Key Concepts for the Lecture of 7Jul03

Kinematic Overview

- The position of a particle is given by the coordinates of the particle in the chosen coordinate system. Position is a vector whose components are the coordinates of the particle:
  \[ \vec{r} = x \, \hat{i} + y \, \hat{j} + z \, \hat{k} \]

- Velocity is the rate of change of position:
  \[ \vec{v} = \frac{d\vec{r}}{dt} \]

- Since the components of the vector velocity are independent, each “scalar component” of velocity obeys its own equation of motion:
  \[ v_x(t) = \frac{dx(t)}{dt} \]

- If the velocity is known, then the position is
  \[ x(t) = \int v_x(t) \, dt \]

- For the special case of constant velocity, the position is
  \[ x(t) = x_0 + v_{x0} \, t \]
  where \( v(t) = v_{x0} = \text{const} \)
  and \( x_0 = x(0) \)

The \( x_0 \) term arises from evaluating the constant of integration. In this case, \( x_0 \) is the initial position. As time increases, the position increases linearly from the initial value. Its rate of increase is the velocity.

- In the more general case, velocity can change as a function of time. Acceleration is the rate of change of velocity:
  \[ \vec{a}(t) = \frac{d\vec{v}(t)}{dt} \]

- Since the components of the vector acceleration are independent, each “scalar component” of acceleration obeys its own equation of motion:
\[ a_x(t) = \frac{dv_x(t)}{dt} \]

- If the acceleration is known, then the velocity is
\[ v_x(t) = \int a_x(t) \, dt \]

- For the special case of constant acceleration,
\[ v_x(t) = v_{x0} + a_{x0} \, t \]

where \( a_{x0}(t) = a_{x0} = \text{const} \)

and \( v_{x0} = v_x(0) \)

The \( x_{x0} \) term arises from evaluating the constant of integration. In this case, \( v_{x0} \) is the initial velocity. As time increases, the velocity increases linearly from the initial value. Its rate of increase is the acceleration.

- Still for the special case of constant acceleration \( a_{x0} \), initial velocity \( v_{x0} \), and initial position \( x_{0} \), the position is
\[ x(t) = x_0 + v_{x0} \, t + \frac{1}{2} a_{x0} \, t^2 \]