Topic 2: Theory of the Firm

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Based Primarily on Varian, Ch. 18-25

The Setup

- A firm
  - produces output $y$, which it can sell for price $p(y)$
    - $p(y)$ is the inverse market demand function
  - from quantities of inputs (factors): $x_1, x_2, \ldots$
  - input cost (per unit): $w_1, w_2, \ldots$
- How can this firm produce?
  - technology
- How should this firm produce?
  - cost minimization
- How much should this firm produce?
  - profit maximization
Technology

Production

Intro: Production

- In our problem, the firm’s production technology is given;
- and: the technology is independent of the market form (market structure):
  - in particular it has nothing to do with competition or firm behavior.
Production Function

- A production function tells you how much output (at most) you can get from given quantities of inputs (factors).
  - Example (Cobb-Douglas): \( f(x_1, x_2) = x_1^a x_2^b \).
  - Here: \( x_1^{0.5} x_2^{0.5} \).

Short-Run Production Function

- In the short run, not all inputs can be varied: at least one input is fixed.
  - Suppose input 2 is fixed at \( x_2 = x_2 \): \( y = f(x_1, x_2) \).
- We can still vary output by varying input 1. This is the short-run production function.
Marginal Product

- Suppose input 2 is held constant: how does output change as we change input 1?
  - The marginal product of input 1 is the partial derivative of the production function with respect to input 1.
  - Example: Holding $x_2$ constant at $x_2 = 2$, how does $u$ change as we change $x_1$ by a little, i.e. what is the slope of the blue line?

\[ \text{Marginal Product, cont'd} \]

- Formally, the marginal product of input 1 of the production function $f(x_1, x_2)$ is:

\[
\text{MP}_1 = \lim_{\Delta x_1 \to 0} \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1} = \frac{\partial f(x_1, x_2)}{\partial x_1}
\]

- That is, at which rate does output increase as this firm uses more of input 1?
Buzz Group: Marginal Product

- What is the marginal product of input 1 of the Cobb-Douglas production function
  \[ f(x_1, x_2) = x_1^{0.5} x_2^{0.5} \]
- Does the marginal product increase or decrease as the firm uses more of input 1?
- Answer:
  \[ MP_1 = 0.5 x_1^{-0.5} x_2^{0.5}. \]
  - This decreases as \( x_1 \) increases, because its slope is negative: \( \frac{\partial MP_1}{\partial x_1} = -0.25 x_1^{-1.5} x_2^{0.5}. \)

Isoquants

- Definition: An isoquant is the locus of all input combinations that yield the same level of output.
Technical Rate of Substitution

- The technical rate of substitution is the slope of an isoquant at a point.
- That is, holding total output constant (remaining on the same isoquant), at what rate can we exchange input 2 for input 1?

TRS, cont’d

- Along an isoquant, output is constant:
  \[ f(x_1, x_2(x_1)) = c \]
- Since this is an identity, we can differentiate both sides with respect to \( x_1 \) to get:
  \[ \frac{df(x_1, x_2(x_1))}{dx_1} = 0 \]
- What is \( df(x_1, x_2(x_1)) / dx_1 \)?
  - First, there is a “direct” effect: \( \frac{\partial f(x_1, x_2)}{\partial x_1} \).
  - Then, there is also an “indirect” effect, through \( x_2 \):
    \[ \frac{df(x_1, x_2(x_1))}{dx_1} = \frac{\partial f(x_1, x_2)}{\partial x_1} + \frac{\partial f(x_1, x_2)}{\partial x_2} \cdot \frac{dx_2(x_1)}{dx_1} \text{ (chain rule)} \]
So we know that
\[
\frac{df(x_1, x_2(x_1))}{dx_1} = \frac{\partial f(x_1, x_2)}{\partial x_1} + \frac{\partial f(x_1, x_2)}{\partial x_2} \frac{dx_2(x_1)}{dx_1}
\]
But we wanted to keep output constant, so
\[
\frac{df(x_1, x_2(x_1))}{dx_1} = 0
\]
So we have:
\[
\frac{\partial f(x_1, x_2)}{\partial x_1} + \frac{\partial f(x_1, x_2)}{\partial x_2} \frac{dx_2(x_1)}{dx_1} = 0
\]
which we can rearrange as:
\[
\frac{dx_2(x_1)}{dx_1} = -\frac{\frac{\partial f(x_1, x_2)}{\partial x_2}}{\frac{\partial f(x_1, x_2)}{\partial x_1}} = \frac{MP_1}{MP_2}
\]

