This lab is done entirely on the computer. The motion of planets and satellites that you have created is displayed on the screen using a "canned" program. "Planets", which behave mathematically as spherical masses, are placed at various points in an otherwise empty space, and the program calculates the ensuing motion. The motion depends, of course, on the relative position of the planets, their masses, and the initial velocities. These three variables, the mass, the position, and the initial velocity, can all be specified by simply typing in the appropriate numbers.

The program, called Gravitation_Ltd_4_0, is available on the Public File Server in the Physics 3 folder in the Courses folder. It will also be available in the laboratory room. You are encouraged to keep a copy of the program to do experiments of your own choosing. (Don't try to run the program while it's still on PUBLIC; it'll take forever. Copy it to a floppy or a hard disk.)

A familiarity with Sections 12-1 through 12-5, and the material up through Chapter 7 of the text, University Physics, plus a reading of the Appendix to this lab handout is necessary for a satisfactory completion of this lab. You should also take note of Kepler's laws on page 329 of the text (Section 12-6), but confronting the math in the rest of that section is optional.

Theory

In most of the computer experiments suggested one places two or masses, \( M_1 \) and \( M_2 \) on the screen, which is an otherwise empty space. These two are coupled by an attractive force given by

\[
F = G \frac{M_1 M_2}{R^2} \tag{1}
\]
where $G$ is the universal gravitational constant, and $R$ is the distance between their centers. If one mass is treated as a large, stationary planet ($M_P$) and the other as a small satellite ($m_s$), the satellite will orbit the planet in an elliptical path, with the planet at one focus. If the satellite is placed into an orbit with a tangential velocity such that the mass times the radial acceleration, $m_s v^2 / R$ is exactly equal to the gravitational force, the orbit will be circular.

If the center of the satellite is initially placed a distance $R_0$ from the center of the planet and given a velocity $v_o$ directly away from the planet, it will travel to a new distance $R_1$ before stopping and falling back to the planet. $R_1$, $R_0$, and $v_o$ are related by

$$v_o^2 = 2GM_P \left( \frac{1}{R_0} - \frac{1}{R_1} \right)$$

(2)

where $M_P$ is the mass of the planet.

One of the most important developments in the history of astronomy was the formulation by Kepler of his three laws governing planetary motion. One of these, the law of periods, can be tested with this program. It relates the period $T$ of a satellite in orbit about a planet of mass $M_P$ with the semi-major axis $R$ of the orbital ellipse:

$$T^2 = \left( \frac{4\pi^2}{GM_P} \right) R^3$$

(3)

Calling the Program

The program will be available in the lab, and you can copy it onto your disk. Click on the icon, which looks like this:
and the screen will open up looking like this:

In this view the scale is 1x, a close-up picture. If you click on Display, and try the 2x and 4x, you will see the white dots pull together. In the dimensions used in the program, the dots are 80 units apart. In the 4x mode (the best to use, usually) the full screen is 1920 units wide and 1080 units high. This means that the x position can be anything from -960 to +960, and the y position anything from +560 to -560.

To get things started, click on Editor and the screen will look like the figure on the next page. The program gives you default values for the mass and position of the first planet. The best thing to do is to place the largest possible planet (M = 900) in the center of the screen (x = 0, y = 0), make it Stationary (click on box at lower left). Then click on Create to create a satellite.
Make the mass of the satellite anything you like, but M = 1 is perfectly good. Place it off to the side of the planet, and give it a tangential velocity. This means that if y = 0, v_x should = 0. Click on Trace Enable, and then on OK. When the Editor clears, click on run.

At this point the best thing to do is play with the program for a while to get familiar with various options, such as the Trace, the Frame Counter, Pause, etc. Try calibrating the velocity units, by placing a single mass on a dot (e.g., let x be a multiple of 80 and y be 0), give the mass a velocity of 1 or 10, or whatever (remember, the speed should not be greater than ±10), and see how long by the frame counter it takes to go a given distance. The speed is the distance units covered divided by the frame counter reading. Some specific experiments are suggested below, each of which is a measurement of G in program units.
Experiment 1:

Create a large stationary planet, \( M_p = 900 \) is suggested, at the center of the screen and place a small satellite into a tangential orbit about it. Adjust the values of the initial velocity and radius to make the orbit circular. When you have found a set of initial conditions that make a circular orbit, set the gravitational force equal to the mass of the satellite times the centripetal acceleration, and use that formula to calculate \( G \):

\[
G = \frac{Rv^2}{M_p} \quad (4)
\]

Try various combinations of \( v \), \( R \), and \( M_p \) to see if the value of \( G \) is constant in this program, as it is in the universe. It won't be because of a computer error called "floating point round-off".

