Introduction

Physics is an experimental science. People talk about the “laws” of nature, and it may appear to you that these laws are engraved in stone somewhere so that all you have to do is learn them.

Actually, it is not like that at all.

Your textbook will tell you that the foundations of Physics 13 are Newton’s “Laws.” That sounds rock solid, until you realize that we cannot prove Newton’s Laws to be true! So, where did these laws come from? From experiment.

Newton spent some time thinking about the results of some actual experiments and developed his “laws” as a set of rules that all the data seemed to obey. That does not constitute a proof of the laws! All we can say is that in thousands (maybe millions by now) of experiments, no one has ever found a violation of Newton’s Laws.

All of this is a long way of introducing the idea that experiments—not textbooks, problem sets, and exams—are the real heart of physics. And the real heart of any experiment is the data. So you can see how it is very important to understand how to think about data: how to understand what it shows you, and what it does not show you.

Now, an Unfortunate Truth: The analysis and treatment of data can require sophistication, wisdom, incredibly ugly mathematics, and the kind of judgment that only comes from years and years of practice.

On the other hand, the Not-So-Unfortunate Truth is that many experiments can be analyzed using straightforward and simple techniques that are easy to understand. The purpose of this lab is to introduce you to a few of these techniques. We’ll leave the ugly math for later (much later).

In particular, this lab will get you acquainted with errors, with how they affect measurements, and with how we deal with them.
Types of Error

Nothing about the physical universe is known exactly. All our knowledge of this universe comes from measurements, and we simply cannot measure perfectly. For example, in this course you will encounter the gravitational constant $G$. We currently think that this constant has the value $6.67259 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-1}$. On the other hand, our uncertainty in this value, which comes from the errors in our experiments, is $8.5 \times 10^{-15} \text{ m}^3 \text{ kg}^{-1} \text{ sec}^{-1}$. A more useful way to think of this uncertainty is to express it as a percent of $G$. Our uncertainty in the value of $G$ is about $0.01\%$, or “one part in $10^4$.” This is actually quite large, considering the sophistication and effort that physicists have put into measuring $G$ in the 300 years or so we have been working on it. It is equivalent to a 6 inch uncertainty in the length of a mile, and any skilled surveyor can do better than that.

This uncertainty in our knowledge of $G$ comes from two sources: random errors, and systematic errors.

A. Random Errors

These occur because there are a large number of unpredictable things that can happen during an experiment. The power line voltage could fluctuate, or the floor could vibrate, either one of which might affect the reading of a delicate instrument. Or, when trying to estimate 10ths of a millimeter while reading from a meter stick, we might make small errors in judgment. The important point about all of these errors is that they are random---which lets us apply some statistical tools. We can’t make the errors go away, but we can develop ideas about what is a reasonable variation in our measurements, and what indicates a problem.

B. Systematic Errors

This type of error is a real problem. For example, someone could give you a meter stick that was printed incorrectly, so all the divisions were only $90\%$ of “true” size. Then, if you use this meter stick to obtain data on a falling object, the meter stick will incorporate a systematic error into your calculation of the acceleration due to gravity, $g$. This difficulty here is that unless you think to check the calibration of your meter stick, there is no way to tell that there is an error! Especially if you are the first person to measure $g$, so there is no “standard answer” to compare to.

Here is another systematic error that you are much more likely to encounter: suppose, when reading with the meter stick, you don’t always look at it straight on. Well, if you do this first to one side, and then to the other, and by slightly different amounts, this error will act like a random
error. But suppose, instead, that you always hold your head to the same side, and always by about the same amount. You would then be introducing a systematic error—all your length measurements would be incorrect in the same direction and by about the same amount. (By the way, this particular problem is known as parallax error.

Unfortunately, systematic errors can be tough to spot, and there is no one prescription for dealing with them. But you should be aware, and always asking yourself if your measurement technique could be introducing systematic errors.

**Uncertainty**

The combined effect of all these errors is to make us “uncertain” about things we measure. But please! The word “uncertainty” has a **VERY** specific meaning in physics. You can be uncertain about how well you did on a test, but that is not the same kind of “uncertainty” as you will encounter in lab.

