Let's calculate the total force on a point with charge $q$ sitting at the upper left corner:

\[
\mathbf{F}_{\text{total}} = \frac{q^2}{r^2} \left( \frac{1}{\sqrt{2}} \mathbf{j} - \frac{1}{\sqrt{2}} \mathbf{x} \right) - \frac{8q^2}{a^2 \sqrt{2}} \left( \frac{1}{\sqrt{2}} \mathbf{j} - \frac{1}{\sqrt{2}} \mathbf{x} \right)
\]

\[
\mathbf{F}_{\text{total}} = \left( \frac{q^2}{a^2} + \frac{8q^2}{2a^2} - \frac{\sqrt{2}q^2}{a^2} \right) \mathbf{j} + \left( \frac{8q^2}{2a^2} - \frac{8q^2}{a^2} + \frac{\sqrt{2}q^2}{a^2} \right) \mathbf{x}
\]

\[
= \frac{q^2}{a^2} \left( 1 + \frac{1}{2\sqrt{2}} - \sqrt{2} \frac{q}{2} \right) \mathbf{j} + \frac{q^2}{a^2} \left( -1 - \frac{1}{2\sqrt{2}} + \sqrt{2} \frac{q}{2} \right) \mathbf{x}
\]

\[
= 0
\]

\[
1 + \frac{1}{2\sqrt{2}} - \sqrt{2} \frac{q}{2} = 0 \quad \Rightarrow \quad \frac{q}{2} = \frac{1 + \frac{1}{2\sqrt{2}}}{\sqrt{2}} = 0.7531
\]

The equilibrium is unstable. For example, if the charge on the upper left corner is moved towards the center, it will keep getting attracted to the center.

1.5

Electric field contributions from segment $Rd\theta$ at $\theta$ & $\pi - \theta$ will have opposite $x$ component, and therefore only $y$ component should be considered.
\[ dE_y = \frac{Q \sin \theta}{R^2} \]  

\[ E_y = \int_0^R \frac{Q \sin \theta}{R^2} \, d\theta = \frac{Q}{2} \left( -\cos \theta \right) \bigg|_0^\pi = \frac{Q \pi}{2R^2} \]

1.8

Pick any ion on the 1-D row and calculate how much
is the potential energy due to interacting with other ions

\[ U_{\text{tot}} = \frac{N}{2} \left( \frac{-e^2}{a} + \frac{e^2}{2a} - \frac{e^2}{3a} + \frac{e^2}{4a} + \cdots \right) \]

\[ = N \frac{-e^2}{a} \left( 1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdots \right) \]

\[ U_{\text{tot}}/N = \frac{-e^2}{a} \cdot \ln 2 \]

1.9

When the sphere is built up to radius \( r \), the
work done to add a layer of thickness \( dr \)
is given by

\[ dU = \frac{Q(r) \rho 4\pi r^2 dr}{r} \]

where \( Q(r) = \frac{4 \pi r^3 \rho}{3} \)

\[ U = \int dU = \int_0^r Q(r) \rho 4\pi r^2 dr \]

\[ = \int_0^r \frac{4\pi r^2 \rho}{3} \cdot 4\pi r^2 dr \]

\[ = \left( \frac{4\pi \rho}{3} \right)^2 \frac{a^5}{5} \]
Use superposition principle and consider the charge distribution as that corresponding to a uniform distribution of $\rho$ throughout the whole sphere of radius $a$, plus a charge distribution of $(-\rho)$ inside the smaller sphere.

At point $A$, the field from the larger sphere is zero. The field from the smaller sphere is $E = \frac{4\pi}{3a} \left( \frac{a}{2} \right)^3 - \frac{\rho}{(a/2)^2}$, or $E = \frac{4\pi a^3 \rho}{3} - \frac{\rho}{(a/2)^2}$.

Therefore, the net field at point $A$ is $E = \frac{4\pi a^3 \rho}{3}$, and is pointing vertically upward.

At point $B$, $E = \frac{4\pi a^3 \rho}{3} - \frac{\rho}{(3a/2)^2}$

$= \frac{4\pi a^3 \rho}{3} - \frac{\rho}{9a^2}$

$= \frac{39}{27} \frac{4\pi a^3 \rho}{3}$; pointing vertically downward.

1.18

Use superposition principle. Electric field

$E_1 = \frac{\rho_1}{2\epsilon_0} = 4.98 \text{ V/cm}$ from an infinite plane of surface charge is given by $E_2 = 2\pi \sigma_2 \text{ cm}$ and points perpendicular to the plane.

In region I, $E = 2\pi \sigma_1 + 2\pi \sigma_2 = 4\pi \epsilon_0 \text{ cm}^2 \text{ and point to the left.}$
In region I, \( E = 2\pi \sigma \text{ esu/cm}^2 \) and points to the right.

In region III, \( E = 4\pi \sigma \text{ esu/cm}^2 \) and points to the right.

\[
\sigma_1 = 6 \text{ esu/cm}^2 \text{ is in the yz plane, and surface charge } \sigma_2 = -4\sigma
\]
is in the xz plane.

The field in the 4 different regions are given as in the left figure.

\[
E = 2\pi \sqrt{\sigma_1^2 + \sigma_2^2} = 2\pi \sqrt{5\sigma^2} \text{ esu/cm}^2 \text{ and angle } \alpha \text{ is given by } \tan \alpha = \frac{\sigma_2}{\sigma_1} = \frac{2}{3}
\]

1.20

Use Gauss law with Gauss surface having a cylindrical shape. From symmetry of charge distribution, we know that \( E \) can only point perpendicular to the pipe.

If we draw a Gauss cylinder inside the pipe, \( E \cdot 2\pi RL = 4\pi \alpha = 0 \) (no charge inside the pipe), or \( E = 0 \).

If we draw a Gauss cylinder outside the pipe, \( E \cdot 2\pi Rl = 4\pi \cdot 2\pi a \sigma \) or \( E = \frac{4\pi \alpha \sigma}{l} = \frac{2}{3} (2\pi a \sigma) \).
Now \( 2\pi a \delta = \lambda \); line charge density.

\[ E = \frac{2\lambda}{r} \]: electric field is equivalent to that of a line charge density.

For a pipe with a square cross section, none of the above statements is true.

\[ E_1 \]

Although \( \int E \, dA = 4\pi \) (total charge enclosed),

\[ E \] on the Gauss's surface has no longer

the same magnitude \( (E_1 \neq E_2) \).

Same argument applies for a Gauss's surface
drawn inside the pipe.

1.2.9

The field for a spherical shell with a small aperture

can be considered as a superposition of the field from a

complete spherical sphere and a disk with opposite charge
density.

\[ E = \frac{4\pi a^2 \delta}{x^2} - 2\pi \delta = 2\pi \delta \]

This result could have been obtained by considering the

force on a surface charge \( F = \frac{1}{2} (E_1 + E_2) \delta \).

\[ E = \frac{F}{\delta} = \frac{1}{2} (E_1 + E_2) = \frac{E_1}{2} \quad (E_1 = 0 \text{ field inside shell}) \]

Now \( E_2 = 4\pi \delta \) \quad \[ E = 2\pi \delta \]