Topic 1: Basic Consumer Theory

Economics 21, Summer 2002
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Based Primarily on Varian, Ch. 2-6, 8, 15

What is this Course about?

Introduction and Outline
What is this Course about?

Economics 01:
Perfect Competition
- Price taking
  - horizontal demand curve
  - no strategic behavior
- Homogeneous products
- Free entry and exit
- Perfect information:
  - perfect foresight
  - full information

Economics 21:
Lifting the Assumptions:
- Price (quantity) setting:
  - Monopoly; Oligopoly
  - Game Theory
- Product differentiation
- Entry deterrence
- Imperfect information:
  - uncertainty
  - asymmetric information

What is this Course about, cont’d

- What is a good?
  - Physical commodity:
    - e.g. apples, oranges, leisure time, public park, pollution, ...
  - Location in time (Intertemporal Choice):
    - e.g. oranges today, oranges on October 3, 2000
  - Contingent consumption (Uncertainty):
    - e.g. umbrella when it is raining, umbrella when it is not raining
  - “Interactions” between agents (Game Theory):
    - e.g. wearing T-shirt and shorts when everyone else wears suits, wearing T-shirt and shorts when everyone else wears shorts
What is this Course about: Outline

- Optimization
  - Example: Basic Consumer Theory

- Consumers
  - Intertemporal Choice
  - Uncertainty, Expected Utility Theory

- Firms
  - Optimization again: Theory of the Firm
  - Market Structure: Price Discrimination, Product Differentiation

- General Equilibrium and Welfare
  - Fundamental Theorems of Welfare Economics
  - Social Welfare Functions

- Game Theory
  - Market Structure: Models of Oligopoly

- Information
  - Adverse Selection
  - Signaling, Screening
  - Moral Hazard

Aims of the Course

- Theory:
  - equip you with the tools you need as a professional economist;
  - relax the assumption of perfect competition.

- Applications:
  - nearly all applications are from Industrial Organization: how do real-life markets and organizations work?
What is Economics about?

The Optimization Principle
(Basic Consumer Theory)

- How do we make decisions?
- **Assumption**: Rational agents always choose to do what they most prefer to do, given the options that are open to them.
- **Questions**:
  - What is “rational”?
  - What is “most preferred”?
  - What is “options open to them”?
Rationality

- **Definition**: A rational agent is someone who has a rational preference ordering over the set of all alternatives (or “consumption bundles”).

- **Definition**: Preference relation: Let \( x \succeq y \) denote: “the bundle \( x \) is (weakly) preferred to \( y \)”
  - Remember: a “bundle” (or “vector”) of goods is a list of quantities of goods:
    - for instance, \( x \) could be: (2 cans of coke, 1 large anchovy pizza, 2 ice creams, …)
    - more generally: \( x = (x_1, x_2, x_3, \ldots, x_n) \)
    - normally two goods are enough: \( x = (x_1, x_2) \)
  - Examples of relations: taller than, older than, …

Rationality, cont’d

- Rational preferences are preferences that are:
  - complete:
    - for all bundles \( x, y \) either: \( x \succeq y \) or \( y \succeq x \) or both
    - aside: if both \( x \succeq y \) and \( y \succeq x \), then we say the consumer is indifferent between \( x \) and \( y \) and denote this by \( x \sim y \)
    - in words: all bundles can be ranked
  - transitive:
    - if \( x \succeq y \) and \( y \succeq z \) then we must have \( x \succeq z \)
Utility and Marginal Utility

“Most preferred:” representing preferences

Utility

- Utility represents preferences: $u(x) \geq u(y)$ whenever $x \succeq y$.
  - In words: whenever $x$ is (weakly) preferred to $y$, then $x$ has a larger utility number associated with it.
  - Implication: $u(x) = u(y)$ whenever $x \sim y$ (indifference).
- This function $u$ (that represents preferences) is called utility function.
- Interpretation of “most preferred”: Economic agents aim to maximize utility.
Review: Functions

- Functions ("transformations") of one variable assign to each value of the independent variable a unique value of the dependent variable.
  - Example: \( f = f(x) \)
    - \( f(x) \) is the rule that assigns to each value of \( x \) a unique value \( f \).

