)).

In this paper, I quantify the degree to which manipulation distorted the Libor between 2005 and 2009. To do so, I estimate a strategic model where the Libor is formed each day in a noncooperative game of incomplete information. The strategic interaction between banks is generated by the aggregation mechanism of the Libor survey. Of the sixteen quoted rates, only the middle eight quotes are used in the resulting average which determines the benchmark. The four highest and lowest quotes

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\(^1\) From the Department of Justice’s investigation of Rabobank, see (DOJ (2013))

\(^2\) Wheatley (2012) estimates \$300 trillion worth of contracts directly reference the Libor when determining interest rate payments. The regulatory bodies of the USA, UK, Switzerland, and the Netherlands have variously fined Barclays, UBS, the Royal Bank of Scotland and Rabobank.

\(^3\) The city of Baltimore, New Britain Firefighter’s and Police Benefit Fund, Atlantic trading USA, and Community Bank&Trust are some of the representative members of the four classes pursuing Libor related damages, according to Perkins Coie (http://www.perkinscoie.com/libor_faqs/). See Dodd (2010) for a description of the losses incurred by municipalities.
are discarded. This means, if a bank was a manipulator, it would need to forecast the quotes of its peers in order to gauge its marginal ability to influence the overall Libor rate. Variation in this marginal ability across banks and trading days allows me to recover each bank’s average portfolio exposure to the Libor and, consequently, what they would have quoted had they had no such exposure.4

I find that the Libor was largely accurate prior to the financial crisis starting in late 2007, but was since distorted downwards by eight basis points. This is substantial given the volume of contracts affected and that manipulators routinely made large gains from single basis point changes.5 I calculate that U.S municipalities, which held $500 billion worth of interest rate swaps in 2010, would have lost $455 million from this eight basis point shift over my sample period.6

The transition between an accurate pre-crisis Libor and a distorted post-crisis Libor was driven by sharp changes in volatility and heterogeneity across banks. Prior to the financial crisis, banks had very similar risk characteristics and typically submitted identical or near-identical quotes to the survey. If the other fifteen banks are all submitting the same, correct rate, what could a potential manipulator achieve by submitting something different? Once the crisis began, however, banks faced newly heterogeneous risks and submitted a broader range of quotes to the survey. This generated a larger interquartile range between the fifth and twelfth highest submissions which gave potential manipulators room in which to work.

With these results in mind, I compare the performance of counterfactual Libor aggregation mechanisms in the presence of active manipulators. This contribution is particularly timely as regulators are currently considering how best to reform the Libor to safeguard it against future manipulation.7 In particular, the Financial Conduct Authority (FCA) is considering increasing the size of the Libor panel, anonymizing quotes for three months, and tying quotes to underlying transactions as much as possible.8

While the FCA also considered changing the current mechanism used to calculate the

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4There are many other benchmark interest rates similar to the Libor, including the Tokyo-based Tibor, the Mumbai-based Mibor, the many Euribor, and others. Since these are also typically calculated using truncated averages, the strategy I employ in this paper could be used to study the manipulation of these benchmarks as well.

5This is a common theme in the regulatory investigations. See for example FCA (2012a): “Barclays’ Derivatives Traders knew on any particular day what their books’ exposure to a one basis point (0.01%) movement in Libor or Euribor was.” A Barclays trader said to the quote submitter, “We have about 80 [billion] fixing for the desk and each [basis point] lower in the fix is a huge help for us.” (Ibid.) See also FCA (2012b) and Snider and Youle (2012) for more examples.

6See Preston (2012) and Dodd (2010) for background on municipal ownership of interest-rate swaps.

7The Libor and other benchmarks are becoming regulated for the first time.

8See Wheatley (2012) for a comprehensive review of the FCA proposals.
Libor from the underlying quotes, they concluded this would not improve the Libor’s accuracy.

I find, on the contrary, that changing the current mechanism for calculating the Libor can make it considerably less vulnerable to manipulation. In particular, changing the Libor to use the median quote removes virtually all of its systematic downwards bias in the sample period I examine. My results differ from those of the FCA analysis because they assume submitted quotes would not change even if the method used to calculate the Libor were changed. This runs contrary to the idea that manipulators take into account the aggregation mechanism when they strategically submit their quotes. Documents revealed by the investigations show manipulators had a very keen awareness of the exact mechanism.9 In my counterfactual analysis, manipulators are perfectly aware of the aggregation mechanism when they submit their quotes.

The counterfactual performance of the median is driven by the difficulty for manipulators to accurately forecast the location of the median on any given day. Even if a manipulator were able to correctly guess the median, they would not be able to skew their quote very far before they were no longer the median submission. In essence, using the median is similar to narrowing the interquartile range. There could also be a feedback effect. If the median mechanism causes most other banks to skew less, manipulators may skew less as they update their median forecasts.

For some values of the model parameters, however, the median can actually perform worse. The is because the distribution of private information plays an important role in my model’s equilibrium. The performance of the median, relative to the interquartile range, depends on this distribution as well as the incentives for banks to manipulate. In particular, under certain conditions, a manipulator may be more or less certain they will be the median quote. In this case, the median mechanism would give them much greater power in determining the final rate, which lead the manipulator to skew more than they would otherwise.10 This ambiguity is why I must bring the model to the data.

I model banks in the Libor panel as playing a noncooperative game of incomplete information. Each bank has a true interbank borrowing cost which depends on publicly observed covariates, as well as an idiosyncratic shock which is private information. Each bank is therefore uncertain of the quotes of the other banks when submitting its own quote. Manipulators are concerned with the quotes of the others because they want

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9See, for example, FCA (2012a)
10Diehl (2013) shows the relative performance of the median-quote Libor is ambiguous in a complete information version of the game introduced in Snider and Youle (2012). This ambiguity persists in my current, incomplete information game.
to forecast the interquartile range within which they can affect the Libor. Banks that aren’t manipulators, however, are not concerned with forecasting the quotes of their peers.

My game is estimated in two steps. First, I nonparametrically estimate each bank’s marginal impact on the expected Libor. This marginal impact is an equilibrium object which depends upon the strategies being played by the other banks. In the second step, I form moments from the model’s first order conditions and use the submitted quotes and the results from the first step to estimate the game’s parameters, which include banks’ incentives to manipulate. From this, I construct a “manipulation free” Libor by calculating what banks would have quoted had they had no such incentives.

It is important to note that portfolio exposure to the Libor was not the only reason banks submitted misleading quotes. The other reason was reputational. Each bank’s quote is publicly revealed after the Libor is computed. Whenever a bank submits a relatively high quote, thereby admitting to a high cost of borrowing funds from its peers, other market participants might infer something is amiss with that bank. In the run-prone environment of the recent financial crisis, it is unsurprising that banks wished to avoid this negative attention. Indeed, regulators have uncovered many documents expressing banks’ desires to avoid being seen as lacking creditworthiness.\footnote{Ibid.}

I do not attempt to meaningfully capture these reputational incentives for banks to submit misleading quotes. Instead, I control for them with a flexible specification of fixed effects. I use bank-quarter effects and use the within-bank, within-quarter variation in marginal impacts upon the Libor to identify my model. Reputational effects are implicitly incorporated into my counterfactual analysis and the manipulation-free quotes I produce. I do not recover the “correct” Libor, only a Libor free of portfolio-driven manipulation.\footnote{These reputational incentives likely would have pushed the Libor downwards even further.} These reputational incentives to misreport will be reduced by the FCA’s new policy of anonymizing individual quotes for three months. This anonymity, however, will exacerbate banks’ portfolio-driven incentives to misreport by reducing the ability of other market participants to examine and monitor the submitted quotes.

Snider and Youle (2012) use a similar model to motivate a test for Libor manipulation. My approach differs from theirs by assuming banks play a game of incomplete information. This relatively minor modeling difference leads to a completely different empirical strategy. Incomplete information creates smoothness in banks’ profit functions and allows the derivation of a system of necessary first order conditions. Estimating these first order conditions lets me measure the size of banks’ long term
average exposures to the Libor. Knowing this size allows me quantify the extent of the Libor’s distortion and examine the accuracy of counterfactual aggregation mechanisms. I am unable, however, to capture manipulation that occurs at a high frequency, which was an important part of the Libor’s recent manipulation, and for which Snider and Youle (2012)’s test is better suited to detect.

