1. A monopolist has the following short-run cost function: \( c(y) = (y - 2)^3 + 12 \). Demand for her product is given by the demand function (demand, \( y \), of course is a function of price, \( p \)), as follows: \( y(p) = 4 - 2p \).

   a. (5 points) What is the monopolist’s fixed cost?
      
      
      answer: Fixed cost is the cost (in the short run) of producing zero output, that is, \( F = c(0) = (0 - 2)^3 + 12 = -8 + 12 = 4 \).

   b. (8 points) What is the monopolist’s profit-maximizing level of output in the short run?
      
      answer: Write up profit and maximize with respect to \( y \). First you need the inverse demand function. From \( y = 4 - 2p \), you get \( p = 2 - (1/2)y \). Then:
      
      \[
      \max [2 - (1/2)y]y - [(y - 2)^3 + 12]
      \]
      
      The first-order condition is:
      
      \[
      2 - y = 3(y - 2)^2, \text{ or}
      \]
      
      \[
      y = 2.
      \]
2. A perfectly competitive firm has the following long-run production function: \( y = x_1^{2/3}x_2^{2/3} \). The input prices for inputs 1 and 2 are \( w_1 \) and \( w_2 \), respectively. Hint: you can answer each of the following subparts without knowing results obtained in other subparts of this question. That is, each subpart is independent of each other.

a. (6 points) For input 1, does the "law of diminishing returns" hold? Calculate the appropriate marginal product and show why this implies (or does not imply) diminishing returns.

answer: Calculate the marginal product of input 1 (partial derivative of production function with respect to input 1):

\[ MP_1 = \frac{2}{3} x_1^{-1/3} x_2^{2/3} \]

This is obviously decreasing with greater \( x_1 \) (the derivative of \( MP_1 = -(2/9)x_1^{-4/3}x_2^{2/3} \) is negative), so there are diminishing returns to input 1.

b. (6 points) Does this production function exhibit increasing, decreasing, or constant returns to scale? Show why.

answer: The definition of increasing returns to scale is that \( f(tx_1, tx_2) > t f(x_1, x_2) \). Here, \((tx_1)^{2/3}(tx_2)^{2/3} = t^{4/3} (x_1^{2/3}x_2^{2/3}) > t (x_1^{2/3}x_2^{2/3})\), so there increasing returns to scale.

c. (5 points) What is the slope of this firm’s isoquant (as always, input 1 is on the horizontal axis and input 2 is on the vertical axis) if this firm were to use 4 units of input 1, and 2 units of input 2?

answer: The slope of the isoquant is known as the TRS (technical rate of substitution). It is (minus) the ratio of the marginal products, i.e. \(-MP_1/MP_2\).

Calculating the marginal products, the TRS = \(-x_2/x_1\). For the given quantities of inputs, that is \(-1/2\).

d. (5 points) Write down the Lagrangean you would use to find the answer to the firm’s cost-minimization problem. Just write down the Lagrangean - you do not need to take the first-order conditions and you do not need to calculate the solution.

answer: \( L = w_1x_1 + w_2x_2 - \lambda (x_1^{2/3}x_2^{2/3} - y) \)
e. (5 points) The cost function that you could derive from part (c) of this question might look like this: $c(y) = \frac{4}{3} \cdot y^{3/4}$. Suppose (for this part of the question only) that $w_1 = $2 and $w_2 = $18. What is this perfectly competitive firm’s profit-maximizing level of output when the price of output is $90 per unit of output?

answer: With the given input prices, cost is $c(y) = 12 \cdot y^{3/4}$. A perfectly competitive firm maximizes profit (price times output minus cost), i.e. $\text{max } 96y - 12y^{3/4}$.

