In frame $F$, where $\sigma$ is stationary, $B = 0$ and $E = 2\pi \sigma$.

$$E_{11} = \frac{2\pi \sigma}{\sqrt{2}} \text{ or } E_x = \pm \frac{2\pi \sigma}{\sqrt{2}}$$

$$E_{1} = \frac{2\pi \sigma}{\sqrt{2}} \text{ or } E_z = \pm \frac{2\pi \sigma}{\sqrt{2}}$$

In frame $F'$ moving in the $x$ direction with velocity $0.6 c$ with respect to $F$, the length in $x$ is contracted by the factor

$$y' = \left(1 - 0.6^2\right)^{1/2} = 1.25$$

$$\sigma' = \frac{\Delta \sigma}{A'}$$ where $A$ & $A'$ is the area of the sheet in Frame $F$ & $F'$

$$\sigma' = \frac{E_y \left(l_x^2 + l_z^2\right)^{1/2}}{E_y' \left(l_x'^2 + l_z'^2\right)^{1/2}}$$

Now $E_y' = E_y$, $l_z' = l_z$, and $l_x' = \frac{1}{y} l x$. Also $l_z = 2 b$.

$$\therefore \sigma' = \frac{E_y \sqrt{2} l_x}{E_y l x \left(\frac{1}{y^2} + 1\right)^{1/2}} = \left(\frac{2}{y^2 + 1}\right)^{1/2} \sigma$$

$$E_{11}' = E_{11} = \frac{2\pi \sigma}{\sqrt{2}}$$

$$E_{1}' = y E_{1} = 1.25 E_{1}$$

$$E' = \left(E_{11}'^2 + E_{1}'^2\right)^{1/2} = E_{11} \left(1 + (1.25^2)^{1/2}\right) = E \left(1 + 1.25^2\right)^{1/2}$$
\[ E' = 1.13E \]

\[ \tan \theta' = \frac{h_e'}{l_{e'}} = \frac{h_e}{l_e} = \gamma \quad \text{or} \quad \theta' = 51.34° \]

Angle that electric field \( E' \) makes with surface normal of surface charge density \( \sigma' \) is given by:

\[ 2 \times \theta' - 90° = 12.68° \]

\[ 5.9 \text{ cm} \quad \Delta \varphi = 6000 \text{ V} \]

\[ E_{\text{rel}} = \gamma mc^2 = mc^2 + \text{kinetic energy} = 500 \text{ keV} + 250 \text{ keV} = 750 \text{ keV} \]

\[ \gamma = \frac{750}{500} = 1.5 \]

\[ \beta = \left( 1 - \frac{1}{\gamma^2} \right)^{1/2} = 0.745 \]

\[ p_x = m \gamma u_x = \gamma mc \frac{u_x}{c} = \gamma \beta m c = 1.118 mc \]

Time spent between plates \( t = \frac{4 \text{ cm}}{u_x} = \frac{4 \text{ cm}}{\beta c} = \frac{4}{0.745 \times 3 \times 10^9} \]

\[ = 1.79 \times 10^{-9} \text{ sec} \]

\[ p_y = eEt = eVt \]

\[ \frac{p_y}{mc} = \frac{eVtc}{mc^2} = \frac{eVt}{mc^2} \frac{tc}{d} \]

\[ = \frac{6000 \times 1.79 \times 10^{-9} \times 3 \times 10^9}{5 \times 10^5 \times 0.8} \]

\[ = 0.0806 \]
Py = 0.0806 m·c = \gamma m_0 u_y

u_y = \frac{0.0806 \text{c}}{\gamma} = \frac{1.612 \times 10^9 \text{ cm/sec}}{1.118}

y = \frac{x t^2}{2} = \frac{x t^2}{2} = \frac{u_y t^2}{2} = \frac{(1.612 \times 10^9)(1.39 \times 10^{-10})}{2}

= 0.144 \text{ cm}

\hline

\hline

\hline

\hline

In a frame where the electron is at rest the plates are moving with speed \( \beta c = 0.745 \text{c} \), and the length of the plates contracted by \( \gamma \).