This paper is related to the recent literature on estimating games that occur in financial markets. Cassola et al. (2013) measures banks’ demand for funding through a structural auction model of the EONIA funds service. They use observed bids and the structure of the auction to recover banks’ valuations. In my model, I use observed quotes and the structure of the Libor mechanism to recover banks’ portfolio exposures. Guerre et al. (2009) use an exclusion restriction in an auction setting to separately identify bidders’ risk aversion from their distribution of valuations. I also employ an exclusion restriction to separately identify manipulators’ portfolio exposures from their unobserved interbank shocks.

The rest of the paper is structured as follows. Section 2 describes the history of the Libor and the recent manipulation scandal. Section 3 describes my data. Section 4 introduces the strategic model of manipulation. Section 5 describes my estimation procedure and results. Section 6 discusses my results. Section 7 introduces an algorithm to compute the Bayes-Nash equilibrium and compares counterfactual Libor mechanisms. Section 8 concludes.

2 History of the Libor

The London Interbank Offered Rate (Libor) is a benchmark interest rate that has grown to become a central institution in financial markets. An estimated $300 trillion of contracts use the Libor to determine their obligated interest payments. Many syndicated loans, adjustable rate mortgages, student loans, and other financial products routinely depend upon the Libor in this fashion. An even larger array of interest rate derivatives, such as forwards, futures, and swaps, depend directly upon the Libor.

The Libor is calculated by a daily survey managed by the British Banker’s Association (BBA). A fixed panel of sixteen large banks are asked,

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13 The Libor is quoted in many different maturities and currencies. In this paper I focus on the three month dollar Libor which, along with the six month Libor, is most commonly used by financial contracts.

14 In the period I examine, the BBA was in charge of overseeing the Libor and monitoring submission accuracy. The BBA is an industry group composed largely of the banks sitting on the Libor panel. The Financial Services Authority (FSA) has since required the BBA transfer the governance of the Libor to another private institution.
“At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to eleven a.m. London time?”

The four lowest and four highest rate quotes are then discarded, with the average of the remaining middle eight quotes forming the day’s Libor. I assume in what follows that, for each bank there is a more or less correct answer to this question, and that they are aware of it. These banks are in constant communication with brokers and one another, making this an easy question for them to resolve.

The modern Libor was established in the 1980s to provide a standardized interest rate benchmark. The growth of the Libor’s general use was facilitated by the growth of trading in interest rate derivatives. The Libor was adopted early on in the markets for interest rate derivatives and experienced lock-in due to network effects. It is easier to write and trade contracts using a well established as opposed to obscure benchmarks. Accordingly, the Libor had the distinct advantage of being introduced in the 1980s, well before current rival benchmarks defined using more recent money market interest rates, such as the going rate for repurchase agreements.

While an enormous number of contracts reference the Libor, the London interbank market, for which the Libor is supposed to represent the average price of funds, is small. This market was large in the 1980s, but has been largely been replaced by overnight and collateralized forms of lending. Repurchase agreements, commercial paper, overnight federal funds, and other close money market substitutes are now the primary vehicles through which banks and other financial institutions exchange funds. This has made it increasingly difficult to verify submitted Libor quotes to actual interbank trades as such trades are increasingly uncommon.

Despite the recent manipulation, the Libor is still widely used, and will remain largely unchanged in the medium term. The FCA ultimately decided to reform the Libor rather than replace it with another measure. Such a switch could trigger a wave of lawsuits and costly renegotiation of legacy Libor-referencing contracts, and is opposed by the vast majority of Libor stakeholders (Wheatley (2012)). In the long run, however, it is entirely possible that the market will move towards a new benchmark that is less vulnerable to manipulation. In the short run, regulators and market participants are eager to restore the Libor’s credibility as a benchmark interest rate.
3 Data

I collect the daily Libor quotes for each of the panel banks from a Bloomberg Terminal. I focus on the period beginning October 4th, 2005 and ending October 28th, 2009 for a total of 1,009 trading days. This period includes the movement in the Libor from its high level prior to the onset of the financial crisis in the second quarter of 2007, to its low level in the third quarter of 2009. It also contains some of the most volatile periods of the financial crisis as well as vocal concerns over Libor manipulation in the media.

Table 1 shows the raw quote data for the sixteen panel banks for the week beginning on Monday, December 12th, 2007. Several features are worthy of note. First, the Libor is calculated using the middle eight quotes; the four highest and lowest quotes are discarded. Second, banks typically quote close together. Third, banks’ quotes are persistent; banks do not typically quote far from their previous values. The within and between bank quote variance, however, alters dramatically over my sample period, as the market moves from calm to stressful periods.

The interquartile range of quotes varies considerably over my sample as shown in figure 1. Before the crisis, banks submitted quotes very close to one another. Once the crisis began, however, banks began to change their quotes considerably from day to day, and quotes become different from bank to bank. A clear example of the difference in pre- and post-crisis behavior can be seen in table 2. On January 2, 2007, fifteen of the sixteen banks submitted identical quotes and the interquartile range was zero. On October 21, 2008, the sixteen banks submitted eleven unique quotes and the interquartile range was fifteen basis points.

In figure 2, I show the submitted Libor quotes relative to the day’s average, overlaid with the banks’ CDS quotes, a measure of their credit risk, relative to the day’s average. Prior to the crisis, there was very little difference between banks in both the CDS spreads and the quotes they submitted to the survey. Once the crisis began, however, there was substantial across-bank heterogeneity both in their CDS spreads and their attendant quotes.

The Libor is one of many money market rates which govern short-to-medium-term lending between financial institutions. Other money market rates include certificate of deposit rates, repurchase agreement rates, and commercial paper rates. These rates are generally close substitutes and typically co-move with each other and the Libor. Including these variables allows me to control for the secular change in interest rates that occurs in response to monetary policy undertaken by the Federal Reserve. To

\[ \text{This tight clustering of quotes was noted early on in the suspicion of the Libor by Abrantes-Metz et al. (2012).} \]
that end, I use data from the H.15 interest rate series on market rates for commercial paper. I also collect daily data on the federal funds effective rate, which is the overnight analogue of the interbank market.\textsuperscript{16}

Table 3 shows the frequencies that each bank is either below, inside, or above the day’s interquartile mean of quotes. There are systematic differences between banks, but no bank is either always within or always outside the middle eight. Some banks, however, spend far more time on one side of the quartile than the other. For example, 52.8\% of Norinchukin’s quotes were above the interquartile range while only 0.7\% were below. This is important for the identification of my model, which is discussed further in section five.

4 Model of Strategic Manipulation

I model the Libor panel as a noncooperative game of incomplete information played by the banks in the survey. Banks on the panel consist of a true interbank borrowing cost, which depends on publically observed covariates, and a private idiosyncratic shock. Banks submit quotes simultaneously and are uncertain of the other banks’ quotes due to the others’ private information. Manipulators, therefore, seek to forecast the interquartile range within which they can impact the Libor.

Banks are not necessarily cooperative nor antagonistic. Their attitude towards their peers depends upon their private information and their incentives to manipulate the Libor. In particular, I allow banks to prefer a high Libor, a low Libor, or to be indifferent. Given the flexibility of interest rate derivatives and the secrecy of banks’ net exposures, it is \textit{a priori} as likely that banks would “short” the Libor as the opposite.