- Functions of more than one variable assign to each combination of independent variables a unique value of the dependent variable.
  - Example: \( u = u(x_1, x_2) \)
    - \( u(x_1, x_2) \) is the rule that assigns to each combination of \( x_1 \) and \( x_2 \) a unique value \( u \).

Utility, cont’d

- Example:
  - Cobb-Douglas utility function:
    - \( u(x_1, x_2) = x_1^a x_2^{1-a} \)
  - Here:
    - \( u(x_1, x_2) = x_1^{0.5} x_2^{0.5} \)
Review: Derivatives (one Variable)

- **Definition:** The *derivative* of the function $f(x)$ is defined as:
  $$ \frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} $$

- Sometimes we will write $f'(x)$ for $df(x)/dx$.
- Graphically, the derivative is the slope of the function at a point.

Review: Monotonicity

- A function $f(*)$ is *positive monotonic* if it is strictly increasing everywhere (of interest):
  - that is, if $f'(\ast) > 0$ everywhere.

- **Example:** $f(x) = ax + b$ (where $a > 0$)
  - $f'(x) = a$
    - this function is positive monotonic everywhere

- **Example:** $f(x) = \ln x$ (where $x > 0$)
  - $f'(x) = 1 / x$
    - this function is positive monotonic everywhere
Utility, cont’d

- So far, we can only say things about *ordinal* properties of utility:
  - The *ranking* of alternatives (bundles) is an ordinal property:
    - When we know that \( u(x) \geq u(y) \), we only know that \( x \) is preferred to \( y \). We do not know by how much it is preferred. The difference between the utility numbers, \( u(x) - u(y) \), is meaningless.
    - (Differences are a *cardinal* property of utility.)

Utility, cont’d

- The utility function
  \[
  10 \cdot u(x_1, x_2)
  \]
contains the same information about preferences as
\[
10 \cdot u(x_1, x_2).
\]

- **Example:**
  \[
  10 \cdot (x_1^{0.5} x_2^{0.5})
  \]
Ordinal Properties of Utility

- In fact, every positive monotonic transformation of utility preserves the same preference ordering.
  - This is why ordinal properties of utility are sometimes called properties that are “unique up to positive monotonic transformations.”

- **Example:**
  - The same information contained in \( u(x_1, x_2) \) is also contained in:
    - \( a \cdot u(x_1, x_2) + b \) (where \( a > 0 \))
    - \( \ln(u(x_1, x_2)) \)
    - etc.

Ordinal Properties of Utility, cont’d

- Why does a positive monotonic transformation preserve the ordinal properties of the utility function?
  - Utility represents preferences:
    - \( u(x) \geq u(y) \) whenever \( x \succeq y \).
  - If \( f(*) \) is positive monotonic then:
    - \( u(x) \geq u(y) \) whenever \( f(u(x)) \geq f(u(y)) \).
  - Therefore:
    - \( f(u(x)) \geq f(u(y)) \) whenever \( x \succeq y \).
Marginal Utility

- **Definition:** *Marginal utility (MU)* is the rate of change in a consumer’s utility as the amount of one good she consumes changes (by a little), holding everything else constant.

  \[
  \text{MU}_i = \lim_{\Delta x_i \to 0} \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1} = \frac{\partial u(x_1, x_2)}{\partial x_1}
  \]

- The expression \(\frac{\partial u(x_1, x_2)}{\partial x_1}\) is the *partial derivative of the function* \(u\) *with respect to* \(x_1\).

