Solutions to Midterm Exam

1a) \( \psi(x,t) = A \sin(kx - wt) \); (or \( \cos \ldots \))
\[
\lambda = \frac{2\pi}{k}, \quad f = 2\pi w, \quad v = \frac{w}{k} = f\lambda; \quad \text{moving right. If } (kx + wt), \text{ then left.}
\]
(b) \( \phi(x,t) = A \sin kx \sin wt \);
\[
\lambda = \frac{2L}{n}, \quad v = f\lambda \quad \Rightarrow f = \frac{n\nu}{2L}
\]

2a) From formula on board \((\cos^2 \beta, \beta = \frac{\pi dy}{\lambda \cdot D})\),
or from geometry
\[
\lambda = d \sin \theta \approx \frac{dy}{D}
\]
\[
\lambda = \frac{dy}{D} = \frac{0.06 \text{ cm} \times 1.1 \text{ cm}}{10 \text{ m}} = 6.6 \times 10^{-7} \text{ m} = 660 \text{ nm}
\]
(b) First zero of \( \sin \alpha \) corresponds to \( n = 3 \) maximum of \( \cos \beta \). So \( \lambda = \pi \) when \( \beta = 3\pi \).
So \( \frac{a}{b} = \frac{a}{2a} = \frac{1}{3} \Rightarrow a = 0.20 \text{ mm.} \)

3a) For \( \theta = 0 \) or \( 90^\circ \) you get one-slit pattern because you know which slit the photons went through. For \( \theta = 45^\circ \) or \( 135^\circ \) you get a two-slit pattern because the photons could have gone through either slit with equal probability (you have no knowledge which slit).

(b) The split polarizers remove half of the photons (because \( \cos^2 45^\circ = \frac{1}{2} \)), and the final polarizer selects only the right slit, so \( \frac{1}{4} \) of the incident photons get through (compared to how many get through with no polarization filters).

4a) The photoelectric effect is the ejection of electrons from matter by incident EM radiation (light or UV typically).

(b) There is a threshold frequency \( f_\text{th} \) below which the effect does not occur.

(c) Stopping voltage \( V_s \) is the applied voltage necessary to halt the photoelectric current (in the circuit shown).
\[ eV_s = KE_{\text{max}} = hf - \Phi \]

Photon frequency \( \uparrow \) \( \uparrow \) Work function

\[(d) \]

\[ \frac{\Phi}{e} \uparrow \quad \text{slope } \frac{h}{e} \quad \rightarrow f \]

Plot \( V_s \) against \( f \). Slope is \( \frac{h}{e} \) and intercept (along negative \( V_s \)-axis) is \( \frac{\Phi}{e} \).

5a) Observers on the ice see a contracted sled which loses contact with the ice. The distance it travels is \( L - L\sqrt{1 - \frac{v^2}{c^2}} \), so the time spent over the hole is

\[ t = \frac{1}{v} \left( L - L\sqrt{1 - \frac{v^2}{c^2}} \right). \]

Expanding sq. root, \( \sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{v^2}{2c^2} \),

\[ \Rightarrow t \approx \frac{Lu}{2c^2}. \]

(b) Riders on the sled see a short hole pass under them. Time interval during which sled straddles hole is (same!)

\[ t = \frac{1}{v} \left( L - L\sqrt{1 - \frac{v^2}{c^2}} \right) \approx \frac{Lu}{2c^2}. \]
(6) The Doppler shift occurs because of time dilation (and only because of this). Because time appears to run slow aboard the spaceship,

\[ f' = \frac{f}{\sqrt{1 - v^2/c^2}}. \]

(7) Light is affected by gravity. On terms of the photons' effective mass \( m_g = \frac{hf}{c^2} \) we can apply Newton's Law

\[ F = \frac{dp}{dt} \quad \text{with} \quad F = -m_g \frac{df}{c^2} = -\frac{hf}{c^2} \quad \text{and} \quad p = \frac{hf}{c} \]

\[ -\frac{hf}{c^2} \frac{df}{c} = \frac{hf}{c} \frac{df}{dt} = \frac{hf}{c} \frac{df}{dz}, \quad \text{because} \quad dz = c \, dt. \]

\[ \frac{df}{dz} = -f \frac{g}{c^2} \quad \Rightarrow \quad \Delta f = -f \int \frac{g \, dz}{c^2} \]

Propagation upward \((\Delta z > 0)\) \(\Rightarrow\) \(\Delta f < 0\) , redshift.

(b) \( \lambda' = \lambda + \frac{h}{m_0 c} (1 - \cos 180^\circ) \)

\[ = 100 \frac{h}{m_0 c} + 2 \frac{h}{m_0 c} = \boxed{102 \frac{h}{m_0 c}} \]

(c) \( KE_e = \hbar (f - f') = \hbar c \left( \frac{1}{\lambda} - \frac{1}{\lambda'} \right) = m_0 c^2 \left( \frac{1}{100} - \frac{1}{102} \right) \)

\[ = m_0 c^2 \left( \frac{2}{10200} \right) = \boxed{1.96 \times 10^{-4} m_0 c^2} \]
9a) \[ 2hf = 2 \frac{m_0c^2}{\sqrt{1-v^2/c^2}} = \text{Total energy before/after} \]

\[ \Rightarrow f = \frac{m_0c^2}{h} \cdot \frac{1}{\sqrt{1-v^2/c^2}}. \]

(b) Here we use momentum conservation:

\[ \frac{2m_0V}{\sqrt{1-v^2/c^2}} = \frac{2hf}{c} \cos \Theta = \text{Total momentum} \]

before/after

Using result of part (a) for \( 2hf \),

\[ \frac{2m_0V}{\sqrt{1-v^2/c^2}} = \frac{2m_0c}{\sqrt{1-v^2/c^2}} \cos \Theta \]

\[ \cos \Theta = \frac{V}{c} \]

(note \( V \rightarrow 0 \Rightarrow \Theta \rightarrow \frac{\pi}{2} \), \( V \rightarrow c \Rightarrow \Theta \rightarrow 0 \))