4.1 Setup

Let \( i = 1, \ldots, 16 \) index the sixteen banks on the Libor panel. Let \( t = 1, \ldots, T \) index the trading days in my sample. A bank’s observed covariates are denoted \( x_{it} \), with \( x_{it} = (x_{1t}, x_{2t}, \ldots, x_{16t}) \). A bank’s submitted quote is \( q_{it} \), with \( q_t = (q_{1t}, \ldots, q_{16t}) \). The day’s Libor is the interquartile mean of the quotes, given by:

\[
L(q_t) = \frac{1}{8} \sum_{i=1}^{16} \mathbb{1}_{q_{it} \in IQR(q_t)} q_{it}
\]

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\textsuperscript{16}There are differences in the banking holidays between London and New York. Consequently, the federal funds effective rate is not always computed every day the Libor is computed, and vice versa. This generates missing values in my data. Whenever a value is unobserved in this fashion, I impute the price to be the last quoted price.
where $IQR(q_t)$ is the interquartile range of the submitted quotes $q_t$.\(^{17}\)

Banks play a static game of incomplete information. They each receive a private, independent, and idiosyncratic cost shock $\epsilon_{it}$. These covariates include general market information, in the form of prices in related substitute money markets, and include bank specific information, in the form of credit risk measured by the price of default insurance.\(^{18}\) The residual $\epsilon_{it}$ is the unobserved component of interbank borrowing costs,

$$
\epsilon_{it} = x_{it}\beta_i + \epsilon_{it}
$$

Each bank’s unobserved borrowing cost $\epsilon_{it}$ is distributed according to $F_{\epsilon_{it}|x_t}$. While I assume that the cost shock is mean independent of the unobservables, i.e. that $E[\epsilon_{it}|x_t] = 0$, the distribution of the cost shock can depend upon the observables $x_t$ in its higher moments. This allows costs to be heteroskedastic, which is important given the time-varying volatility evident in my data. The interbank borrowing costs themselves remain correlated through their dependence upon the correlated observable variables ($x_{11t}, x_{21t}, ..., x_{16t}$). Since banks are entangled in the same system of supply and demand of interbank loans and typically share similar risk characteristics, correlated costs are likely.

Banks have strategic incentives to manipulate the Libor stemming from their portfolios. They also don’t want to be seen as manipulating it. I model their payoff function as being additive in these two incentives. Bank $i$’s realized profits, given its characteristics and the quotes of the others, is given by:

$$
\pi_i(q_{it}, q_{-it}, x_{it}, \epsilon_{it}) = v_i(x_{it})L(q_{it}, q_{-it}) - \delta(q_{it} - x_{it}\beta_i - \epsilon_{it})^2 + \psi_i q_{it}
$$

The term $v_i(x_{it})$ represents bank $i$’s portfolio exposure to the Libor, which can depend upon its characteristics.\(^{19}\) The scalar $\delta$ represents the bank’s concern over the consequences of misreporting their true interbank borrowing costs. This is because banks are monitored by regulators, other banks, and market participants. They also face the possibility of being probed by regulators who then levy steep fines upon them for manipulation. I do not attempt to model this underlying monitoring. Instead I

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\(^{17}\)Note that while Libor is increasing and Lipschitz continuous, it is neither continuously differentiable, supermodular, nor concave. Thus the first order conditions are not sufficient for optimality without further assumptions.

\(^{18}\)I use daily one year senior CDS spreads at the bank level as the bank-specific credit risk measure.

\(^{19}\)I can’t recover each bank’s exposure to the Libor at a daily frequency. In principle, there is nothing stopping a trader from setting up a position one day and then rewinding it and setting up the opposite position the next. The test by Snider and Youle (2012) is better suited to detect manipulation driven by such behavior. Since I am trying to recover longer term incentives to skew the Libor, I aim to capture each bank’s average exposure to the Libor over my sample period.
use a simple quadratic functional form to approximate banks’ desire to quote near their costs, a desire that was routinely expressed in documents uncovered by investigators.

The term $\psi_i$ captures bank’s reputational concerns over their quotes. This stems from the fact that banks’ quotes are publicly revealed later in the day. If a bank were to submit a relatively high rate quote it might signal to other market participants that it is not as creditworthy as its peers. This reputational concern might differ across banks and across market conditions. Thus I allow $\psi_i$ to change from quarter to quarter, treating it as a bank-quarter fixed effect in estimation.

The long-run average portfolio exposures $v = (v_1, v_2, ..., v_{16})$ are assumed to be common knowledge. The observed covariates $x_t$ and the parameters $(\delta, \beta_1, ..., \beta_{16}, \psi_1, ..., \psi_{16})$ are also common knowledge. Finally, the distributions of the unobserved shocks $F$ are also common knowledge. Each bank knows these variables and parameters as well as their own private cost shock $\epsilon_{it}$. The only variables bank $i$ is uncertain of when submitting its quotes are the idiosyncratic cost shocks $\epsilon_{-it}$ of the other fifteen banks.

I assume that banks play pure strategies. Bank $i$’s strategy at $t$, denoted $\phi_{it}$, maps its information $(x_t, \epsilon_{it})$ into its quote $q_{it}$ such that $\phi_{it}(x_t, \epsilon_{it}) = q_{it}$. A bank’s expected profit, when the other banks are playing $\phi_{-it}$, is given by

$$\Pi_i(q_{it}, x_t, \epsilon_{it}; \phi_{-it}) = \int_{\epsilon_{-it}} \pi_i(q_{it}, \phi_{-it}(x_t, \epsilon_{-it}), x_{it}, \epsilon_{it})dF_{\epsilon_{-it}|x_t}$$

Banks are assumed to be playing a pure strategy Bayesian-Nash equilibrium.

**Definition:** A (pure strategy) **Bayesian-Nash Equilibrium** is a vector of strategies $\phi_i$ such that, for every bank $i$ and information set $(x_t, \epsilon_{it})$,

$$\phi_{it}(x_t, \epsilon_{it}) \in \arg\max_q \Pi_i(q, x_t, \epsilon_{it}; \phi_{-it})$$

For what follows, it is convenient to define an expected Libor (given $\phi_{-it}$):

$$L_i(q_{it}, x_t; \phi_{-it}) \equiv \int_{\epsilon_{-it}} L(q_{it}, \phi_{-it}(x_t, \epsilon_{-it}), x_{it}, \epsilon_{it})dF_{\epsilon_{-it}|x_t}$$

Banks are only concerned with forecasting the quotes of their peers insofar as they alter the expected Libor. This allows me to rewrite the expected profit equation (3) as,

$$\Pi_i(q_{it}, x_t, \epsilon_{it}; \phi_{-it}) = v_i(x_{it})L_i(q_{it}, x_t; \phi_{-it}) - \delta(q_{it} - x_{it}\beta_i - \epsilon_{it})^2 + \psi_i q_{it}$$

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\(^{20}\)The Single Crossing Condition as defined in Athey (2001) is satisfied and therefore a monotone pure strategy equilibrium exists.
Taking the first order condition and rearranging,
\[ q_{it} = \frac{v_i(x_{it})}{2\delta} \frac{\partial}{\partial q_{it}} L_i(q_{it}, x_t; \phi_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta + \epsilon_{it} \] (6)

This equation has an intuitive interpretation. Banks quote their costs with a skew term similar to bidders in an auction model.\(^{21}\) The degree to which they skew is proportional to the ratio of their manipulation \((v_i(x_{it}))\) and truth telling \((\delta)\) incentives, as well as their marginal impact on the expected Libor. They also skew proportional to their reputational incentives \((\psi_i)\).

### 4.2 Multiplicity

My model admits multiple equilibria.\(^{22}\) This stems from the kinks in the Libor function \(L\) which allows banks to coordinate on the aggressiveness of their skewing. Consider a case where banks have identical incentives. If the other banks skew aggressively in bank A’s favored direction, bank A also has a wider latitude to skew and will do so. If they don’t skew aggressively, bank A is more likely to be excluded from the middle eight by skewing aggressively and will skew less. If the cost distribution has finite support, one can prove the existence of multiple equilibria constructively.

These equilibria, however, are bounded within a set range. This is because the derivative of the expected Libor is always bounded between zero and one eighth, irrespective of the strategies being played by the other banks. This then implies the best responses are bounded as a consequence of the first order condition, as shown in equation 6. Intuitively, these bounds come from two facts. First, no manipulator would ever skew the Libor in a direction opposite of their incentives. Second, no manipulator would ever skew the Libor more than it would if only its quote were used to compute the Libor. I use these bounds when confronting this multiplicity in my counterfactual analysis.

This multiplicity, however, poses no problem for my ability to estimate the model, so long as the equilibrium selection mechanism depends only on the observables. This is because an analogue of the first order condition, which averages across the equilibria and is in terms of objects observable to the econometrician (up to the parameters), is a logical implication of the model. I pursue this idea in the remainder of this section.