\[ l' = \frac{4 \text{ cm}}{\gamma} = 2.67 \text{ cm} \]

The time during which the electron is subjected to the electric field from the plates is

\[ t' = \frac{l'}{\beta c} = \frac{2.67}{0.745 \text{c}} \]

The electric field is \( E' = \gamma E \)

The vertical momentum acquired is

\[ eE't' = e\gamma E \cdot \frac{l}{\beta c} = e\gamma E t' \], which is

\[ \frac{\gamma}{\beta c} = 0.0806 \text{ m·c} \]

the same as in the lab frame.

In this frame the electron is non-relativistic \( E \)

\[ u_y' = 0.0806 \text{c} \]

\[ y' = \frac{u_y't'}{2} = \frac{0.0806 \text{c} \times 2.67}{2 \times 0.745 \text{c}} = 0.144 \text{ cm} \]
Frame $F'$ moves at velocity $V_0$ relative to frame $F$.

Line charge density of positive ions is given by $\lambda = \frac{\gamma \lambda_0}{\beta^2}$ in frame $F'$, where $\gamma = \left(1 - \beta^2\right)^{-1/2}$, $\beta = \frac{v}{c}$ in frame $F'$.

Line charge density of electrons in $F'$ is given by $-\lambda_0$.

\[
E = \frac{Q}{r^2} \frac{1 - \beta^2}{\left(1 - \beta^2 \sin^2 \theta\right)^{3/2}} \beta c \rightarrow \beta c
\]

For the two protons moving parallel to one another a distance $r$ apart, with velocity $\beta c$ in lab frame, $\theta = 0$.

\[
E = \frac{e (1 - \beta^2)}{r^2} = \frac{1}{\beta^2} \frac{e}{r^2}
\]

Consider proton rest frame $F'$ which moves at velocity $\beta c$ relative to $E' = \frac{e}{\beta c}$, electric field due to proton at rest frame.

\[
F_{ii}' = \frac{e^2}{\beta c^2}, \quad \text{force on proton measured in rest frame}
\]

\[
F_{ii} = F_{ii}' = \frac{e^2}{r'^2} \frac{1}{\beta^2}
\]

Now $r' = \sqrt{r^2}$.

\[
F_{ii} = \frac{e^2}{(\sqrt{r^2})^2} = \frac{e^2}{r^2} = e E_{ii}, \quad \text{where} \quad E_{ii} = E_{ii}' = \frac{e}{\beta^2 + 1}
\]
Consider another situation where

\[ E = \frac{e}{\gamma} \frac{1 - \beta^2}{\gamma^2 (1 - \beta^2 \sin^2 \theta)^{3/2}} = \frac{e}{\gamma} \frac{1 - \beta^2}{\gamma^2}, \text{ for } \theta = \gamma/2 \]

Consider proton rest frame \( F' \)

\[ E_i' = \frac{1}{\gamma} E_i = \frac{e}{\gamma} = \frac{e}{\gamma} \frac{1}{\gamma^2} \]

\[ F_i' = \frac{e^2}{\gamma^1} \]

\[ F_e = \frac{1}{\gamma} F_e' = \frac{1}{\gamma} \frac{e^2}{\gamma^1} = \frac{1}{\gamma} \frac{e^2}{\gamma^2} = e E_i + e \beta B \]

Find \( B \) that satisfies above eg.

\[ \beta = \left( \frac{1}{\gamma} \frac{e^2}{\gamma^2} - \frac{e^2 \gamma^2}{\gamma^2} \right) \frac{1}{e \beta} \]

\[ = \frac{1 - \gamma^2}{\gamma} \frac{e^2}{\gamma^2} \frac{1}{e \beta} \]

\[ \gamma^2 = (1 - \beta^2)^{-1} \]

\[ = \frac{1 - \gamma^2}{\gamma} \frac{e^2}{\gamma^2} \frac{1}{e \beta} \]

\[ = \frac{-\beta^2 \gamma^2}{\gamma} \frac{e^2}{\gamma^2} \frac{1}{e \beta} \]

\[ = \frac{-\beta^2 e}{\gamma^2} \frac{1}{e \beta} \]

\[ = -\beta E \]