Marginal Utility, cont’d

- **Example:** Holding \(x_2\) constant at \(x_2 = 2\), how does \(u\) change as we change \(x_1\) by a little?
  - What is the slope of the blue line at any point?
Buzz Group: Partial Derivatives

- What is the partial derivative (with respect to $x_1$) of the utility function $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$?
  - That is, what is $\frac{\partial u(x_1, x_2)}{\partial x_1}$?
  - Holding $x_2$ constant, take the derivative with respect to $x_1$:
    - $\frac{\partial u(x_1, x_2)}{\partial x_1} = 0.5 x_1^{-0.5} x_2^{0.5}$
- And what about $u(x_1, x_2) = x_1 + x_2$?
  - $\frac{\partial u(x_1, x_2)}{\partial x_1} = 1$

Indifference Curves

“Where indifference curves come from” and other stories.
Utility and Indifference Curves

- Indifference curves are a way of representing utility graphically.
- An indifference curve is the collection of bundles between which the consumer is indifferent.
- Implication: An indifference curve is the collection of bundles with the same utility.

Utility and Indiff. Curves, cont’d

- An indifference curve is the collection of bundles with the same utility.
- Indifference curves are the contours of the “utility mountain.”
- Example: Cobb-Douglas
  \[ u(x_1, x_2) = x_1^{0.5} x_2^{0.5} \]
Utility and Indiff. Curves, cont’d

We can draw indifference curves for two goods in a two-dimensional projection of the contours of the “utility mountain.”

“Nice” Indifference Curves

We need more assumptions on preferences (more than just rationality) to give us “nice” (well-behaved) indifference curves:

- monotonicity,
- convexity.
Well-behaved preferences:

- **monotone:**
  - for any two bundles, \( x, y \): if \( x \geq y \), then \( x \succeq y \)
  - in words: if \( x \) has greater (or equal) quantities of all goods than \( y \), then \( x \) must be (weakly) preferred to \( y \): "more is better"

![Diagram](image1.png)

- **convex:**
  - Suppose we know that \( x \sim y \). Then preferences are convex if any weighted average of the bundles \( x \) and \( y \) is preferred to \( x \) (and \( y \)).
    - any "averaged bundle" lies on a straight line between the two bundles (where on the line is determined by the weights)
  - in words: "averages are preferred to extremes"

![Diagram](image2.png)
Utility and Indifference Curves

- Another interpretation of "most preferred": on the highest indifference curve.

Marginal Rate of Substitution

- The marginal rate of substitution (MRS) is the slope of an indifference curve at some point:
  - At what rate is the consumer just willing (while remaining at the same level of utility) to exchange less of $x_2$ for more of $x_1$?
MRS, cont’d

- We want to know how much of x₂ the consumer needs to give up for each small increase in x₁, while holding utility constant.
  - Think of the indifference curve as a function x₂(x₁).
- We want to know dx₂(x₁) / dx₁ such that u(x₁, x₂(x₁)) does not change.

Along an indifference curve, utility is constant:

\[ u(x₁, x₂(x₁)) = c \]

Since this is an identity, we can differentiate both sides with respect to x₁ to get:

\[ \frac{du(x₁, x₂(x₁))}{dx₁} = 0 \]

What is du(x₁, x₂(x₁)) / dx₁?

- First, there is a “direct” effect: \( \frac{\partial u(x₁, x₂)}{\partial x₁} \).
- Then, there is also an “indirect” effect, through x₂:
  \[ \frac{du(x₁, x₂(x₁))}{dx₁} = \frac{\partial u(x₁, x₂)}{\partial x₁} \cdot \frac{dx₂(x₁)}{dx₁} + \frac{\partial u(x₁, x₂)}{\partial x₂} \cdot \frac{dx₂(x₁)}{dx₁} \] (chain rule).
MRS, cont’d

- So we know that
  \[
  \frac{du(x_1, x_2(x_1))}{dx_1} = \frac{\partial u(x_1, x_2)}{\partial x_1} + \frac{\partial u(x_1, x_2)}{\partial x_2} \frac{dx_2(x_1)}{dx_1}
  \]

- But we wanted to keep utility constant, so that
  \[
  \frac{du(x_1, x_2(x_1))}{dx_1} = 0
  \]