Suppose for a given \(x_t\) there is a finite number \(e = 1, ..., E\) of equilibria. The

\(^{21}\) For examples, see Krishna (2009)

\(^{22}\) In the complete information version of my model, studied in Snider and Youle (2012) and Diehl (2013), there is also a considerable multiplicity of equilibria.
econometrician does not know which equilibrium \( e \) is being played on any given day. Let \( \Lambda^e(x_t) \) be the probability equilibrium \( e \) occurs according to some underlying and unknown selection mechanism. Let \( \phi^e_{-it} \) be the strategy profile corresponding to equilibrium \( e \). The econometrician does not observe \( L_{it}(q_{it}, x_t; \phi^e_{-it}) \) directly, but only the following weighted average across equilibria,

\[
L_{it}(q_{it}, x_t) \equiv \sum_{e=1}^{E} \Lambda^e(x_t) L_{it}(q_{it}, x_t; \phi^e_{-it})
\]  

(7)

Similarly, the econometrician does not observe \( \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t; \phi^e_{-it}) \) directly, but only the following weighted average across the equilibria,

\[
\frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t) = \frac{\partial}{\partial q_{it}} \sum_{e=1}^{E} \Lambda^e(x_t) L_{it}(q_{it}, x_t; \phi^e_{-it}) = \sum_{e=1}^{E} \Lambda^e(x_t) \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t; \phi^e_{-it})
\]  

(8)

Finally, the econometrician does not observe quotes conditional on an equilibrium \( e \) but only the unconditional quotes,

\[
E[q_{it}|x_t] = \sum_{e=1}^{E} \Lambda^e(x_t) \int_{\epsilon_{it}} \phi^e_{it}(x_t, \epsilon_{it}) dF_{\epsilon_{it}}
\]  

(9)

However, because both the first order conditions and differentiation are linear, a version of the first order condition is implied by the model,

\[
E[q_{it}|x_t] = \sum_{e=1}^{E} \Lambda^e(x_t) \int_{\epsilon_{it}} \phi^e_{it}(x_t, \epsilon_{it}) dF_{\epsilon_{it}}
\]  

(10)

\[
= \sum_{e=1}^{E} \Lambda^e(x_t) \int_{\epsilon_{it}} \left( \frac{v_i(x_t)}{2\delta} \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t; \phi^e_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta_i + \epsilon_{it} \right) dF(\epsilon_{it})
\]  

(11)

\[
= \sum_{e=1}^{E} \Lambda^e(x_t) \left( \frac{v_i(x_t)}{2\delta} \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t; \phi^e_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta_i \right)
\]  

(12)

\[
= \frac{v_i(x_t)}{2\delta} \sum_{e=1}^{E} \Lambda^e(x_t) \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t; \phi^e_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta_i
\]  

(13)

\[
= \frac{v_i(x_t)}{2\delta} \frac{\partial}{\partial q_{it}} L_{i}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it}\beta_i
\]  

(14)

Where the equation (12) used the first order condition equation (6), and equation (14) used equation (8). The model then implies the following relationship,

\[
E[q_{it}|x_t] = \frac{v_i(x_t)}{2\delta} \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it}\beta_i
\]

(15)
Which is in terms of objects that are observable (up to the parameters) by the econometrician, and only requires the assumption that the selection mechanism \( \Lambda \) depends only on the observables \( x_t \). This assumption is also used in De Paula and Tang (2012) who develop a formal test for multiple equilibria in entry games. It also means that I need not assume the same equilibrium is being played across different trading days. Relaxing this assumption, which is common in the structural estimation of games, is possible due to the linearity of my first order conditions and the fact that they hold in every equilibrium.

5 Estimation

The challenge posed by the possibility of multiple equilibria drives my empirical strategy. A common technique for estimating non-cooperative games involves the explicit computation of equilibria while searching over the parameter space to minimize an empirical criterion function.\(^{23}\) Applying that technique in this setting would require me to make strong assumptions about the form of the equilibrium selection function \( \Lambda \), as well as the functional form of the distribution of the unobservables \( F_{\epsilon_{it}|x_t} \).\(^{24}\) Searching over the parameter space is computationally expensive and would require either simulation methods or the construction of a likelihood function – both complicated by the presence of multiple equilibria.

To avoid these issues, I implement a two-step estimator.\(^{25}\) In the first step, I nonparametrically estimate the equilibrium derivative of the expected Libor \( \mathcal{L} \) with respect to a bank’s quote. I use local linear kernel regression, using the Libor fixes, quotes, and observables from the data. In the second step, I estimate the banks’ necessary first order conditions. Specifically, I estimate equation 6 using this estimated derivative as a covariate. This allows me to recover each bank’s long run average exposure to the Libor \( v_i(x_{it}) \) as a parameter and its interbank shock \( \epsilon_{it} \) as a residual.\(^{26}\)

In the second stage, I use exclusion restrictions to generate additional moment conditions which are necessary to pin down the parameters of my model. This indirect,

\(^{23}\)Many papers use variations of this approach. For an example, see Seim (2006) who circumvents the multiplicity problem by showing the existence of a unique equilibrium under certain conditions.

\(^{24}\)An alternative is to use a bounds approach as in Ciliberto and Tamer (2009).

\(^{25}\)My estimator is loosely inspired by Bajari et al. (2007), Pakes et al. (2007), Aguirregabiria and Mira (2007), and Pesendorfer and Schmidt-Dengler (2008). Unlike these approaches, I do not need to assume the same equilibrium is being played across the data – only that the equilibrium selection mechanism is reasonably well behaved.

\(^{26}\)As the first step is nonparametric and our second parametric, this procedure is a semiparametric regression as discussed in Robinson (1988)
revealed preference approach is necessary as banks carefully guard the secrecy of their portfolio exposures to the Libor. However, as a robustness check, I compare the results of my estimates with the information on bank portfolio positions revealed in the Call Reports.

5.1 Nonparametric First Step

The first step in my estimation procedure is to nonparametrically estimate the derivative of the expected Libor \( \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t) \). I do this by using the definition of a derivative and nonparametrically estimating the expected Libor itself at two nearby points.

The procedure begins by considering how the realized Libor \( L_t \) would have changed had a given bank \( i \) submitted a slightly higher quote. I compute,

\[
L_{it}^\Delta \equiv L(q_{it} + \Delta, q_{-it})
\]

for each bank \( i \) and day \( t \). This measures the resulting Libor had bank \( i \) quoted \( q_{it} + \Delta \) instead of \( q_{it} \). Note that I could use these calculations to compute the realized derivative for each bank as \( (L_{it}^\Delta - L_t)/\Delta \). For a \( \Delta \) small enough, this realized derivative will always be zero or one eighth. Either \( q_{it} + \Delta \) is included in the interquartile range or it isn’t. This is what would be relevant if banks are playing a game of complete information.

However, banks are not perfectly aware of where the pivotal cutoff points of the interquartile range will be located. This is because they are uncertain about the shocks \( \epsilon_{-it} \), and hence the resulting quotes, of their peers. This creates a smooth range over where their marginal impact on the expected Libor can lie. This marginal impact remains bounded between zero and one eighth, but can also lie somewhere in between.

Recall that,

\[
L_{it}(q_{it}, x_t) = \sum_{e=1}^{E} \Lambda^e(x_t) \int_{\epsilon_{-it}} L(q_{it}, \phi_{-it}^e(x_t, \epsilon_{-it}))dF_{\epsilon_{-it}|x_t} = \mathbb{E}[L_t|q_{it}, x_{it}] \quad (17)
\]

where \( L_t \) is the realization of the day’s Libor, which depends upon the underlying equilibrium, quotes, and cost shocks. Recovering \( \mathbb{E}[L_t|q_{it}, x_t] \), however, involves no more than estimating a conditional expectation, as I observe \( (L_t, q_{it}, x_t) \) directly in the data. Similarly, by computing \( L_{it}^\Delta \) as above, using the mechanical rule for the Libor mechanism and the observed quotes, I have the following,
\[ \mathcal{L}_{it}(q_{it} + \Delta, x_t) = \sum_{\epsilon = 1}^{E} \Lambda^\epsilon(x_t) \int_{\epsilon - it} L(q_{it} + \Delta, \phi_{-it}(x_t, \epsilon - it)) dF_{-it|x_t} = \mathbb{E}[L^\Delta_{it}|q_{it}, x_{it}] \] (18)

and again, as \((L^\Delta_{it}, q_{it}, x_{it})\) are all observed, recovering \(\mathbb{E}[L^\Delta_{it}|q_{it}, x_{it}]\) involves nothing more than estimating a conditional expectation.

