First assume that \( q_d \) is very far away, and therefore we can ignore its influence on surface charge redistribution of the conductor \( A \).

On the surfaces of cavities containing charge \( -q_b \) \& \( q_c \), there will be induced a uniform surface charge totaling \(-2q_b - q_c\). This happens such that the charges \( q_b \) \& \( q_c \) are shielded inside the electric field from conductor \( A \). Therefore, the presence of the two cavities will not be noticed outside of them.

Since conductor \( A \) is neutral (total \( Q = 0 \)), a uniform surface charge density with total charge of \( q_b + q_c \) is induced in the outer surface of the conductor.

The force on charge \( q_b \) \& \( q_c \) is zero, since a uniform charge density of spherical symmetry yields \( E = 0 \) inside the sphere.

The uniform surface charge density of the outer surface of conductor \( A \) gives rise to an electric field of \( \vec{E} = \frac{q_b + q_c}{4\pi\varepsilon_0 r^2} \) outside the conductor, as if \( q_b \) \& \( q_c \) were at the center of the spherical conductor.
The force on charge $Q$ is therefore

$$ F = \frac{Q}{d} \cdot \frac{B_0 + B_c}{r^2} $$

Due to Newton's third law, the force on conductor $A$ is equal in magnitude and opposite in direction.

3.3

We determine $R$, so that half of $-Q$, the induced charge on the plane, is contained within the circle of radius $R$.

$$ \frac{-Q}{2} = \int_0^R 0.2\pi r dr $$

$$ 0 = \frac{-Qh}{2\pi (r^2 + h^2)^{3/2}} $$

$$ = -Qh \left( \frac{r}{(r^2 + h^2)^{3/2}} \right) |_0^R $$

$$ = -Qh \frac{1}{2} \left( -2 \cdot \left( \frac{1}{(r^2 + h^2)^{1/2}} \right) \right) $$

$$ = +Qh \left( \frac{1}{\sqrt{R^2 + h^2}} - \frac{1}{h} \right) $$

or

$$ \frac{1}{r} = \frac{h}{\sqrt{R^2 + h^2}} $$

$$ \therefore R = \sqrt{3} \cdot h $$
3.5 \[ W = \frac{Q^2}{2h} \]

- \( Q \) image charge

\[ F_e = -\frac{Q^2}{(2h)^2} \text{ force on charge } Q, \text{ when } Q \text{ is } h \text{ above plane. It is perpendicular to plane.} \]

\[ W = -\int_{h_0}^{\infty} F_e \, dz = \int_{h_0}^{\infty} \frac{Q^2}{(2h)^2} \, dh = \frac{Q^2}{4} \left( 1 - \frac{h_0}{h} \right) \]

\[ = \frac{Q^2}{4h_0} \]

The second student has the right answer. The first answer assumes that we do work on both charge \( Q \) and \( -Q \) to pull them apart. However, \( -Q \) is an image charge. Therefore, the total work done to move charge \( Q \) to \( \infty \) is \( \frac{1}{2} \frac{Q^2}{2h} \).

3.7 \( \Phi_A \)

\[ \oint \mathbf{E} \cdot d\mathbf{s} = 0 \text{ for a closed loop, or in other words, we should reach the same electric potential.} \]

\[ \text{When we go around a loop, } C_6 \text{ we start from pt } A, \text{ held at } \Phi_A, \text{ and go around the square loop CCW, there is a potential drop from B to C,} \]
and a potential drop from C to A. Therefore, after traversing the loop, we don't arrive at the same electric potential. Therefore, such a configuration is physically impossible.

\[ C = \frac{2\epsilon \epsilon_0}{\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)} \]

\[ \epsilon \approx \frac{b^2}{a^2}, \quad \epsilon \ll 1, \quad \text{and} \quad \ln\left(1+\epsilon\right) \approx \epsilon \]

\[ \therefore \quad C \approx \frac{2\epsilon \epsilon_0}{\epsilon - (-\epsilon)} \quad \text{for} \quad b \gg a \]

Let \( C_0 \) be the capacitance of a sphere of unit radius \( a = b = 1 \), and \( C \) the capacitance of a prolate spheroid of equal volume, i.e., \( ab^2 = 1 \) (or \( b = \frac{1}{\sqrt{a}} \))

\[ \epsilon = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1}{a^3}} \quad \frac{C}{C_0} = \frac{\epsilon b^2}{\ln\left(\frac{1+\epsilon}{1-\epsilon}\right)} \]

\[
\begin{array}{cccccc}
\epsilon & 0.19 & 0.4987 & 0.9354 & 0.9985 & \\
C/C_0 & 1.0000 & 1.0018 & 1.1005 & 2.9102 & \\
\end{array}
\]

<table>
<thead>
<tr>
<th>( a )</th>
<th>1.01</th>
<th>1.1</th>
<th>2</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon )</td>
<td>0.19</td>
<td>0.4987</td>
<td>0.9354</td>
<td>0.9985</td>
</tr>
<tr>
<td>( C/C_0 )</td>
<td>1.0000</td>
<td>1.0018</td>
<td>1.1005</td>
<td>2.9102</td>
</tr>
</tbody>
</table>

Capacitance grows as drop is deformed from sphere to a prolate spheroid.