What is the technical rate of substitution (slope of the isoquant) for the Cobb-Douglas production function
- \( f(x_1, x_2) = x_1^{0.5} x_2^{0.5} \)
- … at the point \( x_1 = x_2 = 2 \)?
- … at the point \( x_1 = 4, x_2 = 1 \)?
  - (this is the same isoquant)
Firm Decisions and Market Structure

What to do, when.

Intro: Tools for Firm Decisions

- How should firms make choices (about production, input purchases, etc.)?

- These tools (unlike production technology) depend on market structure:
  - e.g. does the firm operate in a competitive market, or is it a monopolist?

- For instance, we assume that the firm operates in a market, so that there are prices for inputs and output.
Firm Decisions I

Profit Maximization

Profit Maximization

- Firms maximize profits (revenue - cost).
- Why?
  - Shareholders are interested in the value of their shares.
  - Shares reflect the present value of the firm, that is the present discounted value of all future profits.
  - Shareholders therefore will want management to maximize profits.
  - (Is this really true? Can shareholders control management sufficiently? See topic 5.)
Short Run Profit Maximization

- A (competitive) firm’s profit maximization problem is:
  \[ \max_{x_1} p f(x_1, x_2) - (w_1 x_1 + w_2 x_2). \]
- The necessary (first order) condition is:
  \[ p \cdot \frac{\partial f(x_1, x_2)}{\partial x_1} - w_1 = 0, \text{ or:} \]
  \[ p \cdot MP_1 = w_1 \]
- The value of the marginal product of the variable input has to be equal to that input’s price.

Long Run Profit Maximization

- A (competitive) firm’s profit maximization problem is:
  \[ \max_{x_1, x_2} p f(x_1, x_2) - (w_1 x_1 + w_2 x_2). \]
- The necessary (first order) conditions are:
  \[ (i) \ p \cdot \frac{\partial f(x_1, x_2)}{\partial x_1} - w_1 = 0, \text{ or:} \]
  \[ p \cdot MP_1 = w_1 \]
  \[ (ii) \ p \cdot \frac{\partial f(x_1, x_2)}{\partial x_2} - w_2 = 0, \text{ or:} \]
  \[ p \cdot MP_2 = w_2 \]
- The value of the marginal product of each input has to be equal to the input’s price.
Firm Decisions II

Cost Minimization

Cost Minimization

- How should a firm produce?
  - Given that the firm wishes to produce a certain level of output, with what combination of inputs should it produce that output?
  - The firm should use inputs so as to minimize cost.
  - This fits a “delegation story.”
- That is, the firm solves:

\[
\begin{align*}
\min_{x_1, x_2} & \quad w_1 x_1 + w_2 x_2 \\
\text{s.t.} & \quad f(x_1, x_2) = y
\end{align*}
\]
Cost Min., Graphically

A firm’s cost \( c \) is:
- \( c = w_1 x_1 + w_2 x_2 \)
  where \( x_1, x_2 \) are inputs at prices \( w_1, w_2 \)
- rearrange: \( x_2 = \frac{c}{w_2} - \left(\frac{w_1}{w_2}\right) x_1 \)
- This is the equation for the isocost line corresponding to cost level \( c \).

Definition: An isocost line is the locus of input combinations that have the same cost.