I estimate both \(\mathbb{E}[L_t|q_{it}, x_{it}]\) and \(\mathbb{E}[L^\Delta_{it}|q_{it}, x_{it}]\) nonparametrically, using local linear regression.\(^{27}\) This is a kernel regression – essentially a locally weighted average of the Libors, where the observables \((q_{it}, x_{it})\) are used to define what “local” is.

Given estimates of \(\hat{\mathbb{E}}[L_t|q_{it}, x_{it}]\) and \(\hat{\mathbb{E}}[L^\Delta_{it}|q_{it}, x_{it}]\), I estimate the derivative of the expected Libor. First I use the definition of a derivative,

\[ \frac{\partial}{\partial q_{it}} \mathcal{L}_{it}(q_{it}, x_t) = \lim_{\Delta \to 0} \frac{\mathcal{L}_{it}(q_{it} + \Delta, x_t) - \mathcal{L}_{it}(q_{it}, x_t)}{\Delta} \]

The empirical analogue I use for estimation is then,

\[ \frac{\partial}{\partial q_{it}} \mathcal{L}_{it}(q_{it}, x_t) = \frac{\hat{\mathcal{L}}_{it}(q_{it} + \Delta, x_t) - \hat{\mathcal{L}}_{it}(q_{it}, x_t)}{\Delta} = \hat{\mathbb{E}}[L^\Delta_{it}|q_{it}, x_{it}] - \hat{\mathbb{E}}[L_t|q_{it}, x_{it}] \]

I estimate this partial derivative for a fixed \(\Delta\), stipulating that \(\Delta \to 0\) as \(T \to \infty\) but that \(\Delta T \to \infty\) for consistency. In practice, I set the value for \(\Delta\) to be a quarter of a basis point.\(^{28}\)

5.2 Parametric Second Step

In the second step I estimate the first order conditions (15) using the Generalized Method of Moments (GMM), plugging in the derivative recovered in the first step.\(^{16}\)

Plugging in our first stage estimates into equation (15) produces the following estimating equation:

\[ \mathbb{E}[q_{it} - \frac{\psi_i(x_t)}{2\delta} \frac{\partial}{\partial q_{it}} \mathcal{L}_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it} \beta_i | x_t] = 0 \] (19)

where I have implicitly assumed that the measurement error in the first step is mean.

\(^{27}\)For background on local linear regression and its properties, see Fan and Gijbels (1996).

\(^{28}\)Banks submit quotes in smaller increments very rarely. While this may suggest a discrete choice modeling approach, a discretization that would surround all the quotes banks actually submit around the day’s Libor, up to quarter basis point increments, would involve over a hundred bins. Given there are sixteen banks, this means an action space of at least one hundred to the sixteenth power, which generates matrices far beyond my ability to invert.
independent of $x_t$. It is possible that first step measurement error leads to bias in the second step, a regular concern when using two-step estimators to structurally estimate games. From the structure of equation (19) we might be content that the model is identified. However, the situation is more pessimistic for a similar environment examined by Guerre et al. (2009), who examine a first price auction where bidders have an unknown (to the econometrician) distribution of valuations for a good, as well as an unknown (again, to the econometrician) risk aversion parameter. This is largely analogous to my environment, where I have an unknown distribution of interbank borrowing shocks and an unknown portfolio exposure parameter. They find that, without further assumptions, their model is nonidentified.

This makes it likely that the identification coming from equation (19) is primarily due to my parametric assumptions when specifying bank profits. A more general misreporting cost function, rather than my current quadratic form, will likely make the model nonidentified like that of Guerre et al. (2009). As I view my quadratic form as a first approximation to some potentially more complex, unknown, form, I pursue the solution of Guerre et al. (2009), which is to use an exclusion restriction. In particular, they assume that the number of bidders in the auction is unrelated to the distribution of bidder valuations. Seeing how observed quotes vary as the number of competitors varies is what enables them to separately identify the risk aversion parameters from the distribution of valuations.

I invoke a related exclusion restriction. However, as I always have the same number of banks in my sample, I use a different set of variables which are unrelated to banks current, unobserved borrowing shocks. I use lagged quotes of the other banks as such a variable, because they are useful for forecasting their current quotes. The logic is that, from a manipulator’s point of view, it is not directly concerned with the lagged quotes of its peers, but they do help it predict what the others will quote today and, hence, what its marginal impact on the Libor will be. Specifically, I use the lagged interquartile range of the quotes as an excluded variable,

$$E[\epsilon_{it} | IQR_{t-1}] = 0$$ (20)

Which I use to form the following unconditional moment conditions,

$$E[x_{it} (q_{it} - \frac{v_i(x_t)}{2\delta} \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it} \beta_i)] = 0$$

$$E[IQR_{t-1} (q_{it} - \frac{v_i(x_t)}{2\delta} \frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it} \beta_i)] = 0$$

17
which I estimate with GMM for each bank where $\psi_i$ is treated as a bank-quarter fixed effect.

As my specification of bank profits is homogeneous of degree one in $(\nu_i(x_i), \delta, \psi_i)$, it is only identified up to scale. This is typical when studying revealed preferences, where it is standard to normalize a utility or cost parameter, or the utility from receiving an outside option. While I can’t interpret my parameters in terms of dollars, I can use them predict bank behavior, which is what I need to measure the performance of counterfactual Libor mechanisms.

To see how I identify bank’s incentives to manipulate, consider a different world where the Libor was not calculated by an interquartile mean, but instead was a simple average. In that case, $\frac{\partial}{\partial q_{it}} L_{it}(q_{it}, x_t)$ would always be one sixteenth for every $i$, $t$, and $(q_{it}, x_t)$. In this case it would be impossible to recover the average portfolio exposure of a bank to the Libor. The derivative would be collinear with the reputational fixed effect $\psi_i$. It is only by seeing how a bank’s incentives to skew the Libor evolves over time, as they move in and out of the middle eight of the quotes of the other banks, we see variation in their marginal ability to influence the Libor.

The primary assumptions I use to identify my model are (i) the specification of bank profits and (ii) the mean independence of the unobservable cost shock from the observables and (iii) the mean independence of the unobservable cost shock from the lagged interquartile range. Mean independence is weaker than assuming full independence, which would not be appropriate setting given the considerable time-varying heteroskedasticity evident in the data. In this sense, my approach shares a similar feature with the econometric models of time-varying volatility, such as multivariate GARCH models, which are commonly used to model interest rates and other financial variables.\(^{29}\)

## 6 Results

I present the results of the first step of my estimation procedure in a series of figures. Figure 3 shows the marginal impact of all possible quotes for the Bank of Tokyo Mitsubishi on two separate trading days. The blue line corresponds to the realized derivative $(L_{it}^\Delta - L_t)/\Delta$, which would obtain if the Bank of Tokyo knew exactly what the day’s interquartile range would be.\(^{30}\) The red line is the estimated derivative

\(^{29}\)See Bauwens et al. (2006) for a survey of these influential time series models.

\(^{30}\)This corresponds to the assumption in Snider and Youle (2012), and generates a “bunching” prediction they use to develop a test for strategic manipulation.
\[ \frac{\partial}{\partial q_{it}} \mathcal{L} \left( q_{it}, x_{it} \right) \]

resulting from my specification of incomplete information. Uncertainty about the private information, and hence the quotes, of a banks’ peers smooths out the distribution of its marginal influence on the expected Libor through its uncertainty over the exact location of the pivotal points.

