The Hubble Redshift Distance Relation

taken from the ‘Student Manual’ from Project CLEA (Gettysburg College)

Introduction

In the 1920’s, Edwin Hubble measured the distances of the galaxies for the first time, and when he plotted these distances against the velocities for each galaxy he noted something remarkable: The further a galaxy was from the Milky Way, the faster it was moving away. Astrophysicists readily interpreted Hubble’s relation as evidence of a *universal* expansion. The distance between all galaxies in the universe was getting bigger with time, like the distance between raisins in a rising loaf of bread. An observer on ANY galaxy, not just our own, would see all the other galaxies traveling away, with the furthest galaxies traveling the fastest.

This was a remarkable discovery. Today, we know that the expansion is a result of a “Big Bang” which occurred between 10 and 15 billion years ago, a date which we can calculate by making measurements like those of Hubble. The rate of expansion of the universe tells us how long it has been expanding. We determine the rate by plotting the velocities of galaxies against their distances, and determining the slope of the graph. The slope is called the Hubble Parameter, $H_0$, and tells us how fast a galaxy at a given distance is receding from us. So Hubble’s discovery of the correlation between velocity and distance is fundamental in understanding the history of the universe.

Using modern techniques of digital astronomy, we will repeat Hubble’s experiment. The technique we will use is fundamental to cosmological research these days. Measuring distances is fairly difficult, while it is relatively easy to determine the velocity of a galaxy. Astronomers often measure the velocity of a galaxy, and calculate its distance using an assumed value of $H_0$. The velocity distance relation thus continues to help us map the universe in space and time.

Note that velocities are measured in astronomy using the Doppler shift, and that galaxies which are moving away from us have their spectra shifted to the red. Thus, we refer to Hubble’s relation between velocity and distance as the redshift distance relation in astronomy. Your goal in this lab is to find the relationship between the redshift in spectra of distant galaxies and their distance, and to use this to determine and the rate of expansion of the universe and to estimate its age.

References

Review section 7.5, Mathematical Insight 7.3 (page 176) and pages 601 – 610 in your textbook (‘*The Cosmic Perspective*’) before coming to the lab.
Overview of the Lab

This lab is done on a computer, where you will run a program which will simulate an observing session at a telescope. You will not be working in pairs for this lab; each student will collect and analyze the data individually. The software for the CLEA Hubble Redshift Distance Relation laboratory exercise puts you in control of a large optical telescope equipped with a TV camera and an electronic spectrometer. Using this equipment, you will determine the distance and velocity of several galaxies located in selected clusters around the sky. From these data you will plot a graph of velocity (the y-axis) versus distance (the x-axis).

How does the equipment work? The TV camera attached to the telescope allows you to see the galaxies, and “steer” the telescope so that light from a galaxy is focused into the slit of the spectro-meter. You can then turn on the spectrometer, which will begin to collect photons from the galaxy. The screen will show the spectrum — a plot of the intensity of light collected versus wavelength. When a sufficient number of photons are collected, you will be able to see distinct spectral lines from the galaxy (the H and K lines of calcium), and you will measure their wavelength using the computer cursor. The wavelengths will be longer than the wavelengths of the H and K labs measured from a non-moving object (3968.5 Å and 3933.7 Å), because the galaxy is moving away. The spectrometer also measures the apparent magnitude of the galaxy from the rate at which it receives photons from the galaxy. So for each galaxy you will have recorded the wavelengths of the H and K lines and the apparent magnitude. Recall that the apparent magnitude of an object is how bright it appears in the sky, and obviously depends on the distance to the object. The fainter an object is, the larger the apparent magnitude.

From the data collected above, you can calculate both the speed of the galaxy from the Doppler-shift formula (Mathematical Insight 7.3 in your text), and the distance of the galaxy by comparing its known absolute magnitude (assumed to be (-22) for a typical galaxy) to its apparent magnitude. Remember that the absolute magnitude of an object is the apparent magnitude an object would have at a distance of 10 parsec. Knowing a galaxy’s apparent and absolute magnitudes allows one to determine the distance to the galaxy.

You will determine the velocity (in km/sec) and a distance (in megaparsecs, Mpc) for each galaxy. The galaxy clusters you will observe have been chosen to be at different distances from the Milky Way, giving you a suitable range to see the straight line relationship Hubble first determined. The slope of the straight line will give you the value of $H_o$, the Hubble Parameter, which is a measure of the rate of expansion of the universe. Once you have $H_o$, you can take its reciprocal to find the age of the universe.

The details of the measurements and calculations are described in the following sections.

Using the Hubble Redshift Program

Welcome to the observatory! We will simulate an evening’s observation during which we will collect data and draw conclusions on the rate of expansion of the universe. We will gain
a proficiency in using the telescope to collect data by working together on the first object. Collecting data for the other nine objects will be left to you to complete the evening’s observing session. Then you will analyze the data, draw your conclusions, and use the information to predict the age of the universe.