5.18

\[ \rightarrow \beta v c \]

\[ \lambda \]

\[ F \]

\[ \leftarrow -\beta v c \]

\[ \beta v' c \]

\[ \lambda \]

\[ F' \]

\[ \beta v' c = \frac{\beta v c + \beta 0}{1 + \beta v c \beta c / c^2} = \frac{(\beta v + \beta) c}{1 + \beta v \beta} \]
\[ \lambda_{n'} = \frac{\lambda_n}{\gamma_{n'}} \]

Now \[ \gamma_{n'} = \frac{1}{\sqrt{1 - \beta_{n}^2}} = \frac{1}{\sqrt{\frac{1 - (\beta_{n} + \beta)}{1 + \beta_{n} \beta}}} \]

\[ = \left( \frac{1 + \beta_{n} \beta}{1 - \beta_{n}^2} \right) \]

\[ = \left( \frac{1 + \beta_{n} \beta}{1 - \beta_{n}^2} \right)^2 \]

\[ = \frac{(1 + \beta_{n} \beta)^2}{(1 - \beta_{n}^2)(1 - \beta^2)} \]

\[ \therefore \gamma_{n'} = \gamma_{n} \gamma \left( 1 + \beta_{n} \beta \right) \]

\[ \lambda_{n'} = \lambda_n \gamma \left( 1 + \beta_{n} \beta \right) \]

\[ = \gamma \left( \lambda_n + \beta \lambda \right) \]

\[ I_{n'} = \frac{\lambda_n' \beta n' c}{c} = \gamma \left( \lambda_n + \frac{\beta \lambda_n c}{c} \right) \left( \beta_{n} + \beta \right) \]

\[ = \gamma \left( \frac{\lambda_n \beta_{n} c + \lambda_n \beta c}{c} \right) \left( 1 + \beta \right) \]

\[ = \gamma \left( I_n + \beta \left( \lambda_n \right) \right) \]

6.3 \[ B_{z} = \frac{2 \pi b^2 I}{c (b^2 + 2^2)^{3/2}} \]
\[ S = \int_{-\infty}^{\infty} \frac{2\pi b^2 I}{c (b^2 + r^2)^{3/2}} \, dr = \frac{2}{b^2 (b^2 + r^2)^{1/2}} \bigg|_{-\infty}^{\infty} = \frac{2 \pi I}{c (b^2 + r^2)^{1/2}} \bigg|_{-\infty}^{\infty} = \frac{4 \pi I}{c} \]

We can ignore the return path, since \( B \sim \frac{1}{r^2} \) as \( r \to \infty \), and the length of the semicircle for the return path \( \alpha r \). \( S = \int B \cdot ds \) over the semicircle for the return path \( \sim \frac{1}{r^2} \) and as \( r \to \infty \) it goes to zero.
The contribution to $\vec{B}$ from all segments points out of the plane. Also, contribution from lower wire is equal to contribution from upper wire.

$R \sin \theta = r$ or $R = \frac{r}{\sin \theta}$

$d\ell \sin \theta = R d\theta$ or $d\ell = \frac{R d\theta}{\sin \theta}$

$dB = \frac{I d\ell \times \hat{r}}{cr^2}$ or $dB = \frac{Id\ell \sin \theta}{cr^2} = \frac{IRd\theta}{cr^2} = \frac{I \sin \theta d\theta}{cr}$