In figures 4, 5 and 6 I present the first stage estimates for three banks along with the relative locations of their quotes compared to the interquartile range of the quotes submitted by their peers. These three figures show the marginal impacts for the quotes banks chose to submit. I also show the marginal impacts for all of the possible quotes a bank could have chosen to submit through time in figure 8. Here we can see the tight and short distribution of marginal impacts before the financial crisis, which then grows in breadth and height as the crisis progresses, and then spikes around the default of Lehman brothers. The marginal impacts shown in figure 3 are time slices of this contour map for two different days.

The second stage parameter estimates are shown in table 4. Most of the banks wish to push the Libor downwards, but not all. Thus the Libor is the result of a “tug of war” in which some banks wish to skew it upwards, and others wish to skew it downwards. Given banks’ radically different locations, primary currencies of operation, and business models, this may be expected. The winning side ultimately depends upon the distribution of bank borrowing costs.

I am unable to estimate the first order conditions for the Bank of Tokyo, Societe Generale, HBOS, Norinchukin and WestLB because they do not have CDS spreads for enough of the trading days over the period I consider. In my counterfactual analysis I assume they are truth tellers. Only Societe Generale has been indicted for manipulating the Libor, and the others are not known to have any current ongoing investigations.

Bank of America, Deutschbank and UBS have the smallest incentives to skew. Barclays, the first bank to admit to manipulation, has the largest incentives, although the estimate is not statistically significant. This may be due to a change in behavior over the sample, as seems to be suggested by its quoting behavior in 5 and the regulatory investigations. These investigations describe Barclays as moving from portfolio-driven manipulation to a strategy designed to avoid damaging their reputation. See Snider and Youle (2012) for a discussion of Barclays.

### 6.1 Recovering the Manipulation-Free Libor

I have estimated banks’ incentives to misreport \( \beta \equiv v_i(x_i)/(2\delta) \). Now I construct a “manipulation-free” Libor which would have occurred had their been no incentives to
manipulate the Libor by any bank. This is not the “correct” Libor because I have not meaningfully recovered banks’ reputational reasons to misreport. Nevertheless, this remains an interesting object to study. Manipulation driven misreporting is probably the more likely source of future Libor distortion, as quotes will soon be anonymous – a policy change that will mitigate banks’ concerns about their reputations.

These counterfactual manipulation-free quotes $\hat{q}_t$ are related to the observed quotes as follows:

$$\hat{q}_{it} \equiv q_{it} - \beta \frac{\partial}{\partial q_{it}} L_i(q_{it}, x_t)$$

(21)

The attendant manipulation-free Libor is defined from these quotes:

$$\hat{L}_t \equiv L(\hat{q}_t)$$

(22)

The difference between this recovered Libor and the actual Libor, $\hat{L} - L$, as well as a 90% confidence interval using the standard errors I compute in my estimate of the incentives $\beta$, is shown in figure 8. The Libor was largely accurate prior to the financial crisis. Afterwards, however, it diverged downwards as the recovered Libor becomes greater than the actual Libor. Eventually this leads to a nearly eight basis point difference at the end of my sample. This is well after the main storm of the crisis had subsided and the Libor was at a very low level.

The difference between these Libors over time is driven by the evolution of marginal impacts across banks. Generally speaking, most banks want to push it down and marginal impacts rise during the crisis. There was more ability for manipulators to alter the rate as the interquartile range of quotes increased during the financial crisis. This initial increased dispersion was likely driven by the underlying dispersion in interbank borrowing costs and greater market volatility in this period. In more peaceful times, however, the truncation mechanism of the Libor works well and greatly diminishes banks abilities to misreport when their costs are tightly aligned.

7 Alternative Libor Aggregation

The FCA has considered alternative ways to form the Libor from the underlying quotes from the panel banks. In particular, they examined alternative benchmarks calculated by the average and the median of submitted quotes, as well as a random quote selection. Their analysis, however, was limited. They took the submitted quotes as given, and examined how the Libor would have looked if it were computed from these

\footnote{Wheatley (2012)}
quotes in ways other than the interquartile mean. This approach ignores any behavioral responses due to an alteration of the underlying mechanism.

In this section, I analyze these alternatives by recomputing the Bayesian-Nash equilibria under alternative mechanisms. I describe the process for calculating the Bayesian-Nash equilibria of my model in detail.

### 7.1 Calculating Bayes-Nash Equilibria

As discussed above, the details of my model and environment lead to a number of computational barriers. The first challenge is the high dimensionality of my game. The second is the very real possibility of multiple equilibria. My solution involves an algorithm similar to an EM-algorithm in maximum likelihood estimation, avoids discretizing the state space, and can compute the bounds on the resulting Libors across the set of possible equilibria.

I avoid the curse of dimensionality by exploiting the limited form of strategic interaction in my model. A bank is only concerned about the behavior of the other banks insofar as they alter its ability to manipulate the overall Libor. More specifically, the marginal impact on the expected Libor is a sufficient statistic for the action profile of the other banks when considering a given bank’s payoffs. This naturally leads to an algorithm which simulates this marginal impact. My algorithm alternates between iterating on bank’s first order conditions given their beliefs and then updating their beliefs about their marginal impact given the current strategies played by the other banks stored in memory. Rather than having to invert large matrices, my computational procedure allows me to iterate on the first order conditions and then check that they are a global optimum. Such a check is necessary because the Libor function is not concave and the first order conditions are not sufficient for optimality.

The second challenge in performing my counterfactual analysis is the very real possibility of multiple equilibria as discussed previously. The set of equilibria, however, is meaningfully bounded. This is because the best response of every bank is bounded,

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32 I don’t independently consider the random and the mean aggregation mechanisms because my banks are risk neutral. Under random selection, where each bank has an equal probability of being selected, leads to identical incentives for banks to submitting misleading quotes in my model, and would generate identical behavior. The random aggregation mechanism, however, would produce a more volatile Libor.

33 This approach to computing the equilibrium is somewhat similar to computational methods developed for different environments. Krusell and Smith (1998) develops an algorithm to compute macroeconomic models in which agents need to forecast the evolution of the wealth distribution in order to forecast prices and make saving decisions. They find that the first moment of this distribution is a sufficient statistic for households to make nearly optimal decisions. My algorithm to compute the Libor is also inspired by the algorithms of Pakes and McGuire (2001) and Fershtman and Pakes (2012).
and the bounds do not depend on the strategy profiles of the other banks. Intuitively this is because banks face increasing convex costs for misreporting and their marginal impact on the expected Libor is bounded between zero and one eighth. No bank would skew more than they would if there impact on the marginal Libor was always one eighth, and no bank would skew in a direction contrary to these costs. These bounds on the best responses bound the possible equilibria, and are common also to the environment of Snider and Youle (2012) where they are studied further.

It is possible that the multiplicity in my environment is reduced due to the incomplete information version of my game. Bajari et al. (2010) find that an incomplete information version of an entry game has considerably less equilibria as the number of players grow than when information is complete. By the time they reach numbers of players near those in my game, the set of possible equilibria has reduced drastically.

Nevertheless, I aspire to be robust to multiple equilibria in my counterfactual analysis by using the fact that the equilibria are bounded. My approach is to initialize the values in my algorithm in such a way as to find the extremal equilibria. The first minimizes the resulting equilibrium Libor and the latter maximizes it. This then bounds the Libors resulting from all of the other equilibria in my model. I find these extremal equilibria by weighting the banks that want to skew it in that direction a maximal amount, and weighting the banks that wish to skew the opposite direction a minimal amount. This leads the updating procedure to settle on the extremal equilibrium. In the lowest Libor equilibrium banks which “short” the Libor skew as aggressively as is consistent with individual rationality. Banks which are “long” the Libor do the same in the highest Libor equilibrium.

7.2 Iterative Algorithm

I repeatedly iterate upon the necessary first order conditions and use the resulting quotes to update equilibrium beliefs until those beliefs converge. The pseudocode for my algorithm is as follows. For each period t,

1. Initialize the beliefs $G^0$, banks have about their equilibrium marginal impacts on the expected Libor.
2. Begin an inner loop (iterating on $m$):
   
   (a) Simulate $S$ draws of $c^t_i$ using a calibrated cost process and the realized history of costs $\{c_t, \sigma_t\}_{\tau=1}^t$.
   