- So we have:
  \[
  \frac{\partial u(x_1, x_2)}{\partial x_1} + \frac{\partial u(x_1, x_2)}{\partial x_2} \frac{dx_2(x_1)}{dx_1} = 0
  \]

- which we can rearrange as:
  \[
  \frac{dx_2(x_1)}{dx_1} = \frac{\frac{\partial u(x_1, x_2)}{\partial x_1}}{\frac{\partial u(x_1, x_2)}{\partial x_2}} = - \frac{\text{MU}_1}{\text{MU}_2}
  \]

MRS, cont’d

- So we have a connection between the slope of an indifference curve and the concept of marginal utility:
  - MRS = - MU_1 / MU_2.

- Why is this interesting?
  - We can’t observe people’s utility.
    - And: utility is only uniquely determined up to positive monotonic transformations.
  - But we can observe people’s MRS.
    - Remember people’s choices are such that MRS = price ratio.
    - And: MRS does not depend on the scaling of utility.
Constraints

“Options open to you:”
What you can and can’t do.

Constraints

- There are constraints to what we can do: limited resources.
  - Examples:
    - consumers cannot spend more than their total wealth
    - workers cannot supply more than 24 hrs labor per day
    - we cannot borrow without saving
    - etc.
- Interpretation of “options open to them”: Economic agents operate under constraints.
Constraints: An Example

- We cannot spend more on goods than our total wealth.
  - Suppose a consumer has wealth \( m \) and faces prices \( p_1, p_2 \) for goods \( x_1, x_2 \).
  - \( p_1 x_1 + p_2 x_2 \leq m \) defines the budget set (what's available)
  - \( p_1 x_1 + p_2 x_2 = m \) defines the budget line (what's maximally available: all wealth is spent)
    - this can be rewritten \( x_2 = \frac{m}{p_2} - \left( \frac{p_1}{p_2} \right) x_1 \)

Constraints: An Example, cont’d

- Budget line: \( x_2 = \frac{m}{p_2} - \left( \frac{p_1}{p_2} \right) x_1 \)
Choice and Individual Demand

“Doing what you most prefer to do given the options that are open to you.”
Calculus-based Maximization

Choice

- Rational agents always choose to do what they most prefer to do, given the options that are open to them.

- Implication: \( \text{MRS} = - \frac{\text{MU}_1}{\text{MU}_2} = - \frac{p_1}{p_2} \)
Choice: Special Cases

- Sometimes the condition $MRS = -\frac{p_1}{p_2}$ does not hold.
- Example: “kinky” tastes

Choice: Special Cases, cont’d

- Sometimes the condition $MRS = -\frac{p_1}{p_2}$ does not hold.
- Example: boundary (corner) solutions
Choice: Special Cases, cont’d

- Sometimes the condition $\text{MRS} = - \frac{p_1}{p_2}$ is not sufficient.
- Example: nonconvex preferences

![Diagram of indifference curves and budget line]

Choice: Special Cases, cont’d

- We will generally make suitable assumptions so that $\text{MRS} = - \frac{p_1}{p_2}$ really characterizes the optimal choice:
  - We will usually make “smoothness” assumptions about utility (this rules out kinked indifference curves).
  - We will usually restrict ourselves to interior optima (this rules out the boundary [corner] optimum case).
  - (Strict) convexity rules out that the tangency condition is not sufficient.
Choice and Calculus

- Restricting attention to convex, smooth preferences, and interior optima, has the advantage that we can use calculus to find the consumer's optimal choice.
  - Rational agents always choose to do what they most prefer to do, given the options that are open to them.
  - Rational agents always choose to do what maximizes their utility, subject to the (budget) constraint.

- We want to solve ("constrained maximization"): 
  \[
  \max_{x_1, x_2} u(x_1, x_2) \\
  \text{s.t.: } p_1 x_1 + p_2 x_2 = m 
  \]

Choice and Calculus, cont’d

- Example:
  - constraint $x_1 + x_2 = 4$
  - max $x_1^{0.5} x_2^{0.5}$ s.t. $x_1 + x_2 = 4$
Review: Maximization

- $f(x)$ attains its maximum at $x^*$:
  - The maximum is characterized by the fact that at $x^*$, the function has a slope of zero, that is: $f'(x^*) = 0$
- So we know that the solution to $\max f(x)$ is characterized by the (necessary) condition $f'(x^*) = 0$.