Cost Min., Graphically, cont’d

Isocost lines: \( x_2 = \frac{c}{w_2} - \left(\frac{w_1}{w_2}\right) x_1 \)
Cost Min., Graphically, cont’d

- Firms choose to produce a given level of output (on a given isoquant) using the least cost combination of inputs (they minimize cost).
  - Firms choose to locate on the lowest isocost line, subject to the constraint of producing a given level of output.
- Implication: \( TRS = - \frac{MP_1}{MP_2} = - \frac{w_1}{w_2} \)

Cost Min., Calculus

- Write down the Lagrangean:
  \[
  L = w_1x_1 + w_2x_2 - \lambda \left( f(x_1, x_2) - y \right)
  \]
- Write down the necessary (first-order) conditions:
  \[
  w_1 - \lambda \frac{\partial f(x_1, x_2)}{\partial x_1} = 0 \quad \text{or} \quad w_1 = \lambda \frac{\partial f(x_1, x_2)}{\partial x_1}
  \]
  \[
  w_2 - \lambda \frac{\partial f(x_1, x_2)}{\partial x_2} = 0 \quad \text{or} \quad w_2 = \lambda \frac{\partial f(x_1, x_2)}{\partial x_2}
  \]
  \[
  f(x_1, x_2) - y = 0 \quad \text{or} \quad f(x_1, x_2) = y
  \]
- Then solve for \( x_1 \) and \( x_2 \).
  - Without a specific functional form for \( f(\cdot, \cdot) \) we cannot solve for \( x_1 \) and \( x_2 \). But the necessary conditions are still informative.
Cost Min., Calculus, cont'd

- Necessary conditions:
  (i): \( w_1 = \lambda \frac{\partial f(x_1, x_2)}{\partial x_1} \)
  (ii): \( w_2 = \lambda \frac{\partial f(x_1, x_2)}{\partial x_2} \)

- Divide (i) by (ii) to obtain the familiar:
  \[
  \frac{w_1}{w_2} = \frac{\frac{\partial f(x_1, x_2)}{\partial x_1}}{\frac{\partial f(x_1, x_2)}{\partial x_2}} = \frac{MP_1}{MP_2}
  \]

Buzz Group: Cost Minimization

- Find the firm’s cost minimizing choices of inputs 1 and 2, when it produces with the following (Cobb-Douglas) production function:
  - \( f(x_1, x_2) = x_1^a x_2^b \).

- Given that the firm chooses inputs in a cost-minimizing way, what is its (minimum) cost for any level of output?
Buzz Group: Cost Min., cont’d

(i) \( w_1 = \lambda ax_1^{a-1}x_2^{b} \)
(ii) \( w_2 = \lambda bx_1^{a}x_2^{b-1} \)
(iii) \( x_1^{a}x_2^{b} = y \)

• multiply (i) by \( x_1 \) and (ii) by \( x_2 \) to get:
  (i) \( w_1 = \lambda ax_1^{a}x_2^{b} \) \( \text{or} \quad w,x_1 = \lambda ay \) \( \text{or} \quad x_1 = \frac{\lambda a y}{w_1} \)
  (ii) \( w_2 = \lambda bx_1^{a}x_2^{b} \) \( \text{or} \quad w_2x_2 = \lambda by \) \( \text{or} \quad x_2 = \frac{\lambda b y}{w_2} \)

• now put these into (i):
  \[ w_1 = \lambda \left( a \frac{y}{w_1} \right)^{a-1} \left( b \frac{y}{w_2} \right)^b \]
  \( \text{or} \quad w_1 = \lambda a^{a-1}a^b y^{a+b-1}w_2^{-1}w_1^{1-a-b} \)

\[
\lambda = \frac{a^{a-1}b^a}{a^{a-1}b^a w_1^{a+b}w_2^{a+b}y^{a+b}}
\]
Buzz Group: Cost Min., cont’d

Putting \( \lambda \) into the expressions for \( x_1 \) and \( x_2 \):

\[
x_1 = \lambda \frac{a}{y_w} \]
\[
x_2 = \lambda \frac{b}{y_w}
\]

so this firm’s cost (when it uses the cost minimizing quantities of inputs) is:

\[
c(y) = w_1x_1 + w_2x_2
\]

\[
c(y) = w_1a^{a|b} \frac{a}{a+b} w_1 a^{a|b} w_2 a^{a|b} y^{a|b} + w_2 a^{a|b} b^{a|b} w_1 a^{a|b} y^{a|b}
\]

\[
c(y) = (a^{a|b} + a^{a|b} b^{a|b}) w_1 a^{a|b} w_2 a^{a|b} y^{a|b}
\]
(Long Run) Total Cost

- Out of the firm’s cost minimization problem comes its (long run) total cost function:
  - The long run total cost function tells you what the cost of producing different output level is, given that you use inputs in the best possible (cost minimizing) way.