Experiment 2:

Place a large stationary planet at the far left of the screen, which is \( x = -960 \) for 4x zoom. Place a second mass (the rocket) a short distance to the right of this and give it a positive \( x \) velocity. About 200 to 500 units to the right of the stationary planet is suggested as a first trial launching point for the rocket, placing it at \(-760 < x < -460\). If you choose properly, it will go most of the way across the screen, like a rocket trying to escape the earth, before stopping and being pulled back to the planet. Measure how far it gets before stopping; keeping the trace on makes this measurement easier, because the stopping point \( (R_1) \) as marked. Then use Eq. (2) to calculate \( G \):

\[
G = \frac{v_o^2 R_1 R_0}{2M_p (R_1-R_0)} \quad (5)
\]

You will find that the extent rocket's flight is very sensitive to the initial conditions. Make it too massive, too close or launch it too slowly, and it will quickly be pulled back to the planet. Change things just a bit, and it flies off the screen.
Experiment 3.

In a recent midterm exam, one problem concerned a planet being orbited by two moons, as shown below:

The moons each have half the mass of the planet, and both are in a circular orbit about it on opposite sides. In principle, there is no net force on the planet at any time, because the satellites are always pulling in opposite directions. Each moon has a centripetal force due to the gravitational attraction of the planet and the opposite moon, and the sum of these forces is equal to the mass of the moon times the radial acceleration:

$$\frac{mv^2}{R} = G \frac{2m^2}{R^2} + G \frac{m^2}{4R^2} = G \frac{9m^2}{4R^2}$$

The question was to find the orbital speed of the planets, and the answer, which can be found by solving this equation for \(v\), was:

$$v = \frac{3}{2} \sqrt{\frac{Gm}{R}}$$

Do an experimental test of the solution to the problem by duplicating the situation shown in the figure above. In the program units \(G = 5\), so if you make \(R = 400\) and \(m = 80\), setting \(v = 1.5\) should put the two moons into the same circular orbit. You should notice that it is not necessary to make the planet stationary, as there is no force on it.
Experiment 4

The final experiment is an experimental test of Kepler's law relating the period of an elliptical orbit to the semi-major axis, expressed in Eq.(3).

Put planets of different masses into some elliptical orbits about the same stationary sun. If the orbits are nearly circular, place the sun near the center of the screen; if they are highly elliptical, place the sun nearer the edge. Measure the period of revolution (T) with the frame counter, by counting the time of one or more revolutions. Measure the semi-major axis (A) directly off the screen (The semi-major axis is one half the length of the longest axis of the ellipse). Then see if $T^2/A^3$ is a constant, as implied by Eq.(3).

You will probably find that you values of G don't agree, but they should all be about 5, in program units. I have consulted with the guy who wrote the program, and we discussed various reasons why the computing algorithm would do this, but came to no firm conclusion.

Report

Many of the trials that you make will have to be aborted, because the satellite flies off the screen, or some other unwanted result. In your lab notebook keep a record of the conditions of each 'run' that did give you some measurable result. Show the calculation that you used to choose the values entered in Editor. The TA's will be looking for evidence that you tutored yourself successfully about the law of universal gravitation, and its consequences.
What is it?

Gravitation Ltd. is a graphic, two dimensional orbital simulation. Users can enter custom designed solar systems or run previously created ones. Systems can be views at varying zoom levels, recorded and played back at high speed, and edited via the graphic solar system editor.

A Gravity Primer

Gravity is the force of attraction between any two objects. The strength of that force depends on the distance between the two objects, the mass of each object, and a gravity constant. This force can be represented by the formula: \( \text{Force} = \frac{(G M_1 M_2)}{R^2} \)

where \( G \) = gravity constant, \( M_1 \) is the mass of the first object, \( M_2 \) is the mass of the second object, and \( R \) is the distance between them. Accordingly, as the two objects come closer the gravitational pull increases. Also, as the mass of either or both objects increases, the gravitational pull will increase.

How much a given gravitational pull affects an object depends on that object's mass. A small pull on a heavy object will cause very little change in the object's speed and position (a small acceleration). A large pull on a light object will cause a strong acceleration.

Using Gravitation Ltd.

File Operations

Solar systems can be opened or saved via the OPEN and SAVE commands in the File menu.

About the Editor

The Graphic Editor, located under the Edit menu, is used to design or modify model solar systems. It allows you to do the following:

Create and delete planets (Max. 10 Planets).
Specify Mass for each planet. Mass between 1 and 900.
Specify X and Y location for each planet. Units are pixels, with 0,0 being center screen.
Specify X Velocity and Y Velocity for each planet (10<V<10). Units are pixels per frame.
Specify Trace Enable for each planet. This selection will leave a trail behind the planet when applicable.
Specify Stationary for each planet. Stationary will lock that object in place so that it is unaffected by the gravitational pull of other objects.

Two displays in the editor allow you to see what effects your changes will have. The first and larger one is the Planet Display, which is a reduced scale view of the screen as it will look when you leave the editor. The second is the Velocity Vector Display. This display lets you see the starting direction and speed of the currently selected planet.

Using the Editor
Buttons

Create: Creates a new planet and selects it.
Delete: Erases currently selected planet and selects next planet.
Next: Select next planet.
Prev: Select previous planet.
O.K.: Leave the editor and return to main program.
Trace: Will leave a trail behind the currently selected planet.
Stationary: Locks the current planet in position.