Review: Maximization, cont’d

- Remember that $f'(x^*) = 0$ is only a necessary, not a sufficient condition for the maximum!
  - This function $f(x)$ has several ("stationary") points at which $f'(x) = 0$,
  - but only one of them is the (global) maximum;
  - one is a local maximum;
  - and one is not a maximum at all but a (local) minimum.
Review Buzz Group: Maximization

- Find the (global) maximum of the function
  \[ f(x) = 16x - 4x^2. \]
- Now find the (global) maximum of the following positive monotonic transformation \( g(\bullet) \) of \( f(x) \):
  \[ g(f(x)) = \ln (f(x)) = \ln (16x - 4x^2). \]
  - (Assume that only \( x \) values between 0 and 4 are admissible.)
- What have we learned from this?

Multi-Variate Maximization

- Functions of two variables:
  - Example: \( u(x_1, x_2) \)
    \[ \max_{x_1, x_2} u(x_1, x_2) \]
- What are the appropriate necessary conditions for a maximum?
  - \( \frac{\partial u(x_1, x_2)}{\partial x_1} = 0 \)
  - \( \frac{\partial u(x_1, x_2)}{\partial x_2} = 0 \)
Constrained Maximization

- Recall that we wanted to solve:

\[
\max_{x_1, x_2} u(x_1, x_2) \\
\text{s.t.: } p_1 x_1 + p_2 x_2 = m
\]

- Here we are not just maximizing a function of two variables, but we have to be careful that the values of \(x_1\) and \(x_2\) we choose obey the constraint.

- The easiest method for solving maximization problems with one or more equality constraints is the method of Lagrange multipliers.

Constrained Maximization, cont’d

- (1) rewrite the constraint as: \(\ldots = 0\).
  - \(\max u(x_1, x_2)\) s.t. \(p_1 x_1 + p_2 x_2 - m = 0\)

- (2) form the following function (Lagrangean):
  - \(L(x_1, x_2, \lambda) = u(x_1, x_2) - \lambda (p_1 x_1 + p_2 x_2 - m)\)
  - \(\lambda\) is called the Lagrange multiplier

- (3) the necessary conditions for a maximum are:
  - (i) \(\frac{\partial L(x_1, x_2, \lambda)}{\partial x_1} = 0\)
  - (ii) \(\frac{\partial L(x_1, x_2, \lambda)}{\partial x_2} = 0\)
  - (iii) \(\frac{\partial L(x_1, x_2, \lambda)}{\partial \lambda} = 0\)

- (4) solve equations (i) - (iii) for \(x_1\) and \(x_2\).
  - This gives us \(x_1(p_1, p_2, m)\) and \(x_2(p_1, p_2, m)\), the consumer’s demand functions for goods 1 and 2.
Constrained Maximization, cont’d

- The general case of two goods:
  - write down the maximization problem:
    - \( \max u(x_1, x_2) \text{ s.t. } p_1x_1 + p_2x_2 - m = 0 \)
  - write down the Lagrangean:
    - \( L(x_1, x_2, \lambda) = u(x_1, x_2) - \lambda (p_1x_1 + p_2x_2 - m) \)
  - write down the necessary (first-order) conditions:
    - (i) \( \frac{\partial u(x_1, x_2)}{\partial x_1} - \lambda p_1 = 0; \text{ or: } \frac{\partial u(x_1, x_2)}{\partial x_1} = \lambda p_1 \)
    - (ii) \( \frac{\partial u(x_1, x_2)}{\partial x_2} - \lambda p_2 = 0; \text{ or: } \frac{\partial u(x_1, x_2)}{\partial x_2} = \lambda p_2 \)
    - (iii) \( p_1x_1 + p_2x_2 - m = 0 \)
  - we cannot solve explicitly for \( x_1 \) and \( x_2 \) - but we can divide (i) by (ii) to obtain the familiar:
    - \( \frac{\partial u(x_1, x_2)}{\partial x_1} \bigg| _{x_1} = \frac{\lambda p_1}{\lambda p_2} \)