$B = 2 \int_0^{\pi/2} \frac{I \sin \theta d\theta}{cr} + \int \frac{Id\ell}{cr^2}$

contribution from straight wire

contribution from semicircle

$= 2I \left(-\cos \theta\right) \bigg|_0^{\pi/2} + \frac{I \cdot \pi r}{cr^2}$

$= \frac{2I + \pi I}{cr} \frac{1}{cr}$

$= \frac{(2\pi + \pi)I}{cr}$

$= \frac{(2\pi + \pi)I}{cr}$

Apply same approach as in problem

$(x, y)$

$dB_z = \frac{I \sin \theta d\theta}{cr}$

$B_z = \int_0^{\pi - \theta_2} \frac{I \sin \theta d\theta}{cx} + \int_{\theta_2}^{\pi} \frac{I \sin \theta d\theta}{cy}$
\[ B_{x} = \frac{I}{cx} \left( -\cos \theta \right) + \frac{I}{cy} \left( -\cos \theta \right) \]

\[ = \frac{I}{cx} (1 + \cos \theta) + \frac{I}{cy} (1 + \cos \theta) \]

\[ = \frac{I}{cx} \left( 1 - \frac{x}{\sqrt{x^{2} + y^{2}}} \right) + \frac{I}{cy} \left( 1 + \frac{y}{\sqrt{x^{2} + y^{2}}} \right) \]

6.14

\[ \vec{B} \text{ cannot have } z \text{ components, since it has mirror symmetry about } \hat{z} = 0 \text{ plane, if we set } z = 0 \text{ to be the midpoint of the height of the torus. Due to the donut shape, the only field direction in the } xy \text{ plane that follows that symmetry is a } \vec{B} \text{ along the circumferential direction.} \]

Assume there is a field outside the torus. Using symmetry argument \( \vec{B} = B \hat{r} \). Apply Ampere's law

\[ \oint \vec{B} \cdot d\vec{L} = \frac{4\pi}{c} I_{\text{enclosed}} \]

\[ B \cdot 2\pi r = 0 \quad (\text{since some amount of current in direction } +\hat{z} (\text{ or } -\hat{z}) \]

\[ \therefore \vec{B} = 0 \text{ outside of torus.} \]

Inside torus, so \( \vec{I} \) is encircled, and by applying circumferential symmetry \( \vec{B} = 0 \) inside of torus.

\[ \oint \vec{B} \cdot d\vec{L} = \frac{4\pi}{c} I_{\text{enclosed}} \]

\[ B \cdot 2\pi r = \frac{4\pi}{c} NI \quad \text{or} \quad B(\hat{r}) = 2NI \]
The only region where $E \neq 0$ is between the two cylinders.

$$E(r) = \frac{2\lambda}{r}, \quad a < r < b$$

$$\Phi(r) = -2\lambda \ln r + \text{const}, \quad a < r < b$$

$$\Phi(a) - \Phi(b) = -2\lambda \ln \frac{a}{b}$$

$$\lambda = \frac{\Phi(a) - \Phi(b)}{2\ln \frac{a}{b}} \quad \text{statvolts} \times \frac{2\pi}{\text{cm}} = 87 \text{ cm} \times \text{cm}$$

$$J = \pi \sigma \nu = \pi \nu \sigma$$ where $\nu$ is vol. charge density

$$J = \pi \sigma \nu = \sigma \nu$$ where $\sigma$ is surface charge density

When inner cylinder is rotating at a speed of 30 revolutions per second we can consider it as a surface current density of $J = \sigma \nu = \frac{\lambda}{2\pi a f} = \frac{\lambda f}{2\pi a}$

The resulting B field is

$$B = \frac{4\pi I}{c}, \quad r < a$$

$$B = 0, \quad r > a$$

$$B = 0$$ where $\nu$ is the # of turns/unit length.

$$I_n = J$$

$$B = \frac{4\pi I}{c} = \frac{4\pi \lambda f}{c} = \frac{4\pi (87)(30)}{3 \times 10^{10}} = 1.09 \times 10^{-6} \text{ Gauss}$$

out of paper
If both cylinders are rotated in the same direction at 30 revolution/sec, use superposition principle.

\[ B = 4\pi r G, \quad a < r < b \]

\[ B = \frac{4\pi r^2 n}{c} = \frac{9.5 \times 10^{-3}}{c} \quad B = 0 \quad r < a \]

\[ r > b \]

\[ G \]