LMC, LAC

- (Long run) marginal cost is the ratio at which cost increases with output:
  \[ \text{LMC}(y) = \frac{dc(y)}{dy} \]

- (Long run) average cost is the per-unit production cost:
  \[ \text{LAC}(y) = \frac{c(y)}{y} \]
Derive LMC and LAC for the long run total cost function:
\[ c(y) = \left( \frac{b}{a^{a+b}b^{a+b}} + a \frac{b}{a^{a+b}b^{a+b}} \right) ^{a} \left( \frac{1}{w_1^{a+b}w_2^{a+b}} \right) y^{a+b} \]

Answer:

Long Run and Short Run Cost

- The long run total cost function tells you what the cost of producing different output levels is, given that you use inputs in the best possible (cost minimizing) way.
  - This implies that you can vary all inputs: in the long run, all inputs can be varied. Production function \( f(x_1, x_2) \).
  - Long run cost: \( c(y) \)
- In the short run, not all inputs are variable: some (at least one) inputs are fixed.
  - Remember: short run production function \( f(x_1, \bar{x}_2) \).
  - Short run cost: \( c_s(y, \bar{x}_2) \)
In the short run, not all factors are variable.
- Suppose $x_2$ is fixed at $\bar{x}_2$.
- How does short run cost compare to long run cost?

For every level of output $y$,
- there is an optimal level of input 2, $x_2(y)$
- (and an optimal level of input 1, $x_1(y)$.)

So: the long run cost for output level $y$, $c(y)$ is just the same as:
- short run cost if the fixed input is fixed at the optimal level $x_2(y)$, that is at the level of that input that would be optimal for that level of output $y$.

That is:
- $c(y) \equiv c_s(y, x_2(y))$. 
L-R and S-R Cost, cont’d

- In the diagram on slide 37, the optimal level of input 2 for an output level of $y_3$ would have been $\bar{x}_2$.
  - Therefore, when in the short run input 2 was fixed at $\bar{x}_2$, the short run cost function for output $y_3$ was the same as the long run cost function for output $y_3$.
  - But for all other output levels, the short run cost was higher than the long run cost, because input 2 was fixed at the "wrong" level.

(Short Run) Marginal Cost

- The short run total cost function is: $c_s(y, \bar{x}_2)$.
- *Marginal cost* is the rate at which short run cost increases as output is increased. So:
  $$\text{MC}(y) = \frac{\partial c_s(y, \bar{x}_2)}{\partial y}$$
There is a close connection between long run marginal cost and (short run) marginal cost:

- Remember that: \( c(y) \equiv c_s(y, x_2(y)) \).

Differentiate both sides with respect to \( y \) to obtain:

\[
\frac{dc(y)}{dy} = \frac{\partial c_s(y, x_2)}{\partial y} + \frac{\partial c_s(y, x_2)}{\partial x_2} \frac{dx_2(y)}{dy}
\]

\( LMC = MC + \text{stuff} \)

We want to know what LMC is at the output level \( y^* \) for which \( x_2^* \) is just the right level of input 2:

- \( \frac{\partial c_s(y^*, x_2^*)}{\partial x_2} = 0 \) when \( x_2 \) is chosen optimally (i.e. such that it minimizes cost), so at that point:

\[
\frac{dc(y^*)}{dy} = \frac{\partial c_s(y^*, x_2^*)}{\partial y}
\]
Short Run Total and Variable Cost

- Since in the short run some inputs are fixed, we can split up short run total cost $c_s(y, x_2)$ into
  - fixed cost (the cost coming from the fixed input), that is: $F = w_2x_2$, and
  - variable cost (the cost that comes from the input that can be varied in the short run), that is: $c_v(y)$.
- Even when producing nothing ($y = 0$), in the short run there is a cost, $F$ (the fixed cost).
  - That is why the short run total cost function does not “start” at the origin:

S-R Total and Var. Cost, cont’d

- $c_s(0, x_2) = F$
  - Even when no output is produced, in the short run there is a cost: the fixed cost.
- The variable cost is the part of short run cost that varies with output, that is, it is $c_s(y, x_2) - F$.
  - Variable cost is just short run total cost less the fixed cost.
**S-R Average Cost**

- Average Fixed Cost:
  \[ AFC = \frac{F}{y} \]
- Average Variable Cost:
  \[ AVC = \frac{c_v(y)}{y} \]
- Average Total Cost
  \[ ATC = \frac{cs(y, x_2)}{y}, \text{ or:} \]
  \[ \frac{cs(y, x_2)}{y} = \frac{c_v(y)}{y} + \frac{F}{y} \]
\[ ATC = AVC + AFC \]

- Marginal Cost intersects AVC and ATC at their minimum points.

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**Buzz Group: Cost**

- If the short run total cost curve is:
  \[ c_s(y, 2) = 5y^2 + 20 \]
  - What is Average Total Cost?
    \[ \text{ATC}(y) = \]
  - What is Fixed Cost?
    \[ F = \]
  - What is Variable Cost?
    \[ V_C(y) = \]
  - What is Average Variable Cost?
    \[ \text{AVC}(y) = \]
  - What is Marginal Cost?
    \[ \text{MC}(y) = \]
Remember that:
- $c(y^*) = c_s(y^*, x_2^*)$, and $c(y) \leq c_s(y, x_2)$.

So we have the following relationship between (short run) average total cost and (long run) average cost:
- $AC(y^*) = c(y^*) / y^* = c_s(y^*, x_2^*) / y^* = ATC(y^*)$
- $AC(y) = c(y) / y \leq c_s(y, x_2) / y = ATC(y)$

That is:
- Short run ATC is always above long run AC ...
- and the two are equal at the output level where short run cost = long run cost (which is also the output level at which marginal costs are equal).

Short run ATC is always above long run AC ... 

… and the two are equal at the output level where short run cost = long run cost
S-R Average Total Cost, cont'd

... and there is one ATC cost curve for each level of the fixed input:

- The (long run) average cost curve is the lower envelope of the (short run) average total cost curves.

Short Run and Long Run Cost

... and long run average cost equals short run average total cost at the output level where long run marginal cost equals short run marginal cost.
Returns to Scale: Const. Returns

Firm Decisions III

Profit Maximization again
Supply

- A firm aims to supply the quantity of output that maximizes its profit:
  - $\max_y \text{ revenue - cost}$
- for each perfectly competitive firm:
  - competitive firms are “price takers”: they face a horizontal demand curve:
  - $\max_y p \cdot y - c_s(y, x_2)$
- for a monopolist:
  - a monopolist faces the (inverse) market demand curve $p(y)$:
  - $\max_y p(y) \cdot y - c_s(y, x_2)$

Firm Decisions: Competitive Firms

ex pluribus unum
Supply: Competitive Firm

- A perfectly competitive firm's problem is to:
  - max \( p \cdot y - c_s(y, x_2) \)
- The necessary (first-order) condition for maximization is:
  - \( p - \frac{\partial c_s(y, x_2)}{\partial y} = 0 \)
  - \( \frac{\partial c_s(y, x_2)}{\partial y} = p \)
  - \( MC(y) = p \)
  - (but only when it's better to produce in the short run than to shut down, that is when:
    - \( p \cdot y - c_s(y, x_2) > -F \)
    - \( p \cdot y - VC(y) - F > -F \)
    - \( p > \frac{VC(y)}{y}, \text{ or: } p > AVC(y). \)

Supply: Competitive Firm, cont'd

- A competitive firm's supply curve is its marginal cost curve above AVC.
Buzz Group: Competitive Firm

- If the short run total cost curve is:
  \[ c_s(y, 2) = 5y^2 + 20 \]
- What is the firm’s profit? At what output is it maximized?
  - \[ \pi(y) = p \cdot y - (5y^2 + 20) \]
  - \[ \pi'(y) = p - 10y \]
  - profit maximum: \[ \pi'(y) = 0, \text{ or: } p = 10y, \text{ or: } y = p/10 \]
- If the firm produces that output, what is its (maximum) profit?
  - \[ p \cdot (p/10) - (5 \cdot (p/10)^2 + 20) = p^2/20 - 20 \]