Edit boxes

Used to enter Mass, X and Y location, and X and Y velocity for each planet.

Planet Display

The planet display, the large grid in the upper right of the editor, gives a graphical representation of the current solar system as it will appear on your screen. This display is also a control in the sense that planets can be both clicked on to select them and view their data, and also dragged around the screen to move them. The selected planet is shown with a rectangle around it. The three round buttons located below the planet display, 1x, 2x and 4x, control the viewing distance for the planet display. (1x = close up view, 4x = viewing from far away. - Try it, you'll see).

Velocity Vector Display

The velocity vector display complements the planet display by providing a graphical representation of the velocity (Speed) and direction of the selected planet. This display can also be changed by clicking in it. The VVD will reflect the settings of the X velocity and Y velocity text boxes and vice versa.

Although all of this may seem complicated, I think it you will find it all very clear when you see it and work(Play?) with it. Just click around and try things.

• Go Menu Commands •

Run: Starts the current simulation.

Set Pause At...

Set pause at brings up a dialog box that allows the simulation to automatically stop, stop and beep, or repeat at a specified frame. The preset frame number is the maximum number of frames that can be recorded by the Instant Replay device (see below). Using a larger number will mean that early replay tape will be overwritten.
Limit Speed

Limit Speed will limit the simulation to a maximum of 30 frames per second. This will keep simulations from going too fast, especially when run on powerful Macintoshes. Users of standard 68000 based Macs will rarely need to use this feature. Limit speed applies to normal Run mode and also to Instant Replay Play mode (see instant replay below).

Anti Drift

Anti drift calculates a Zero Momentum Frame. That is, anti drift will zero the total velocity of the model solar system. This will help keep the system from drifting off of the screen. Anti Drift is a one time calculation done only at frame zero. Therefore, for it to take effect, you must be at frame zero or use the Reset command or enter/exit the editor to get to frame zero. Anti drift will have no effect if any planets are locked in place with stationary. Indeed, the whole purpose of the anti drift feature is to avoid having to lock any planets in place, yet still have the simulation stay centered on the screen.

Reset

Reset clears the screen and returns the simulation to its starting point (frame zero). All replay tape is erased.

•Instant Replay Menu Commands•

Instant Replay

Selecting instant replay will bring up an instant replay palette in the upper right corner of the screen. This device will let you review the previous 2400 frames. The five buttons in the palette from left to right are...
- Rewind: Rewinds the "tape" back to the beginning at high speed.
- Stop: Stops the tape.
- Play: Runs the tape forward at normal speed (limited by the limit speed command).
- Fast Forward: Moves the tape forward at high speed.
- Exit Playback: Returns to the main simulation.

The primary purpose of the replay device is to allow you to review simulations at speeds greater than are possible when first calculating them. The replay device does not have to calculate as it displays frames and is therefore much faster. Remember, only 2400 frames can be stored. Additional frames will take the place of the oldest frames. The scroll bar on the bottom of the replay box can be clicked on to quickly move through the replay tape.

Normal Play

Returns the program to the main simulation at the frame it was at when instant replay was entered. This is the same as clicking on the Exit Playback button.
•Display Menu Commands•

Trace On
Trace On activates the tracing of all planets that have been selected as Trace Enable in the editor. This will leave a trail of dots behind those planets.

Clear Trace
Clear trace will erase all previous trace dots.

Normal Zoom
Zoom Out 2x
Zoom Out 4x
These three items control the scale at which the solar system is displayed. Normal Zoom is a close up view, and Zoom Out 4x is a distant view which will allow you to see a larger area.

Frame Counter
This command brings up a counter in the upper left of the screen which shows how many frames have passed since the simulation began.

•Collisions etc.•
When two or more planets collide their mass and momentum will be added to form a single new planet. Stationary or trace attributes will be combined into the Or of the original planets. Because the number of calculations required increases geometrically as the number of planets is increased, large simulations will run slowly. Use instant replay to review them at high speed after they have been calculated.

•Gravitation Documents.•
Numerous Gravitation Ltd. solar systems are available on bulletin boards and on-line services. Please take the time to upload solar systems that you design to any available services. This gives others a chance to share in your creativity. If your service has a keyword search capability, please include the keyword "Gravitation" with your file so that users can quickly find all available files. If your on-line service does not have a copy of Gravitation Ltd. 4.0, please upload it - this would be especially appreciated. Thank you.

•Technical Stuff•
Compatibility:
Gravitation Ltd. 4.0 should run on 512K, 512KE, Plus, SE, Mac II.

Memory usage and Multifinder.
Gravitation Ltd. 4.0 requires about 400 K to run. Although large screen Macs may require more memory (Up to about 650 K.). If insufficient memory is available the program should alert you when you start it up. Within these memory guidelines, it should be fine with Multifinder. Future compatibility, both with software and hardware, has been strived for but cannot be guaranteed (Naturally).