Choice and Calculus, cont’d

- Example: (Cobb-Douglas)
  - \( \max_{x_1, x_2} x_1^{0.5}x_2^{0.5} \text{ s.t. } x_1 + x_2 = 4 \)
  - Write the Lagrangean:
    - \( L = x_1^{0.5}x_2^{0.5} - \lambda (x_1 + x_2 - 4) \)
  - Necessary conditions:
    - (i) \( 0.5x_1^{-0.5}x_2^{0.5} - \lambda = 0 \)
    - (ii) \( x_1^{0.5}0.5x_2^{-0.5} - \lambda = 0 \)
    - (iii) \( x_1 + x_2 - 4 = 0 \)
  - Now solve for \( x_1 \) and \( x_2 \):
    - (i’): \( 0.5x_1^{-0.5}x_2^{0.5} = \lambda \)
    - (ii’): \( x_1^{0.5}0.5x_2^{-0.5} = \lambda \)
    - (i’)/(ii’): \( x_1^{-1}x_2^{1} = 1 \)
    - or: \( x_2 / x_1 = 1 \)
    - or: \( x_1 = x_2 \)
    - or: \( x_1 + x_2 = 2x_2 \)
    - from (iii): \( 4 = 2x_2 \)
    - or: \( x_2 = 2 \)
    - hence: \( x_1 = 2 \)
Calculus and Indifference Curves

- This is how our maximization problem connects up with the usual indifference curve story:

![Calculus and Indifference Curves Diagram]

Buzz Group: Choice

- Now solve the more general example (C-D):
  \[
  \max_{x_1, x_2} x_1^a x_2^{1-a}
  \]
  s.t. : \( p_1 x_1 + p_2 x_2 = m \)
  
  - \((a \text{ is between } 0 \text{ and } 1)\)

- **Hint**: remember that the positive monotonic transformation \( \ln(x_1^a x_2^{1-a}) \) contains the same information as \( x_1^a x_2^{1-a} \):
  - \( \ln(x_1^a x_2^{1-a}) = a \ln(x_1) + (1-a) \ln(x_2) \) makes your life (much) easier.
Choice and Demand Functions

- The solutions to
  \[
  \max_{x_1, x_2} x_1^{a} x_2^{1-a} \\
  \text{s.t. } p_1 x_1 + p_2 x_2 = m
  \]
- are:
  - \( x_1 = a m/p_1 \)
  - \( x_2 = (1-a) m/p_2 \).
- The relationship between \( x \) and \( p \) is the consumer's demand function for the good.
- The relationship between \( x \) and \( m \) is the consumer's Engel curve for the good.

Demand

- Varying own price. Example: \( x_1(p_1; p_2, m) = a m/p_1 \)
Demand, cont’d

- Normally, demand for a good decreases as price increases:
  - $\frac{\partial x_1(p_1; p_2, m)}{\partial p_1} < 0$.
  - Example (Cobb-Douglas): $x_1(p_1; p_2, m) = \frac{a m}{p_1}$
    - $\frac{\partial x_1(p_1; p_2, m)}{\partial p_1} = -\frac{a m}{(p_1)^2} < 0$
- For Giffen goods, demand increases as price increases:
  - $\frac{\partial x_1(p_1; p_2, m)}{\partial p_1} > 0$.

Engel Curve

- Varying income. Example: $x_1(m; p_1, p_2) = a \frac{m}{p_1}$
Engel Curve, cont’d

- For normal goods, demand increases as income increases:
  - $\partial x_1(m; p_1, p_2) / \partial m > 0$.
  - Example (Cobb-Douglas): $x_1(m; p_1, p_2) = a m / p_1$
    - $\partial x_1(m; p_1, p_2) / \partial m = a / p_1 > 0$ (for positive prices)

- For inferior goods, demand decreases as income increases:
  - $\partial x_1(m; p_1, p_2) / \partial m < 0$.