   (b) Generate the corresponding optimal quotes for each bank, using the necessary first order condition and the current value of $G$:
\[ q^g_{it} = \frac{v_i(x_i)}{2\delta} G_{i}^{m-1} + e^g_{it} \]

(c) Update the value of \( G^m \) using the simulated using fixed weights \( w \in (0,1) \) according to:

\[
G^m_i = wG^{m-1}_i + (1 - w) \sum_{s=1}^{S} L(q^g_{it} + \Delta, q^g_{i-1}) - L(q^g_{it}, q^g_{i-1}) \]

(d) Continue until \( G^m \) converges.

3. Compute \( q_t \) using the converged \( G^m \) and do a grid search to check the quotes are truly optimal as the first order conditions are merely necessary. If they not optimal, I begin the process again for a different initialization of \( G^0 \).

How \( G^0 \) is initialized will typically determine the resulting equilibrium. The choice of weights \( w \) govern the speed and reliability of the convergence of beliefs. I am able to compute lowest-Libor equilibria by initializing those banks incentives to push the Libor down with large \( G^0_i \)'s and those with incentives to push the Libor up with \( G^0_i \)'s of zero. Every bank with \( V_i < 0 \) will skew their quotes downwards as much as possible. The other banks won’t skew at all. This will likely not be consistent with equilibrium and banks will revise their beliefs and consequently skew less. Over time this will converge to the lowest possible Libor value that is consistent with individual rationality. I perform the converse exercise to compute the highest-Libor equilibrium.

### 7.3 Alternative Libor Aggregation Mechanisms

My counterfactual results are presented in table 5. The mean has the largest systematic bias, the median has the least, and the interquartile range is intermediate. The poor performance of the mean is due to the fact that manipulators are always able to skew irrespective of the other banks on the panel. Under the mean, there is no limit to skewing aggressively, as there is no relationship between how far a bank skews from its peers and its marginal impact. With the interquartile range the marginal impact of a bank’s quote on the expected Libor is decreasing the more extreme the quote is, relative to the others. This is especially so for the median. This is likely the reason why the median performs so much better in the Bayes-Nash equilibria I compute.

An additional reason for the excellent accuracy of the median may be from a strategic complementarity. I find that most banks wish to push the Libor downwards. If other banks skew aggressively downwards, you are more willing to skew also, as this increases the marginal impact of your rate when moving downwards. Under a median, all banks skew less because their incentives are lessened, which then has knock on
affects due to this complementarity.

In table 5 I also calculate the yearly losses accruing to U.S. Municipalities under these alternative mechanisms. This is based on the estimate that municipalities held $500 billion notional value of interest rate swaps during this period, in order to hedge the floating rate municipal bonds, whose rates are determined in a competitive market. It is also important to note that, as far as swaps market is concerned, this is a small amount. Banks on the Libor panel routinely have trillions of dollars of notional value interest rate swaps, as shown in their call reports. Nevertheless, these municipalities net exposures’ are clear and they are a prominent class of institutions that suffered from a depressed Libor.

8 Conclusion

The main problem with the Libor is that the interbank market in which it is determined has become increasingly small. If instead the interbank market were thick and competitive, banks would not have the ability to modify interest rates in a transactions-defined Libor, nor would they have wide latitude to use expert judgment in a survey-defined Libor. Regulators could also easily compare banks submitted quotes with actual transactions. Unfortunately, the interbank market has been largely replaced by overnight and collateralized forms of financing, and is unlikely to ever return to its prior, active status.

The Libor, on the other hand, will remain an important benchmark for third party contracts for the medium term. In the long run, however, it is entirely possible the market will substitute away from contracts defined on the Libor towards close money market substitutes, such as rates for repurchase agreements or certificates of deposit. In the short run, many institutions which hold large portfolios of Libor denominated contracts are eager to restore the credibility of the Libor (Wheatley (2012)).

I also suggest the Libor mechanism be modified to use only the median of the submitted quotes. I find this can increase the accuracy of the Libor by over 70% in equilibrium. This is because the largest manipulators are able to manipulate less on average, and by smaller amounts.
Table 1: Submitted Quotes (3M USD Libor; Week of 12/17/2007)

<table>
<thead>
<tr>
<th>Bank</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barclays</td>
<td>5.03</td>
<td>5.03</td>
<td>5.00</td>
<td>4.99</td>
<td>4.95</td>
</tr>
<tr>
<td>HBOS</td>
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<td>4.98</td>
<td>4.91</td>
<td>4.95</td>
<td>4.95</td>
</tr>
<tr>
<td>Deutsche Bank</td>
<td>4.97</td>
<td>4.96</td>
<td>4.93</td>
<td>4.90</td>
<td>4.85</td>
</tr>
<tr>
<td>Norinchukin</td>
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<td>4.92</td>
<td>4.90</td>
<td>4.88</td>
</tr>
<tr>
<td>Bank of Tokyo</td>
<td>4.95</td>
<td>4.94</td>
<td>4.91</td>
<td>4.89</td>
<td>4.87</td>
</tr>
<tr>
<td>WestLB</td>
<td>4.95</td>
<td>4.94</td>
<td>4.92</td>
<td>4.89</td>
<td>4.87</td>
</tr>
<tr>
<td>RBC</td>
<td>4.95</td>
<td>4.93</td>
<td>4.91</td>
<td>4.88</td>
<td>4.85</td>
</tr>
<tr>
<td>Bank of America</td>
<td>4.94</td>
<td>4.93</td>
<td>4.92</td>
<td>4.88</td>
<td>4.86</td>
</tr>
<tr>
<td>UBS</td>
<td>4.94</td>
<td>4.91</td>
<td>4.90</td>
<td>4.88</td>
<td>4.86</td>
</tr>
<tr>
<td>Citigroup</td>
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<td>4.91</td>
<td>4.90</td>
<td>4.89</td>
<td>4.85</td>
</tr>
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<td>4.93</td>
<td>4.91</td>
<td>4.81</td>
<td>4.87\frac{1}{2}</td>
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<td>4.92</td>
<td>4.92</td>
<td>4.85</td>
<td>4.85</td>
</tr>
<tr>
<td>HSBC</td>
<td>4.92</td>
<td>4.91</td>
<td>4.90</td>
<td>4.87</td>
<td>4.83</td>
</tr>
<tr>
<td>Rabobank</td>
<td>4.92</td>
<td>4.87</td>
<td>4.87</td>
<td>4.80</td>
<td>4.78</td>
</tr>
<tr>
<td>RBOS</td>
<td>4.91</td>
<td>4.90</td>
<td>4.90</td>
<td>4.85</td>
<td>4.80</td>
</tr>
<tr>
<td>J.P. Morgan</td>
<td>4.90</td>
<td>4.87</td>
<td>4.89</td>
<td>4.86</td>
<td>4.84</td>
</tr>
<tr>
<td>3Month USD Libor</td>
<td>4.94\frac{1}{5}</td>
<td>4.92\frac{5}{8}</td>
<td>4.91</td>
<td>4.88\frac{3}{5}</td>
<td>4.85\frac{3}{4}</td>
</tr>
</tbody>
</table>

Snapshot of the raw quote data for the sixteen banks which compose the Libor panel in the period I consider. This is for the three month U.S. Dollar Libor. This rate, along with the six month, is the most important for third party contracts. The Libor is calculated using the average of the middle eight submitted quotes on each day. The bolded quotes are those which determine Monday’s Libor rate.
Table 2: Comparison of Pre- and Post-Crisis Interquartile Ranges

