4.4 (a) \( \rho = 3 \times 10^{-6} \text{ ohm-cm} \)

\[
R = \frac{1}{t} \frac{\rho}{7} \text{ resistance in parallel}
\]

\[
R = \frac{\rho L}{A} = \frac{(3 \times 10^{-6} \text{ ohm-cm})(3 \times 10^8 \text{ cm})}{12 \left( \frac{3.3 \times 10^{-2} \text{ cm}}{2} \right)^2}
\]

\[= 2.2 \times 10^5 \text{ ohm} \]

\( R_{\text{tot}} = 3.1 \times 10^4 \text{ ohm} \)

(b) At the central part of the ocean, the current can spread out over a cross section of \( 10^3 \text{ km} \) wide and 1 km deep. Assume a \( 10^3 \text{ km} \) long path.

\[
R = \frac{\rho L}{R} = \frac{(25 \text{ ohm-cm})(10^8 \text{ cm})}{10^8 \text{ cm} \times 10^5 \text{ cm}}
\]

\[= 2.5 \times 10^{-4} \text{ ohm} \text{ negligible}
\]

The resistance of the return path is determined by the region where the current funnels in. If \( a \) is a typical dimension of the electrode, through which current funnels in,

\[
R = \frac{\rho a}{2} = \frac{\rho}{a^2}
\]

If \( a \approx 1 \text{ m} \), then \( R = 0.25 \text{ ohm} \).
4.5

\[ J = \frac{\sigma_1}{E_1} + \frac{\sigma_2}{E_2} \]

\[ I = JA \]

\[ J = \sigma_1 E_1 = \sigma_2 E_2 \]

\[ \therefore \frac{E_2}{E_1} = \frac{\sigma_1}{\sigma_2} \text{ or } E_2 = \frac{\sigma_1}{\sigma_2} E_1 \]

Imagine a Gauss surface enclosing the intersection between the two different materials.

\[ \int_{S} \mathbf{E} \cdot d\mathbf{a} = (E_2 - E_1) A = 4\pi \sigma A, \text{ where } A \text{ is the cross sectional area of the intersection and } \sigma \text{ is the surface charge density.} \]

\[ Q = \sigma A = \frac{1}{4\pi} (E_2 - E_1) A \]

\[ = \frac{1}{4\pi} \left( \frac{\sigma_1}{\sigma_2} - 1 \right) E_1 A \]

\[ = \frac{1}{4\pi} \left( \frac{\sigma_1}{\sigma_2} - 1 \right) \frac{JA}{\sigma_1} \]

\[ = \frac{1}{4\pi} \left( \frac{1}{\sigma_2} - \frac{1}{\sigma_1} \right) I \]

4.7

\[ \sigma_{\text{Ag}} = 7.2 \ \text{ for Ag} \]

\[ \sigma_{\text{Au}} = \frac{1}{2} \ \text{ for Au} \]

\[ \sigma_{\text{Ag}} = 7.2 \ \text{Ag} \]

\[ \sigma_{\text{Au}} = \frac{1}{2} \ \text{Au} \]
For vertical currents the layers are in series and
resistivity adds
\[ R_1 = \frac{1}{3} \sigma_{Ag} + \frac{2}{3} \sigma_{Sn} = \frac{1}{3} \sigma_{Ag} + \frac{2}{3} \times 7.2 \sigma_{Ag} \]
\[ = 5.1 \sigma_{Ag} \quad \text{or} \quad \sigma_1 = 0.2 \sigma_{Ag} \]

For parallel currents, the layers are in parallel
and conductivity add
\[ \sigma_{||} = \frac{1}{3} \sigma_{Ag} + \frac{2}{3} \sigma_{Sn} = \frac{1}{3} \sigma_{Ag} + \frac{2}{3} \times \frac{1}{7.2} \sigma_{Ag} \]
\[ = 0.43 \sigma_{Ag} \]

\[ \sigma_1 / \sigma_{||} = \frac{0.2 \sigma_{Ag}}{0.43 \sigma_{Ag}} = 0.46 \]