Let’s begin:

1. Open the Hubble Redshift program by double clicking on the Hubble Redshift Icon (pink). Click on Log in... in the MENU BAR and enter your name. Click OK when ready.

2. The title screen of exercise appears. In the MENU BAR, the only available (bold-faced) choices are to start or to quit, click on Start.

The Hubble Redshift Distance Relation program simulates the operation of a computer-controlled spectrometer attached to a telescope at a large mountaintop observatory. It is realistic in appearance, and is designed to give you a good feeling for how astronomers collect and analyze data for research.

The screen shows the control panel and view window as found in the “warm room” at the observatory. Notice that the dome is closed and tracking status is off.

3. To begin our evening’s work, first open the dome by clicking on the dome button.

The dome opens and the view we see is from the finder scope. The finder scope is mounted on the side of the main telescope and points in the same direction. Because the field of view of the finder scope is much larger than the field of view of the main instrument, it is used to locate the objects we want to measure. The field of view is displayed on screen by a CCD camera attached on the finder scope. (Note that it is not necessary for astronomers to view objects through an eyepiece.) Locate the Monitor button on the control panel and note its status, i.e. finder scope. Also note that the stars are drifting in the view window. This is due to the rotation of the earth and is very noticeable under high magnification of the finder telescope. It is even more noticeable in the main instrument which has even a higher magnification. In order to have the telescope keep an object centered over the spectrometer opening (slit) to collect data, we need to turn on the drive control motors on the telescope.

4. We do this by clicking on the tracking button.

The telescope will now track in sync with the stars. Before we can collect data we will need to do the following:

(a) Select a field of view (one is currently selected).

(b) Select an object to study (two from each field of view).

5. To see the fields of study for tonight’s observing session.

Click once on the change field item in the MENU BAR at the top of the control panel. Note that you can only change fields of study when you are using the finder scope.
The items you see are the fields that contain the objects we have selected to study tonight. An astronomer would have selected these fields in advance of going to the telescope by: (a) selecting the objects that will be well placed for observing during the time we will be at the telescope; and (b) determining the coordinates for each object field from a catalog.

This list in the **change field** menu item contains 5 fields for study tonight. You will need to select two galaxies from each field of view and collect data with the spectrometer (a total of 10 galaxies).

To see how the telescope works, change the field of view to Ursa Major II. Note that RA and DEC are listed for each field of view. RA and DEC are celestial coordinates (similar to longitude and latitude) that astronomers use to specify the positions of objects in the sky.

Notice the telescope “slews” (moves rapidly) to the RA and DEC coordinates we have selected. The view window will show a portion of the sky that was electronically captured by the charge coupled device (CCD) camera attached to the telescope.

The view window has two magnifications (see Figure 1):

**Finder View** is the view through the finder scope that gives a wide field of view and has a cross hair and outline of the instrument field of view.

**Spectrometer View** is the view from the main telescope with red lines that show the position of the slit of the spectrometer. Note that you slew the telescope to a new object field while in spectrometer view.

As in any image of the night sky, stars and galaxies are visible in the view window. It is easy to recognize bright galaxies in this lab simulation, since the shapes of the
brighter galaxies are clearly different from the dotlike images of stars. But faint, distant galaxies can look similar to like stars, since we can’t see their shape.

6. Now change fields by clicking on Ursa Major I in the lists of selected galaxies to highlight the field. Then click the OK button.

7. Locate the Monitor button in the lower left hand portion of the screen. Click on this button to change the view from the Finder Scope to the Spectrometer. Using the Spectrometer view, carefully position the slit directly over the object you intend to use to collect data—any of the galaxies will be fine. Do this by “slewing”, or moving, the telescope with the mouse and the N, S, E or W buttons. Place the arrow on the one of the direction buttons and click on the left mouse button to move the telescope in that direction. To move continuously, press and hold down the left mouse button. Notice the red light comes on to indicate the telescope is “slewing” in that direction.

As in real observatories, it takes a bit of practice to move the telescope to an object. You can adjust the speed or “slew rate” of the telescope by using the mouse to press the slew rate button. (1 is the slowest and 16 is the fastest). When you have positioned the galaxy accurately over the slit, click on the take reading button to the right of the view screen.

The more light you get into your spectrometer, the stronger the signal it will detect, and the shorter twill be the time required to get a usable spectrum. Try to position the spectrometer slit on the brightest portion of the galaxy. If you position it on the fainter parts of the galaxy, you are still able to obtain a good spectrum but the time required will be much longer. If you position the slit completely off the galaxy, you will just get a spectrum of the sky, which will be mostly random noise.