The energy stored in the electric field is given by \( \frac{Q^2}{2C} \). Therefore, energy will decrease.
\[ P_1 = (0, 0, 2d) \]

\[ \psi_{P_1} = \int_{d}^{0} \frac{\lambda \, dz}{2d - z} \quad u = 2d - z \]

\[ = \int_{d}^{0} -\lambda \, du = -\lambda \ln \left( \frac{d}{3d} \right) = \lambda \ln 3 \]

\[ P_2 = (x, 0, 0), \quad \psi_{P_2} = \int_{-d}^{d} \frac{\lambda \, dz}{\sqrt{x^2 + z^2}} \]

\[ = 2 \int_{0}^{d} \frac{\lambda \, dz}{(x^2 + z^2)^{1/2}} \]

\[ = 2 \lambda \ln \left( x + (x^2 + d^2)^{1/2} \right) \bigg|_{0}^{d} \]

\[ = 2 \lambda \ln \frac{d + (x^2 + d^2)^{1/2}}{x} \]

\[ \psi_{P_1} = \psi_{P_2}, \quad \lambda \ln 3 = 2\lambda \ln \frac{d + (x^2 + d^2)^{1/2}}{x} \]

\[ \Rightarrow \sqrt{3} = \frac{d + (x^2 + d^2)^{1/2}}{x} \]

\[ \therefore x = \sqrt{3}d \]

The distances from \( P_1 \) to the ends of the rod is \( d + 3d = 4d \).

\[ A_1 = (0, 0, d) \]

The sum of the distances from \( P_2 \) to the ends of the rod is \( 2d + 2d = 4d \).

\[ A_2 = (0, 0, -d) \]
Therefore $P_1$ & $P_2$ lie on the ellipse defined by the foci at $(0,0,-d)$ & $(0,0,d)$.

Point $P_3 = \left( -\frac{2d}{2}, 0, d \right)$ lies on the same ellipse.

\[
\left| A_1 P_3 \right| + \left| A_2 P_3 \right| = \sqrt{\left(\frac{3d}{2}\right)^2} + \sqrt{\left(\frac{3d}{2}\right)^2 + (2d)^2}
\]
\[
= \frac{3d}{2} + \frac{5d}{2}
\]
\[
\Rightarrow \left( \frac{3d}{2}, 0, d \right) \quad \text{lies on the same ellipse}
\]

\[
\phi \left( \frac{3d}{2}, 0, d \right) = \int_0^d \frac{\lambda \, du}{\sqrt{\frac{(3d)^2}{4} + u^2}}
\quad \text{with } -u = d - u
\]
\[
= \int_0^d \frac{-\lambda \, du}{2d \sqrt{\frac{(3d)^2}{4} + u^2}}
\]
\[
= \lambda \ln \left( u + \sqrt{\frac{(3d)^2}{4} + u^2} \right) \bigg|_0^{2d}
\]
\[
= \lambda \ln \frac{2d + \frac{5d}{2}}{\frac{3d}{2}}
\]
\[
= \lambda \ln 3 \quad \Rightarrow \text{same potential as } P_1 \text{ & } P_2
\]

Now calculate potential $\phi$ at a general pt. $(x, 0, z)$.

\[
\phi(x, 0, z) = \int_0^d \frac{\lambda \, du}{d \sqrt{x^2 + (2u - z)^2}}
\quad \text{with } u = 2 - u'
\]
\[
= \int_{2-d}^{2+d} \frac{-\lambda \, du}{\sqrt{x^2 + u^2}}
\]
\( \varphi(x, 0, z) = \lambda \ln \left( \frac{x + \sqrt{x^2 + u^2}}{z - d} \right) \)

\[
\begin{align*}
\varphi(x, 0, z) &= \lambda \ln \left( \frac{x + \sqrt{x^2 + (z+d)^2}}{z - d + \sqrt{x^2 + (z-d)^2}} \right) \\
&= \lambda \ln \left( \frac{a(2+d) + a^2 + 2d}{a(z-d) + (a^2 - 2d)} \right) \\
&= \lambda \ln \left( \frac{(a+d)(z+a)}{(a-d)(z+a)} \right) \\
&= \lambda \ln \left( \frac{a+d}{a-d} \right) : \text{indep. of } x, z \\
\therefore \text{ Ellipse is an equipotential surface.}
\end{align*}
\]