4.18

\[ E \]

\[ I = \frac{E}{R + R_i} \]

\[ P = I^2 R \]

\[ = \frac{E^2 R}{(R_i + R)^2} \]

\[ dP = \frac{E^2}{(R_i + R)^2} - \frac{2E^2 R}{(R_i + R)^3} \]

\[ = \frac{E^2 (R_i + R - 2R)}{(R_i + R)^3} \]

\[ = 0 \] \quad \text{when} \quad R = R_i

To check if it is a maximum,
\[ \frac{d^2p}{dR^2} = -\frac{e^2}{(R_i+R)^3} - \frac{3e^2(R_i-R)}{(R_i+R)^4} \]

\[ = -\frac{e^2(R_i+R+3R_i-3R)}{(R_i+R)^4} \]

\[ = -\frac{e^2(4R_i-2R)}{(R_i+R)^4} \]

\[ \frac{d^2p}{dR^2} \bigg|_{R=R_i} < 0 \quad \therefore \quad p \text{ is a maximum at } R = R_i \]

**Y-19**

\[ A \quad R_A = 20 \Omega \]

\[ B \]

\[ C \]

\[ E = 1.5V \]

\[ R_2 \]

The voltage drop between C and B:

\[ V_{CB} = I_AR_A = (I-I_A)R_i \]

or

\[ I_A(R_A+R_i) = IR_i \]

\[ \therefore \quad I_A = \frac{R_i}{RA+R_i} \]

From the condition that adding a 15Ω resistance R, I_A drops from 50mA to 25mA, we know that I is halved by the introduction of 15Ω. This happens if the total resistance of the circuit is of order 15Ω before the addition of R.

For \( R = 0 \),

\[ I = \frac{1.5V}{15\Omega} = 0.1A \Rightarrow 50mA \]
Most of the current coming to point C in the circuit flows through \( R_4 \)

\[ R_4 \ll R_A = 20 \Omega \]

In the limit \( R_1 \ll R_A \)

\[ I_a = \frac{R_1}{R_A} I \]

and

\[ I = \frac{\varepsilon}{R_2 + R} \]

For \( R = 0 \)

\[ I_a = \frac{R_1}{R_A} \cdot \frac{\varepsilon}{R_2} = 50 \mu A \]

For \( R = 15 \Omega \)

\[ I_a = \frac{R_1}{R_A} \cdot \frac{\varepsilon}{R_2 + 15} = 25 \mu A \]

\[ R_2 = 15 \Omega \]

\[ R_1 = \frac{50 \mu A}{(1.5V/15\Omega)} \times R_A = 10^{-2} \Omega \]

For \( R = 5 \Omega \)

\[ I_a = \frac{10^{-2} \Omega \cdot 1.5V}{20 \Omega \cdot (15+5) \Omega} = 3.75 \times 10^{-5} A = 37.5 \mu A \]

For \( R = 50 \Omega \)

\[ I_a = \frac{10^{-2} \Omega \cdot 1.5V}{20 \Omega \cdot (15+50) \Omega} = 1.15 \times 10^{-5} A = 11.5 \mu A \]

\[ R_{ab} = R_1 + R_2 = 10 + 20 = 30 \Omega \]

\[ R_{ac} = R_1 + R_3 = 10 + 50 = 60 \Omega \]

\[ R_{bc} = R_2 + R_3 = 20 + 50 = 70 \Omega \]
\[(R_{ab})^{-1} = \frac{1}{R_1 + \frac{1}{R_2 + R_3}} = \frac{R_1 + R_2 + R_3}{R_1 \cdot (R_2 + R_3)}\]

\[R_{ab} = \frac{R_1 \cdot (R_2 + R_3)}{R_1 + R_2 + R_3}\]

\[R_{ac} = \frac{R_2 \cdot (R_1 + R_3)}{R_1 + R_2 + R_3} = \frac{85 \times (34 + 170)}{34 + 85 + 170} = 60 \Omega\]

\[R_{bc} = \frac{R_3 \cdot (R_1 + R_2)}{R_1 + R_2 + R_3} = 70 \Omega\]