We are about to collect data from the object. We will be looking at the spectrum from the galaxy in the slit of the spectrometer. The spectrum of the galaxy will exhibit the characteristic H & K calcium lines which would normally appear at wavelengths 3968.47 Å and 3933.67 Å, respectively, if the galaxies were not moving. However, the H & K lines will be red shifted to longer wavelengths depending on how fast the galaxy is receding.

Photons are collected one by one. We must collect a sufficient number of photons to allow identification of the wavelength. Since an incoming photon could be of any wavelength, we need to integrate for some time before we can accurately measure the spectrum and draw conclusions.

The more photons collected, the less the noise in the spectrum, making the absorption lines easier to pick out. To initiate the data collection, press start/resume count.

8. To check the progress of the spectrum, click the stop count button. The computer will plot the spectrum with the available data (see Figure 2). Clicking the stop count button also places the cursor in the measurement mode. Using the mouse, place the arrow anywhere on the spectrum, press and hold the left mouse button. Notice the arrow changes to a cross hair and the wavelength data appears in the lower right area of the window. As you hold the left mouse button, move the mouse along the
spectrum. You are able to measure the wavelength and intensity at the position of the mouse pointer.

![Figure 2: Spectrometer Reading Window](image)

Also notice other information that appears in the window:

**Object:** the name of the object being studied

**Apparent magnitude:** the visual magnitude of the object

**Photon count:** the total number of photons collected so far, and the average number per pixel

**Integration (seconds):** the number of seconds it took to collect data

**Wavelength (angstroms):** wavelength as read by the cursor in the measurement mode

**Intensity:** relative intensity of light from the galaxy at the position marked by the cursor in the measurement mode

**Signal-to-noise Ratio:** A measurement of the quality of the data taken to distinguish the H and K lines of calcium from the noise. Try to get a signal-to-noise ratio of 10 to 1. For faint galaxies, this may take some time.

9. Click **start/resume** count from the menu bar in the Spectrometer Reading Window. Continue to collect photons until you can easily see the H & K lines of calcium in the display. These lines are approximately 40 Å apart. They should stand out from the noise. If not, continue to count photons. If you are not sure about the data, check with a TA to help you interpret the data.

Additional information is needed to complete the analysis of the information that is not displayed in the spectrometer reading window. They are the following:
(a) The absolute magnitude (M) for all galaxies in this experiment is $-22$
(b) The laboratory wavelength of the K line of calcium is 3933.67 Å.
(c) The laboratory wavelength of the H line of calcium is 3968.847 Å.

10. Record the object, photon count, apparent magnitude, and the measured wavelength of the H & K lines of calcium on the data sheet located at the end of this exercise. The H & K lines measured should be red shifted from the laboratory values depending on the galaxies motion.

11. To collect data for additional galaxies, press **Return**. Change the **Monitor** to display **Finder**. Record a spectrum for an additional galaxy in this field (for a total of 2 galaxies per field). Then follow steps 6 through 10 to obtain data for galaxies in other fields.

**Procedure**

1. Record the number of the computer that you are using at the top of your data sheet. Using the computer simulated telescope, measure and record the following quantities on your data sheet: the wavelengths of the calcium H and K lines for two galaxies in each of the five fields. Record the object name, apparent magnitude and photon count for each object on your data sheet. Collect enough photons (usually around 40,000) to determine the wavelength of the line accurately.

2. Use your measured magnitudes and the assumed absolute magnitude for each galaxy and derive the distance, ($D$). Note that the absolute magnitude is the apparent magnitude of an object at 10 pc. Using this definition, along with the fact that magnitudes are defined on a logarithmic scale and that the intensity of light falls off as $1/D^2$, one can derive the formula (don’t worry, you don’t have to do the derivation): 

\[ M = m + 5 - 5 \log D \]  

where $M$ is the absolute magnitude of the galaxy, $m$ is the apparent magnitude and $D$ is the distance in parsecs. This equation may be written as

\[ \log D = \frac{m - M + 5}{5}. \]  

Express your distances in both parsecs and megaparsecs in the appropriate places on your data table. Note that equation (2) tells you how to find the log of the distance, to find the distance, $D$, you must take the anti log, i.e.

\[ D = 10^{\log D} \]  

3. Use your measured wavelengths given above to calculate the redshifts for each line, $\Delta \lambda_H$ and $\Delta \lambda_K$. The redshift is defined to be $\Delta \lambda_H = \lambda_{H\text{ measured}} - \lambda_H$ for the calcium H line, with a similar definition for the K line. Record each on your data table.
4. Use the Doppler shift formula

\[ v = c \frac{\Delta \lambda}{\lambda} \]  

(4)

where \( c \) is the speed of light and \( v \) is the velocity of the galaxy, to determine the velocities as measured by both the H and K lines. There is a place on the data table for each of these figures:

5. Calculate and record the average velocity of the galaxy (from the H and K line velocities). Estimate an error in your velocity measurement by using difference in the velocity between the H and K lines for each galaxy. Is there a relationship between your velocity error and the total photon count?