Substitutes and Complements

- In general, the solution to the consumer’s maximization problem gives us $x_1(p_1, p_2, m)$ for good 1 and $x_2(p_1, p_2, m)$ for good 2.
  - (In the Cobb-Douglas example $x_1$ did not depend on $p_2$ because of the special form of the utility function.)

- One way of defining substitutes and complements is: how does demand for good 1 change as the price of good 2 changes?
  - Definition: good 1 is a (gross) substitute for 2 if:
    - $\partial x_1(p_2; p_1, m) / \partial p_2 > 0$.
  - Definition: good 1 is a (gross) complement for 2 if:
    - $\partial x_1(p_2; p_1, m) / \partial p_2 < 0$. 

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Behind Individual Demand

Income and Substitution Effects:
Hicks v Slutsky

What happens as price falls?

- The good is now relatively cheaper (relative to other goods).
  - Typically, the consumer will substitute away from other goods, and towards the good for which the price has fallen.
  - This is the substitution effect.
- The consumer is now “wealthier” (she could still buy the same bundle and have money left over).
  - Typically, this will lead the consumer to buy more of that good as her wealth increases.
  - This is the income effect (wealth effect).
The Hicks Decomposition

- A price fall has made the consumer “wealthier:” to isolate the (Hicks) substitution effect, take away just enough income to make the consumer equally as well off as before the price change.

Hicks Income, Substitution FX

- price fall
Hicks Income, Substitution FX

A price fall has made the consumer “wealthier:” to isolate the (Slutsky) substitution effect, take away just enough income to make the consumer be able to afford the same bundle as before the price change.
Slutsky Income, Substitution FX

price fall

Slutsky Income, Substitution FX

price increase
Slutsky Equation

- Suppose you initially consume
  - bundle \((\bar{x}_1, \bar{x}_2)\)
  - at prices \((\bar{p}_1, \bar{p}_2)\)
  - and with income \(\bar{m}\)

- Define the following function:
  \[
  x_1^*(p_1, p_2, \bar{x}_1, \bar{x}_2) = x_1(p_1, p_2, \bar{p}_1 \bar{x}_1 + p_2 \bar{x}_2)
  \]
  This is your demand function for good 1 when you have just enough income to be able to buy \((\bar{x}_1, \bar{x}_2)\).

- Now differentiate both sides of this identity w.r.t. \(p_1\):
  \[
  \frac{\partial}{\partial p_1} x_1^*(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{\partial x_1}{\partial p_1}(p_1, p_2, \bar{m}) \bar{x}_1 + \frac{\partial x_1}{\partial m}(p_1, p_2, \bar{m}) \bar{x}_1
  \]

Slutsky Equation, cont’d

- We have just derived:
  \[
  \frac{\partial}{\partial p_1} x_1^*(p_1, p_2, \bar{x}_1, \bar{x}_2) = \frac{\partial x_1}{\partial p_1}(p_1, p_2, \bar{m}) \bar{x}_1 + \frac{\partial x_1}{\partial m}(p_1, p_2, \bar{m}) \bar{x}_1
  \]
- Which we can rewrite as:
  \[
  \frac{\partial x_1}{\partial p_1}(p_1, p_2, \bar{m}) = \frac{\partial x_1^*}{\partial p_1}(p_1, p_2, \bar{x}_1, \bar{x}_2) - \frac{\partial x_1}{\partial m}(p_1, p_2, \bar{m}) \bar{x}_1
  \]
- This is the Slutsky Equation:
  - It tells us that the effect on demand of a price change …
  - is made up of a substitution effect (keeping purchasing power constant, i.e. allowing the consumer to buy the original bundle) …
  - and an income effect.
We know which way the income effect operates:
- this depends on whether the good is a normal or inferior good.

