Physics 17: Introductory Physics III, Winter 2003

Problem set 4 due 4/25 (from Eisberg and Resnick):
Ch 4: 7,13,16,19,23,25

Hint on 16: the dipole moment is defined as \( \mu = IA \) where \( I \) is the current associated with the orbiting electron (that is, \( I = e/T \) where \( T \) is the orbital period) and \( A \) is the area of the orbit.

Extra problem: First verify that the energy and DeBoiglie relations give

\[
\omega = \sqrt{m_0^2c^4/\hbar^2 + c^2k^2}
\]

and show that this result in the non-relativistic limit gives

\[
\omega = m_0c^2/\hbar + \hbar k^2/(2m_0) + \ldots
\]

In what region of \( k \) is this valid?

Next, substitute the expanded form of \( \omega \) into the formula

\[
\psi(x,t) = \int_{-\infty}^{\infty} A(k) e^{i(kx - \omega t)} dk \quad \text{with} \quad A(k) = A_0 \exp[-(k - k_0)^2/(4\Delta k^2)]
\]

where \( k_0 \) and \( \Delta k \) are constants. Show that the breakdown of the expansion of \( \omega \) at large \( k \) in the integral will negligible if \( k_0 \) and \( \Delta k \) are both sufficiently small. Then, carry out the integral with the formula

\[
\int_{-\infty}^{\infty} e^{-ak^2+bk} dk = \sqrt{\pi/a} e^{b^2/(4a)}
\]

to show that

\[
\psi = A_0 \sqrt{4\pi\Delta k^2f(t)} \exp[-im_0c^2t/\hbar - k_0^2/(4\Delta k^2) - (\Delta k x - ik_0/(2\Delta k))^2f(t)]
\]

where \( f = 1/(1 + it/\tau) \), \( \tau = m_0/(2\hbar \Delta k^2) \). Then, show this gives the probability density:

\[
|\psi|^2 = \psi \psi^* = |A_0|^2(2\pi\Delta k/\Delta x(t)) \exp[-(x - v_0 t)^2/(2\Delta x(t)^2)]
\]

where \( v_0 = p_0/m_0 = \hbar k_0/m_0 \) and \( \Delta x(t) = (1 + t^2/\tau^2)^{1/2}/(2\Delta k) \). Since \( \Delta x(0) = 1/(2\Delta k) \), \( \Delta x(t) \) can also be written as

\[
\Delta x(t) = (1 + t^2/\tau^2)^{1/2} \Delta x(0)
\]

with \( \tau = 2m_0(\Delta x(0))^2/\hbar \). Qualitatively describe the main features of \( |\psi|^2 \) as a function of \( x \) and \( t \). What does this imply physically? Given the expression for \( \Delta x(t) \), calculate the spreading time \( \tau \) for an electron as a function of \( \Delta x(0) \). If \( \tau = 1 \text{ sec} \), what is \( \Delta x(0) \)? If instead of an electron one assumes \( m_0 = 1 \text{ kg} \) and \( \Delta x(0) = 10^{-10} \text{ m} \) (the scale of a typical atom), what is \( \tau \)?

Note: Problem sets due at 11:15 am in box near Wilder main entrance