A very specific example should help. Suppose we have a balance that will let us measure the mass of an object. Now suppose the smallest division on the balance is .1 kg. We weigh an object three times, and get 20.1 kg, 20.1 kg, and 20.3 kg for the three measurements. Clearly we now have to think—we have to decide what to write down for the mass of the object, and whatever we do write down can’t possibly be “exact.” So...what is our best estimate of the mass of this object? And what is our uncertainty in that estimate? Well, it turns out (and this is probably not surprising) that the best guess is the **average**, which a calculator will tell you is

20.1666666666666666666666666666666666666666666666666666666666666666666666....

There is a problem with this result! To write it with complete accuracy would require an infinite number of digits—but more to the point, more digits than our measuring device is even capable of producing. That means that most of the sixes in the answer are completely meaningless!! But, we can deal with this awkward number if we estimate the **uncertainty** in the result we get by taking the average. How do we do that? Look at the original data. All the data points are (roughly) within 0.1 kg of the average. So the uncertainty can’t be zero, but is can’t be huge either—an uncertainty of 0.1 kg seems about right. (We will have a slightly more precise way of doing this later on, but it requires more data points.)

Now look at the average—we have just encountered the dreaded problem of: **significant figures**.
There are lots of ways to make significant figures complicated, but let’s try for a way to make it simple:

**Any measured quantity cannot be written with more precision than its uncertainty.**

In our example the uncertainty is ±0.1 kg, so we round our result for the mass to the nearest 0.1 kg, and dutifully write “20.2±.1 kg” in our lab notebooks.

Well, that's all for an introduction. Now it's time to go out and get some data.

**References**


**Procedure**

This lab has two parts, each of which require some very simple measurements, and some thought. Be sure to answer ALL of the questions that are asked, and WRITE YOUR ANSWERS IN YOUR LAB BOOK.

I. The Foucault Pendulum

Out in the Fairchild Tower, which connects Wilder, Steele, and Fairchild Laboratories, there is a very tall pendulum with a large brass bob. Take a stopwatch and your lab notebook out there and do the following.

A. Time a single period of the pendulum. (A period is the time required to swing back and forth.) Write it down in your lab book. Also write down an estimate of your uncertainty in this single measurement. Give an explanation for how you arrived at this uncertainty. Don't be shy—you can't get this part wrong! But you must make an uncertainty estimate now because you will need it later.

B. Now make at least 20 single measurements of the period of the pendulum. That sounds like a lot, but it won't actually take very long. You may, in fact, want to make more measurements
than that. How do you decide if 20 is-or is not-enough? By making a histogram of your data. (If you don't know how to do this, see the Appendix to these instructions.) Your histogram should have a “bell curve” shape: a hump in the middle with “wings” or “tails” on either side. If your data looks like it might do that, but you can't really tell, then you need to take more data points. It might help to be building up the histogram as you take each point, but be sure to actually write the value of each measurement down!

When you are satisfied with your bell curve, you can make a new estimate of the period of the pendulum-it's just the average value: the middle of the hump. You can also estimate your uncertainty from the histogram. The usual choice is to pick a value so that 2/3 of the data points are within the uncertainty of your average. To put that another way, if your average is $a$, then pick the uncertainty $u$ so that:

$$a - u \leq \frac{2}{3} \text{ of the data} \leq a + u$$

(There are mathematical justifications for these choices of average value and uncertainty. Take a look at the books listed in the References section if you would like to know more about this.)

Compare the new uncertainty estimate with the one you made in part A. Which strikes you as more reasonable? BE SURE TO DEFEND YOUR ANSWER.

C. Here is one final method. Time 10 periods of the pendulum, and divide to get the value for a single period. Repeat this enough times to get what you think is a reasonable estimate of the uncertainty. (But not anything like 20 times!)

Which to you think gives a better estimate of the period: measuring many single periods, or making a single measurement of many periods? And why? Again, please BE SURE TO DEFEND YOUR ANSWER.

II. The Wall

Look at the walls of the laboratory room. They're made from cinderblocks. These blocks are manufactured so that they are all more or less identical. The bricklayer who assembled the wall tried to make the layer of mortar between the blocks a uniform thickness, so if we define the “length” of a block to be the sum of the length of the actual block plus the thickness of one layer of mortar, we should always find a constant value. Let's see....
A. Measure the length of one block (from now on, “one block” will always mean “one block plus one layer of mortar”). Do this in the hallway outside the lab because the blocks are easier to reach there. Measure the block in the long direction. Write this value in your lab book. Also write down an estimate of your uncertainty in this single measurement. Give an explanation for how you arrived at this uncertainty. Don't be shy—you can't get this part wrong! But you must make an uncertainty estimate now because you will need it later.