We also know which way the substitution effect operates:
- it always goes in the opposite direction to the price change:
- For a price fall, the substitution effect says: consume more of the good.

Substitution and income effects for a Giffen good and a (non-Giffen) inferior good.
**From Individual Demand to Market Demand**

Adding Up.

**Individual to Market Demand**

- Market demand (total demand for one particular good) is just the sum of individual demands functions:
  - A’s demand function for good 1: $x_1^A(p_1, p_2, m^A)$
  - B’s demand function for good 1: $x_1^B(p_1, p_2, m^B)$
  - C’s demand function for good 1: $x_1^C(p_1, p_2, m^C)$
  - ...

- Market demand:
  - $X_1(p_1, p_2, m^A, m^B, m^C, \ldots) = x_1^A(p_1, p_2, m^A) + x_1^B(p_1, p_2, m^B) + x_1^C(p_1, p_2, m^C) + \ldots$
Individual to Market Demand, cont.

Price Elasticity of Demand

- The price elasticity of demand measures the responsiveness of demand for a good with respect to changes in the price of that good.

- **Definition**: The *price elasticity of demand* is the percentage change in the quantity demanded that results from a 1 percent change in price.

- Precisely, it is: \[ \eta = \frac{\Delta X / X}{\Delta p / p} \]

- or, rewritten: \[ \eta = \frac{\Delta X / X}{\Delta p / p} = \frac{\Delta X}{\Delta p} \cdot \frac{p}{X} = \frac{dX}{dp} \cdot \frac{p}{X} \]
Price Elasticity of Demand, cont’d

\[ \eta = \frac{dX}{dp} \cdot \frac{p}{X} \]

- The price elasticity of demand at some point on the demand curve is
  - the derivative of demand with respect to price,
  - times the ratio of price to quantity at that point on the demand curve.

- Implications: the price elasticity of demand is
  - (probably) different at every point on the demand curve;
  - nonpositive for non-Giffen goods.

Price Elasticity of Demand, cont’d

- We call demand (at some point) elastic, if the quantity demanded is relatively responsive to changes in price.
  - Definition: demand is elastic whenever \( \eta < -1 \).

- We call demand (at some point) inelastic, if the quantity demanded is relatively unresponsive to changes in price.
  - Definition: demand is inelastic whenever \(-1 < \eta < 0\).

- We call demand (at some point) unit elastic, if the quantity demanded changes proportionately to changes in price.
  - Definition: demand is unit elastic whenever \( \eta = -1 \).
Buzz Group: Elasticity

- Calculate the price elasticity of demand of the following demand curve:
  - $X(p) = p^{-a}$,
  - (where $a > 0$),
- and sketch the demand curve.
- Economists draw “inverse demand curves,” that is they draw price as a function of quantity $p(X)$.

Elasticity and Marginal Revenue

- Suppose you can sell your product in a market with the inverse demand curve $p(X)$.
  - That is, if you sell $X$ units of your product, you will make $p(X)$ for each unit you sell.
  - Your revenue is: $R(X) = X \cdot p(X)$.
- How does your revenue change as you change price?
  - (using the product rule):
    $$\frac{dR(X)}{dX} = p(X) + X \frac{dp(X)}{dX}$$
Elasticity and MR, cont’d

- So marginal revenue (the change in revenue when price changes) is:
  \[ MR = \frac{dR(X)}{dX} = p(X) + X \frac{dp(X)}{dX} \]
- But remember that elasticity is \((dX/dp) \cdot (p/X)\):
  \[ MR = p + X \frac{dp}{dX} = p + p \frac{X \frac{dp}{dX}}{p} = p \left(1 + \frac{X \frac{dp}{dX}}{p} \right) = p \left(1 + \frac{1}{\eta} \right) \]
- If demand is inelastic: MR is negative
  - Revenue decreases when you increase output (lower price)
- If demand is elastic: MR is positive
  - Revenue increases when you increase output (lower price)