<table>
<thead>
<tr>
<th>Date</th>
<th>1/2/2007</th>
<th></th>
<th>10/21/2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barclays</td>
<td>5.36</td>
<td>Barclays</td>
<td>4.10</td>
</tr>
<tr>
<td>HBOS</td>
<td>5.36</td>
<td>Credit Suisse</td>
<td>4.00</td>
</tr>
<tr>
<td>Deutsche Bank</td>
<td>5.36</td>
<td>Norinchukin</td>
<td>3.95</td>
</tr>
<tr>
<td>Norinchukin</td>
<td>5.36</td>
<td>RBOS</td>
<td>3.95</td>
</tr>
<tr>
<td>Bank of Tokyo</td>
<td>5.36</td>
<td>WestLB</td>
<td>3.9</td>
</tr>
<tr>
<td>WestLB</td>
<td>5.36</td>
<td>Bank of Tokyo</td>
<td>3.9</td>
</tr>
<tr>
<td>RBC</td>
<td>5.36</td>
<td>HBOS</td>
<td>3.85</td>
</tr>
<tr>
<td>Bank of America</td>
<td>5.36</td>
<td>Deutsche Bank</td>
<td>3.85</td>
</tr>
<tr>
<td>UBS</td>
<td>5.36</td>
<td>UBS</td>
<td>3.85</td>
</tr>
<tr>
<td>Citigroup</td>
<td>5.36</td>
<td>RBC</td>
<td>3.82</td>
</tr>
<tr>
<td>Credit Suisse</td>
<td>5.36</td>
<td>Bank of America</td>
<td>3.75</td>
</tr>
<tr>
<td>Lloyds</td>
<td>5.36</td>
<td>HSBC</td>
<td>3.75</td>
</tr>
<tr>
<td>HSBC</td>
<td>5.36</td>
<td>Lloyds</td>
<td>3.7</td>
</tr>
<tr>
<td>Rabobank</td>
<td>5.36</td>
<td>Rabobank</td>
<td>3.6</td>
</tr>
<tr>
<td>J.P. Morgan</td>
<td>5.36</td>
<td>J.P. Morgan</td>
<td>3.55</td>
</tr>
<tr>
<td>RBOS</td>
<td>5.35$\frac{1}{2}$</td>
<td>Citigroup</td>
<td>3.5</td>
</tr>
<tr>
<td>Libor</td>
<td>5.36</td>
<td>Libor</td>
<td>3.83$\frac{3}{8}$</td>
</tr>
</tbody>
</table>

A comparison of quotes and interquartile ranges for two different days in the sample period. Before the crisis, banks submitted very similar quotes and the interquartile range was often small, as seen in the left panel. After the crisis, the interquartile range expanded as banks submitted more heterogeneous quotes, as seen on the right.
| Bank of Tokyo | 0.4% | 56.7% | 42.9% |
| Bank of America | 15.7% | 63.4% | 20.9% |
| Barclays | 6.1% | 60.0% | 33.9% |
| J.P. Morgan | 35.9% | 58.7% | 5.4% |
| Citigroup | 22.5% | 76.7% | 0.8% |
| Credit Suisse | 2.1% | 84.9% | 13.0% |
| DeutschBank | 24.5% | 63.0% | 12.5% |
| HBOS | 6.0% | 85.1% | 8.9% |
| HSBC | 28.7% | 70.4% | 0.9% |
| Lloyds | 10.6% | 87.7% | 1.7% |
| Norinchukin | 0.7% | 46.5% | 52.8% |
| Rabobank | 27.5% | 69.4% | 3.1% |
| RBC | 5.3% | 87.5% | 7.2% |
| RBOS | 33.5% | 42.4% | 24.1% |
| UBS | 15.9% | 77.3% | 6.8% |
| WestLB | 2.8% | 83.7% | 13.5% |

The frequencies that the banks in the Libor panel participate below, within, and above the interquartile range of the day’s submitted quotes. My identification strategy involves exploiting the variation in this probability of participation over time, so it is comforting to see there are no banks which either always or never participate.
Table 4: Estimating Necessary First Order Conditions; Second Stage

<table>
<thead>
<tr>
<th></th>
<th>Bank of Tokyo-Mitsubishi</th>
<th>Point Estimates $\left(\frac{V_i}{20}\right)$</th>
<th>Average Skew (bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Bank of America</td>
<td>-0.66</td>
<td>-3.04 (1.19)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.26)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Barclays</td>
<td>-5.29</td>
<td>-13.47 (24.47)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(9.62)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>J.P.Morgan</td>
<td>-3.52</td>
<td>-7.90 (2.81)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.25)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Citigroup</td>
<td>-1.05</td>
<td>-6.19 (2.04)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.35)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Credit Suisse</td>
<td>0.91</td>
<td>5.34 (2.00)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.34)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Deutschbank</td>
<td>0.34</td>
<td>1.51 (0.75)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.17)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Societe Generale/HBOS</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>HSBC</td>
<td>-0.96</td>
<td>-4.34 (1.33)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.30)</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Lloyds</td>
<td>-1.28</td>
<td>-8.41 (3.68)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.56)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Norinchukin$^1$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>Rabobank</td>
<td>-0.82</td>
<td>-4.00 (4.63)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.95)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Royal Bank of Canada$^2$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>Royal Bank of Scotland</td>
<td>0.70</td>
<td>2.15 (0.80)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.26)</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>UBS</td>
<td>0.38</td>
<td>2.36 (0.58)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>WestLB$^3$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Fixed Effects; Bank-Quarter Effects

Additional Controls: 1 Year Senior CDS Spreads by bank, Commercial Paper Rate, Fed Funds Effective Rate
Table 5: Alternative Libor Mechanisms (Q3 2009)

<table>
<thead>
<tr>
<th></th>
<th>Average of Middle Eight</th>
<th>Random or Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average distortion from counterfactual Libor</td>
<td>-3.0bp</td>
<td>-4.7bp</td>
<td>0.0bp</td>
</tr>
<tr>
<td>Variance</td>
<td>0.57</td>
<td>0.77</td>
<td>0.53</td>
</tr>
<tr>
<td>Sum of squared errors from counterfactual Libor</td>
<td>6.8bp</td>
<td>17.1bp</td>
<td>1.3bp</td>
</tr>
<tr>
<td>Loss to U.S. Municipalities (per year)</td>
<td>$150 mill.</td>
<td>$235 mill.</td>
<td>$0</td>
</tr>
</tbody>
</table>

The counterfactual analysis of alternative Libor aggregation mechanisms. A Bayes-Nash equilibria is computed where manipulators have rational expectations and full awareness of the exact mechanism.
The interquartile range of the quotes submitted by the banks on the Libor panel, relative to the day’s Libor fix. The quotes are tightly packed together prior to the financial crisis, but then disperse.
The top graph shows the sixteen submitted quotes over time, relative to the daily average of the quotes. The bottom graph shows the corresponding sixteen one year senior CDS spreads, a measure of credit risk, relative to the daily average.
Bank of Tokyo Mitsubishi’s marginal ability to influence the Libor based on two separate trading days. The actual derivative would occur in a game of complete information where BoTM knew with certainty what the other banks would quote. The kinks occur at those quotes where BoTM gets kicked out of the interquartile range. With incomplete information, BoTM is uncertain where these kinks will be, and smooths out its marginal ability to influence the rate.
This figure shows two aligned graphs. The top graph shows the quotes of the bank in question relative to the interquartile range of the quotes of its peers, normalized to the day’s Libor. The bottom graph shows the results of my first stage estimations, in which I nonparametrically recover the bank’s marginal impact upon the Libor. The bank’s marginal impact varies over time as it near and further relative to the interquartile range of its peers. Towards the ends of my sample it submits relatively high quotes and has a very low marginal impact on the Libor.
A similar figure to that of 4 but now for Barclays. This bank used to submit high quotes relative to the others, and thus had a low marginal impact on the Libor. Its quoting behavior changed in 2009 and moved downwards relative to the quotes of its peers, and typically had a great marginal impact on the Libor.
Figure 6: First Stage Results for Bank Five (Citigroup)

A similar figure to that of 4 but now for Citigroup. This bank is positioned relative to the others such that it typically has a high marginal impact on the Libor.
This is a contour map of the first stage nonparametric estimates for bank 1 (Bank of Tokyo Mitsubishi). Results for the other 15 banks are similar. The Y axis is the distance of the bank’s submitted quote from the day’s realized Libor. The Z-axis, or height, is the estimated derivative of the expected Libor in the bank’s quote. The bank has little expected influence on the Libor for extreme quotes, but more so when quoting near “the middle of the pack.”
The difference between the “manipulation-free” Libor I estimate and the reported Libor. As the series is positive and growing, manipulation distorted the Libor downwards. See section 6 for a discussion.
References