Supply: Competitive Firm, cont’d

- In competitive markets, there is free entry into, and exit from, the market.
  - If firms in this market make positive profits, there is an incentive for other firms to enter the market …
  - … which lowers price …
  - … and erodes profits.
- When is there no opportunity for firms to make positive profits (i.e. when will entry no longer occur)?
  - When \[ \pi(y) = p \cdot y - c_s(y, x_2) = 0, \text{ and no other choice of } x_2 \text{ can give you lower cost.} \]
  - That is, when \[ p = c_s(y, x_2) / y = ATC(y), \text{ and you are operating on the lowest possible ATC curve.} \]
  - That is, when you are operating at the lowest point on LAC.
In long-run equilibrium, all profit has been eliminated through entry into the market:
- each firm in this industry produces at the lowest point of its long-run average cost curve.

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"The best of all monopoly profits is a quiet life."

Supply: Monopolist

- A monopolist's problem is to:
  - \( \max_y p(y) \cdot y - c_s(y, x_2) \)
- The necessary (first-order) condition for maximization is:
  - \( \frac{dp(y)}{dy} \cdot y + p(y) - \frac{\partial c_s(y, x_2)}{\partial y} = 0 \)
  - \( \frac{\partial c_s(y, x_2)}{\partial y} = p(y) \left[ 1 + \frac{dp(y)}{dy} \cdot \frac{y}{p(y)} \right] \)
  - \( MC(y) = p(y) \left[ 1 + \frac{1}{\eta} \right] = MR(y) \)
- Mark-up pricing:
  - \( p(y) = \left[ \frac{1}{1+\frac{1}{\eta}} \right] \cdot MC(y) \)

Supply: Monopolist, cont’d

- Example:
  - linear (inverse) demand \( p(y) = a - by \)
  - \( \pi(y) = (a - by) \cdot y - c_s(y, x_2) = ay - by^2 - c_s(y, x_2) \)
    - \( ay - by^2 \) is revenue
  - \( \pi'(y) = a - 2by - \frac{\partial c_s(y, x_2)}{\partial y} \)
    - \( a - 2by \) is marginal revenue
  - profit maximum: \( a - 2by = MC(y) \)
Supply: Monopolist, cont’d

Linear demand $p(y) = a - by$, so $\text{MR}(y) = a - 2by$

Market Structure I

Monopoly Behavior:
Price Discrimination and
Two-Part Tariffs
Price Discrimination

- First-degree (perfect) price discrimination:
  - different prices for different units of output, and
  - different prices for different consumers.
- Second-degree price discrimination (non-linear pricing):
  - different prices for different units of output, and
  - same prices for similar customers.
- Third-degree price discrimination:
  - same prices for different units of output, but
  - different prices for different customers.

First-Degree Price Discrimination

- The perfectly discriminating monopolist knows each consumer's demand curve.
- The monopolist prices each unit of output at each consumer's marginal willingness to pay.
First-Degree Price Disc., cont’d

- Perfectly discriminating monopolist would like to sell:
  - to consumer A: \( x_1 \) at price \( A + A' \); to consumer B: \( x_2 \) at \( B + B' \)
- All consumer surplus is extracted by the monopolist.
- First-degree price discrimination is efficient.
- But: informationally demanding.

First-Degree Price Disc., cont’d

- What limits first-degree price discrimination:
  - unobservable preferences:
    » “informationally demanding”;
  - competition (or the threat of entry):
    » instead, the monopolist may try to limit entry (more on entry deterrence in Topic 7);
  - arbitrage (resale):
    » Example: Suppose my marginal willingness to pay is low (i.e. I pay a low price for the quantity I buy). Since my consumer surplus is zero, there are gains from trade if I sell to you (your willingness to pay is high);
  - administrative costs.
The monopolist could achieve the same outcome by charging a two-part tariff: 
- charge a one-off fee of $A$ (consumer surplus), and 
- charge each unit bought at marginal cost.

The consumer will then buy $x_1$ units (i.e. up to where price = willingness to pay), and all consumer surplus is extracted.

