Simple Harmonic Motion of Pendula

I. Purpose

The purpose of this lab is to study the motion of a simple pendulum, a physical pendulum, and a torsional pendulum and to compare this motion with the simple harmonic motion predicted by theory for small oscillations of these systems. Finally, some standard techniques of data analysis will be emphasized.

II. Theory

For small displacements from a stable equilibrium one can approximate the restoring force as being proportional to the displacement. Using Newton's law

\[ F - m \left( \frac{d^2 x}{dt^2} \right) = 0 \]

so

\[ -kx - m \left( \frac{d^2 x}{dt^2} \right) = 0 \]

which has as solutions \( \sin (\sqrt{k/m} \, t) \) and \( \cos (\sqrt{k/m} \, t) \). These represent oscillations with a period \( T = 2\pi\sqrt{m/k} \). For torsional pendulums a similar equation if instead of Newton's law one states with torque \( \tau = d(I\omega)/dt = I \, d^2\theta/dt^2 \). For specific examples it is simply a matter of determining the form of the restoring force (e.g., for a simple pendulum \( K = mg/l \) and hence \( T = 2\pi\sqrt{l/g} \)). This is done for a number of examples in Eisberg and Lerner and is summarized below. Please read the relevant sections in the text before your lab.

A. Simple Pendulum: \( T^2 = 4\pi^2 l/g \) with

1 = distance from support to center of mass of the bob.

(pg. 219-222 in text)

B. Physical Pendulum: \( T^2 = \frac{4\pi^2}{g} \left( \frac{I_0 + Ml^2}{Ml} \right) \)

where \( I_0 \) = moment of inertia about the center of mass,
1 = distance from support to center of mass,
M = mass.

(pg. 406-408 of text, and parallel axis theorem)

C. Torsional Pendulum: \( T^2 = 4\pi^2 (I_0/k) \)

where \( I_0 \) is the moment of inertia about the support and \( k \) is a constant.

(pg. 409-410 of text). Nonlinear effects are discussed in Mechanics - Berkeley Physics
III. **Apparatus**

A. String and various weights to use as bobs in setting up a simple pendulum.
B. A brass rod with six holes drilled in it to serve as a physical pendulum.
C. A torsional pendulum with different sliding weights to vary the moment of inertia.
D. A stopwatch to time the oscillations.

IV. **Procedure**

Using the apparatus provided, you must test the formula given above for each type of pendulum. Some things to check are:

1. The period of the simple pendulum as a function of $l$ and $m$.
2. The period of the torsional pendulum as a function of $I_0$.
3. The period of the physical pendulum as a function of $l$.
4. The effects of large amplitudes.

In all cases, accuracy requires measuring the total time for many oscillations and dividing to determine the period. You might also repeat your measurement several times and take the average.

V. **Analysis**

Compare theory with the results of your experiments. Be sure to carefully consider the type of graph you would like to use. How do you know you are in the small amplitude limit? Do large amplitudes mean smaller or greater periods?

With respect to calculating $I_0$ for the torsional pendulum, this can be done exactly. For the physical pendulum, $I_0$ is very difficult to calculate.

You might use $I_0$ for a cylinder without holes, or you might try to do a more accurate calculation. In any case, you can adjust $I_0$ to produce a best fit between theory and experiment and compare this $I_0$ with your calculations.

Since much of the data involves averaging, it is important to do the statistics correctly. Is the difference between theory and experiment reasonable, (i.e. within your estimated error)? What are the largest sources of error? Do they account for your results?