With 3 resistors, for each R configuration (I or II), there is a unique solution of R1, R2, R3 when R_{ab}, R_{bc}, R_{ac} is specified. There is no other possibility. Furthermore, there is no measurement that distinguishes between the two configurations.

\[E \quad \frac{\text{Open circuit voltage is given by } V_1 - V_2 \text{, when A & B is open.}}{R} \]

\[V_1 \quad \frac{R}{V_2} \]
Voltage drop by going from $V_1$ to $V_2$, and from $V_2$ to $V_1$

(a) \[ V_1 - IR + \varepsilon - IR = V_2 \quad \ldots \quad (a) \]

(b) \[ V_2 - IR + \varepsilon - IR + \varepsilon - IR = V_1 \quad \ldots \quad (b) \]

(a) \[ V_1 - V_2 = 2IR - \varepsilon \]

(b) \[ V_1 - V_2 = 2\varepsilon - 3IR \]

\[ 2IR - \varepsilon = 2\varepsilon - 3IR \quad \text{or} \quad 3\varepsilon = 5IR \]

\[ I = \frac{3\varepsilon}{5R} = \frac{4.5V}{500\Omega} = 9 \times 10^{-3} \text{ A} \]

\[ V_1 - V_2 = 2IR - \varepsilon = \frac{6\varepsilon}{5} - \varepsilon = \frac{1}{5} \varepsilon = 0.3V \]

\[ E_{eq} \text{ Open circuit voltage is 0.3 V} \]

From the circuit, we know that

\[ \frac{1}{R_{eq}} = \frac{1}{R+R} + \frac{1}{R+R+R} = \frac{1}{2R} + \frac{1}{3R} = \frac{5}{6R} \]

\[ \therefore R_{eq} = \frac{6R}{5} = 120 \Omega \]

We can get $R_{eq}$ by calculating the short circuit current.

\[ V_8 + 2\varepsilon - 3I_1R = V_A \]

or \[ V_8 - V_A = 0 = 3I_1R - 2\varepsilon \]

\[ I_1 = \frac{2\varepsilon}{3R} \]

Short circuit \[ \Rightarrow V_A = V_8 \]

\[ \frac{3V}{300\Omega} = 10^{-2} \text{ A} \]
\[ V_A + \varepsilon - 2I_z R = V_B \]
\[ \text{or } V_A - V_B = 0 = 2I_z R - \varepsilon \]
\[ I_z = \frac{\varepsilon}{2R} = \frac{1.5V}{200\Omega} = 7.5 \times 10^{-3} A \]
\[ \text{. Short circuit current } = I_1 - I_2 = 2.5 mA \]
\[ \text{Reg} = \frac{E_0}{2.5mA} = \frac{0.3V}{2.5mV} = 120 \Omega \]
\[ \varepsilon \]
\[ 0.3V \]
\[ \text{4.25 } I = \frac{V_o}{R} e^{-t/RC} \]
\[ I^2 R = \frac{V_o^2}{R} e^{-2t/RC} \]
\[ \int_0^\infty I^2 R \, dt = \frac{V_o^2}{R} \left[ -\frac{e^{-t/RC}}{-RC} \right]_0^\infty \]
\[ = \frac{V_o^2}{R} \left[ \frac{-e^{-t/RC}}{RC} \right]_0^\infty \]
\[ = \frac{CV_o^2}{2C} \]
\[ Q = CV_o e^{-t/RC} \]

Assume a \text{ LiF} capacitor charged to 1V at \( t = 0 \)
\[ Q = CV = 10^{-6} C \]
Charge of electron is \( 1.6 \times 10^{-19} C \).
\[
\frac{Q}{Q_0} = 1.6 \times 10^{-19} = e^{-\frac{t}{RC}}
\]

Assume \( R = 10^6 \Omega \)

\[
RC = 10^6 \times 10^{-6} = 15
\]

\[
\ln \left( \frac{Q}{Q_0} \right) = -\frac{t}{RC}
\]

or \( t = -RC \ln \left( \frac{Q}{Q_0} \right) = -1 \times \ln \left( 1.6 \times 10^{-13} \right) = 29.5 \)

It takes 30 sec to discharge the capacitor.