As before, this is efficient, but informationally demanding.

**Two-Part Tariff: Examples**

- How does economic theory (the theory of two-part tariffs) explain features of the real world?
- Amusement parks: 
  - admission fee + marginal cost per ride.
- Telephone line: 
  - connection charge + marginal cost per call.
- Xerox photocopiers: 
  - rental fee + marginal cost per copy.
Second-Degree Price Disc.

- Suppose a monopolist cannot observe each customer’s marginal willingness to pay.
  - But: she can observe the quantity demanded by customers.
  - She could sell different price-quantity “packages”, aimed at customers with different marginal willingness to pay: customers will *self-select* into buying the “package” designed for them.

- An example of an asymmetric information problem (more in Topic 5).

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Third-Degree Price Discrimination

- The monopolist charges different prices to different customers (i.e. in different elasticity markets).
  - Examples: private/business telephony, student discounts, business/economy class air travel, …
  - (The monopolist must be able to observe a customer’s demand elasticity.)

- Marginal cost equals marginal revenue in each market. (Monopoly pricing in each market.)
  - (argument by contradiction)
Third-Degree Price Disc., cont’d

- Suppose there are two markets, with (inverse) demand curves $p_1(y_1)$ and $p_2(y_2)$. The monopolist’s problem is to:
  - $\max_{y_1,y_2} y_1 \cdot p_1(y_1) + y_2 \cdot p_2(y_2) - c(y_1 + y_2)$

- The necessary (first order) conditions for this are:
  - (i) $y_1 \cdot \frac{dp_1(y_1)}{dy_1} + p_1(y_1) - c'(y_1 + y_2) = 0$
  - (ii) $y_2 \cdot \frac{dp_2(y_2)}{dy_2} + p_2(y_2) - c'(y_1 + y_2) = 0$

- Rewrite (i): (and analogously for (ii))
  - $c'(y_1 + y_2) = p_1(y_1) \left[ 1 + \left( \frac{y_1}{p_1(y_1)} \right) \cdot \frac{dp_1(y_1)}{dy_1} \right]$, or:
  - $p_1(y_1) = \frac{c'(y_1 + y_2)}{1 + \left( \frac{1}{\eta} \right)}$

- The more elastic demand in a market, the lower price in that market and vice versa.

Third-Degree Price Disc., cont’d

- Example:
  - Low elasticity market: demand $D_1$ (e.g. private telephony)
  - High elasticity market: demand $D_2$ (e.g. business telephony)
  - Price where marginal cost = marginal revenue

- Price is high in the low elasticity market, and low in the high elasticity market.
The welfare effects of third-degree price discrimination (compared with standard monopoly pricing) are ambiguous:

- Two inefficiencies:
  - Output is too low:
    - The monopolist charges the monopoly price in each market.
      (She restricts output below the efficient level.)
  - Misallocation of goods:
    - Goods are allocated to the wrong individuals.
    - Example: I value a theater ticket at $20, you value it at $10. You get a student discount (ticket for $8) and buy the ticket. I have to pay the normal price ($28) so I don’t buy the ticket. But my valuation is higher than yours!

- And a welfare improvement over monopoly: ...

If the monopolist were not allowed to (third-degree) price-discriminate, she might only sell in one market:

- Pricing uniformly in both markets may be less profitable than selling in only one market.
- In this example, the monopolist would only sell in market 1: profit in market 1 is greater than profit in market 2
Market Structure II

Monopolistic Competition: Differentiated Products and the Hotelling Model

Product Differentiation

- Monopolistic Competition: every firm faces a downward-sloping demand curve (i.e. has some degree of monopoly power).
- In an industry with non-homogeneous products, how do firms choose their products’ characteristics?
  - Example: cars, economics courses, …
- Imagine one product characteristic that can be chosen continuously: e.g. location of two ice-cream vendors along a beachfront.
Product Differentiation: Location

- Hotelling’s “principle of minimum differentiation”: both ice-cream vendors locate in the middle of the beach.
  - This is not welfare maximizing (the location choice in the left-hand panel in the diagram is).
- More examples: political parties, radio stations, ...