Now for \((x, z)\) on an ellipse,

\[
\frac{x^2}{(a^2 - d^2)} + \frac{z^2}{a^2} = 1, \quad \text{where foci is at } z = \pm d, \quad \text{and major axis is } a.
\]

Then,

\[
x^2 = \left(1 - \frac{z^2}{a^2}\right)(a^2 - d^2) = \frac{(a^2 - z^2)(a^2 - d^2)}{a^2}
\]

\[
x^2 + (z+d)^2 = \frac{(a^2 - z^2)(a^2 - d^2) + a^2(z+d)^2}{a^2}
\]

\[
= \frac{(a^2 + 2d)^2}{a^2}
\]

For \(x^2 + (z-d)^2 = \frac{(a^2 - zd)^2}{a^2}\)

\[
\therefore \varphi(x, 0, z) = \lambda \ln \left( \frac{x + (a^2 + zd)}{z - d + \frac{(a^2 + zd)}{a}} \right) \\
= \lambda \ln \left( \frac{a(z+d) + a^2 + 2d}{a(z-d) + (a^2 - 2d)} \right) \\
= \lambda \ln \left( \frac{(a+d)(z+a)}{(a-d)(z+a)} \right) \\
= \lambda \ln \left( \frac{a+d}{a-d} \right) : \text{indep. of } x, z
\]
3.22 We know from 2.11 that a prolate spheroid is an equipotential surface

with \( \phi = 2 \frac{\ln(a+d)}{a-d} \)

Now \( Q = 2 \cdot 2d \): total charge on rod of length 2d, which is equivalent to the total charge on conductor of prolate spheroid shape.

\[ Q = 4d \left( \frac{\ln(a+d)}{a-d} \right) \]

\[ C = 2d \left( \frac{\ln(a+d)}{a-d} \right)^{-1} \]

In an ellipse \( b^2 = a^2 - d^2 \), \( b \) minor axis, \( a \) major axis, \( d \) foci

\[ d = \sqrt{a^2 - b^2} = a \sqrt{1 - \frac{b^2}{a^2}} = aE, \text{ where } E = \sqrt{1 - \frac{b^2}{a^2}} \]

\[ C = \frac{2aE}{\ln\left(\frac{a+ae}{a-ae}\right)} = \frac{2aE}{\ln\left(\frac{1+e}{1-e}\right)} \]

3.24 If a point charge is located between the plates \( y=0, h \) \( j = 5 \), at \( y=b \), the total surface charge on the inner surface of both plates is \( -Q \)
This is due to the fact that \( E = 0 \) inside the 2 conductors, and when we do a surface integral \( \int E \cdot d\sigma = 0 \) for total charge enclosed.

\[ \text{Gauss surface } \quad \text{total charge enclosed} = 0 \]

\[ \Rightarrow \text{surface charge on both conductors} = -Q. \]

Since the total surface charge induced at \( y=0 \) \( y=a \) by any number of charges is independent of their position on plane, imagine the charge \( Q \) having a uniform surface distribution \( \sigma = \frac{Q}{A} \). The total surface charge induced on inner surfaces of both plates is still \( -Q \). For this uniform charge distribution, we can easily calculate the electric field between the plates.

\[ E_1 = \frac{Q_2 - Q_1}{b}, \quad E_2 = \frac{Q_2 - Q_1}{a} \]

Electrical potential at left \( \phi_i \) right plate is the same \( \phi_f \).

\[ \sigma_1 + \sigma_2 = -\sigma \text{, where } \sigma_1 \text{ is surface charge density on left plate } \& \sigma_2 \text{ is surface charge density on right plate.} \]
\[
\begin{align*}
\sigma_1 &= \frac{E_1}{4\pi} = \frac{Q_2 - Q_1}{4\pi b} \quad \sigma_2 &= \frac{E_2}{4\pi} = \frac{Q_2 - Q_1}{4\pi(s-b)} \\
\frac{Q_1}{Q_2} &= \frac{\sigma_1}{\sigma_2} = \frac{s-b}{b} \\
Q_1 + Q_2 &= -Q \\
\left(\frac{s-b}{b} + 1\right) Q_2 &= -Q \\
\therefore \quad Q_2 &= -\frac{b Q}{s} \\
Q_1 &= -\left(\frac{s-b}{s}\right) Q
\end{align*}
\]