6. Now plot a Hubble diagram by graphing the velocity of a galaxy in km/sec (y-axis) vs. the distance in megaparsecs (x-axis) on a sheet of graph paper (attached). Draw a straight line through the origin that best fits all the data points. The slope of the line is the Hubble Parameter (\( H_o \)). To calculate the slope of the line, measure a value of \( D \) and \( v \) from a point near the upper right end of the line you drew (do not use one of the points that you plotted, just use a point on the line that you drew).

Determine \( H_o \) using the following equation:

\[ H_o = \frac{v}{D} \]  

(5)

where \( H_o \) is the Hubble Parameter in km/sec/Mpc; \( v \) is the velocity measured from your line \( D \) is the distance measured from your line.

(a) Record your value for the Hubble Parameter on the graph paper

(b) Mark the point you used on your graph

(c) Label the axis of your graph, and give it a meaningful title.

(d) How big is the scatter of the individual points about your best fit line? What do you think this scatter is due to?

(e) Estimate an error in your measurement of the Hubble Parameter by fitting two more lines to the data, one line with a slope smaller than your best fit line, the other line with a slope larger than your best fit line. These two lines should still do a reasonable job of fitting your data. Determine \( H_o \) from each of these two lines in the same manner that you used for your best fit line. The difference between these two values of \( H_o \) yields an estimate of the error in your value of \( H_o \).

**Determining the Age of the Universe**

The Hubble Law, equation (5), can be used to determine the age of the universe. We need to go through some math to do this. Include all of your calculations (in an organized fashion) in your lab report (along with some notes indicating exactly what you are doing).
Using the average value of $H_o$ that you determined from the graph and equation (5), calculate the recessional velocity $v$ of a galaxy which is 800 Mpc away. Verify your velocity by looking it up on your Hubble diagram. You now have two important pieces of information:

1. How far away the galaxy is
2. How fast it is going away from us

You can visualize the process if you think about a trip in your car. If you tell a friend that you are 120 miles away from your starting point and that you traveled 60 miles per hour, your friend would know you had been traveling TWO hours. That is your trip started two hours ago. You know this from the relationship:

$$\text{Distance} = \text{velocity} \times \text{time}$$

we can write as

$$D = vt$$

or

$$t = \frac{D}{v}.$$ 

Thus,

$$2 \text{ hrs} = \frac{120 \text{ mi}}{60 \text{ mi/hr}}.$$ 

Now let’s determine when the universe “started its trip”. The distance is 800 Mpc, but first convert Mpc into km because the rate, or velocity, is in km/sec. Recall that 1 pc = 3.26 light-years, that the speed of light is $3 \times 10^5$ km/sec and that in 1 year there are $3.15 \times 10^7$ seconds. Using this distance in km and your determination for the velocity of the galaxy, determine how many seconds ago the universe started. Convert your answer to years. How does this age compare to the age of the Earth? The oldest stars in the Milky way (as determined from main sequence fitting) are $(12.8 \pm 1.7) \times 10^9$ years old. How does this compare to the age you determined for the universe based on its expansion velocity?

**Lab Report**

Your lab report should include:

1. A brief statement describing the purpose of the lab.
2. Your data sheet, graph and calculations. Your calculations should be presented in an organized fashion, and should include a brief description of what you are doing in each step.
3. Answers to the questions asked in the Procedure section.
### Data Sheet

<table>
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<th>Galaxy Field Name</th>
<th>Abs Mag</th>
<th>Photon Count</th>
<th>App Mag</th>
<th>Dist in pc</th>
<th>Dist in Mpc</th>
<th>$\lambda_{\text{measured K line}}$</th>
<th>$\lambda_{\text{measured H line}}$</th>
<th>$\Delta \lambda_{\text{H}}$</th>
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**Useful Equations and Quantities:**

1 Mpc = $1 \times 10^6$ pc  
$c = 3 \times 10^5$ km/sec

\[
M = m + 5 - 5 \log D  \quad \Delta \lambda_K = \lambda_{\text{K measured}} - \lambda_K  \quad v_K = c \frac{\Delta \lambda_K}{\lambda_K}  \quad \lambda_K = 3933.67 \, \text{Å}
\]

\[
\log D = \frac{m - M + 5}{5}  \quad \Delta \lambda_H = \lambda_{\text{H measured}} - \lambda_H  \quad v_H = c \frac{\Delta \lambda_H}{\lambda_H}  \quad \lambda_H = 3968.47 \, \text{Å}
\]