B. Now measure the lengths of 10 different blocks. Make a histogram of the data. In part I of this lab you estimated the uncertainty from the shape of a histogram like the one you just made. In this part, we'll do it the mathematically correct way.

If we can assume that all the errors that crept into your measurements of the blocks were truly random, then we can make the following two statements:

1. The best estimate for the length of the block is the average of the 10 values.
2. The best estimate for the uncertainty in the result is the standard deviation.

So, calculate the average and write it in your lab book. Next calculate the standard deviation, which you do by the following recipe. Suppose you have \( N \) data points. Let's name them \( x_1, x_2, x_3, \ldots, x_i, \ldots, x_N \). Then the standard deviation \( \sigma \) is defined to be

\[
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x - x_i)^2}
\]

where \( \bar{x} \) is the average of the \( N \) data points.

In words: Take the difference between each data point and the average, and square it. Add up all these squared numbers and divide the total by the total number of data points (notice that we are computing an average of these squared differences). Finally, take the square root.

Write your value for the standard deviation in your lab book and answer the following questions.

1. What fraction of your 10 data points are within the standard deviation of the average? Compare the standard deviation to the uncertainty estimate you made in part A.
2. In your opinion, would it be reasonable to use the standard deviation as an estimate of the uncertainty in the length of a block? BE SURE TO DEFEND YOUR ANSWER!

C. Finally, let's introduce a powerful technique we'll use again and again in P13: the straight line fit. The idea here is very simple, instead of measuring individual blocks, we'll measure the position of each block in a row of them. Hold the tape measure full length along the wall. Without moving it, note the position of the ends of as many blocks as the tape measure will cover. You can think of the tape measure as the $x$-axis in a coordinate system, and you are just noting the $x$-coordinate of the start of block 1, end of block 1, end of block 2, etc.

Make a graph of this data in your notebook. Use an ENTIRE PAGE for this graph. Put the positions that you measured on the vertical axis and the corresponding block number on the horizontal axis. The data should make a straight line, and the slope of this line should prove an excellent estimate for the length of a block. Here is how you get that estimate: use the clear plastic ruler to draw the “best” straight line through the data—the line which you think best approximates the data. The slope of this line is then your best estimate for the length of a block. (Very important: use the line, not a pair of data points, to find the slope. Otherwise, what is the point of drawing the line?)

How to estimate the uncertainty now? Draw two more lines on the graph: draw a “worst” line through the data with a slope greater than the slope of your “best” line. This step requires some judgment. You can’t just draw any old ridiculous line—it still has to do a reasonable job of representing the data. Draw a second “worst” line through the data, only this time with a slope less than your best line. The spread in slope between the two “worst” lines provides an estimate of the uncertainty of the slope of the “best” line.

How does the value for the length of a block you determined in this part compare with your results from parts A and B?

How do the uncertainties in your results from parts A, B, and C compare?

Which method gives the best estimate for the length of a block? BE SURE TO DEFEND YOUR ANSWER.

**A Final Question**

Since you will be expected to apply what you’ve learned in this lab to the rest of the labs in this course, write a paragraph summarizing what you actually did learn in this lab.
Appendix 1: Making Histograms

Not as grand as making history...but useful anyway.

To make a histogram, you divide a set of data into “bins,” and make a plot with the bins along the horizontal axis, and the number of data points in each bin along the vertical axis.

An example will help. The 20 members of the Tall Professors Club have the following heights (given in cm)

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<th>Height (cm)</th>
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<tbody>
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<td>183.9</td>
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<td>192.5</td>
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The first step is to decide on a useful bin size. 0.1 cm would be too small—every data point would be in its own bin. 10 cm would be too large: the data would be almost all in two bins. 1 cm wide bins looks like a good bet. So, for example, all the data $x$ such that

$$188 \leq x < 189$$

would be considered to lie within the same bin. Note where there is a “$\leq$” and where there is a “$.<.$” This is important. (You could do it like this: $188 < x \leq 189$, or like this: $188.5 \leq x < 189.5$. It’s up to you. You just have to make sure your bins are all the same size.

So, here is what a histogram of this data looks like:

![Histogram of Tall Professors Heights](image)
See if you think it has been made correctly!