The first-order condition is: $90 = 12 \cdot (3/4) \cdot y^{-1/4}$, or $90 = 9 \cdot y^{-1/4}$, or $10 = y^{-1/4}$. Solving for $y$ you get:

$$y = 0.0001.$$ 

f. (5 points) Suppose (for this part of the question only) that in the short run, input 2 is fixed at $x_2 = 8$. Suppose further that that $w_1 = $2 and $w_2 = $18. What is the firm’s short-run cost function?

answer: The short-run production function is $y = x_1^{2/3}x_2^{2/3}$, $y = 4x_1^{2/3}$. That is, to produce $y$ units of output, you have to use (solve out for $x_1$) $x_1 = (y/4)^{3/2}$ units of input 1. That is, your cost function is:

$$c_s(y) = w_1 \cdot (y/4)^{3/2} + w_2 \cdot 8,$$

or, for the given values of $w_1$ and $w_2$,

$$c_s(y) = 2 \cdot (y/4)^{3/2} + 144 = (1/4) \cdot y^{3/2} + 144.$$ 

3. (5 points) For a perfectly competitive firm with the short-run cost function $c(y) = 5 \cdot y^2 + 20$, calculate the profit-maximizing level of output when the price of output is $50, and also check if the firm indeed wants to produce that level of output in the short run or whether it prefers to shut down in the short run.

answer: Profit is $50y - 5y^2 - 20$, and the first-order condition for maximization is $50 = 10y$, that is, $y = 5$. This is the profit-maximizing quantity of output. Next, check if the firm would prefer to shut down. The variable cost is $5y^2$. The average variable cost is $5y$. At the level of output chosen ($y = 5$), average variable cost is 25. This is below the price of 50, so the firm indeed wants to produce $y = 5$ and NOT shut down in the short run.
4. (10 points) We know you are risk averse. I offer you the following lottery: with probability 1/3 you will win $600 and with probability 2/3 you will win $60. You can have either that lottery or $200 for certain. Will you choose the lottery, choose the $200 for certain, or can we not tell what you will do without further information? Explain your reasoning.

answer: Risk aversion is defined as: you prefer the expected value of a lottery to the lottery. Here, the expected value is \((1/3)\times600 + (2/3)\times60 = 240\). So you prefer $240 to playing the lottery, that's all we know. Will you also prefer $200 to the lottery? We don't know, and therefore the answer is: we can't tell.

5. (10 points) You are thinking about the following lottery: you will lose $100 with probability 1/2, and you will win $100 with probability 1/2. For you, the certainty equivalent of this lottery is $10. Are you risk-averse, risk-neutral, or risk-loving? Illustrate with a diagram.

answer: The expected value of this lottery is $0. The certainty equivalent is greater than the expected value, and therefore you are risk-loving. See the following diagram:
6. (10 points) At the moment you are uninsured. Here is the setup of the problem:

- If you remain healthy (the “good state”), you will earn $100. If you get sick (the “bad state”) you will earn only $10.
- Your probability of getting sick is 0.2.
- Your von Neumann-Morgenstern utility function (defined over certain money payments, \( m \)) is as follows: \( u(m) = \ln(m) \) [note that “\( \ln \)” denotes the natural logarithm].
- You can choose how much insurance to buy (the insurer offers you insurance at a rate \( \gamma \)), so that your budget constraint is \( \gamma \cdot m_b + (1 - \gamma) \cdot m_g = \gamma \cdot c_b + (1 - \gamma) \cdot c_g \), where \( m_g \) and \( m_b \) is your income in the good and in the bad state, and \( c_g \) and \( c_b \) is your consumption in the good and in the bad state.
- You are offered insurance at a rate \( \gamma = 0.5 \) (this is not actuarially fair insurance!).

Use the Lagrangean method to determine how much consumption you will choose in the good state, and how much in the bad state.

answer: The budget constraint for the values of \( m_g = \$100 \) and \( m_b = \$10 \) and \( \gamma = 0.5 \) is:

\[
55 = 0.5 \cdot c_b + 0.5 \cdot c_g.
\]

The Lagrangean is: 

\[
L = 0.2 \cdot \ln(c_b) + 0.8 \cdot \ln(c_g) - \lambda (0.5 \cdot c_b + 0.5 \cdot c_g - 55)
\]

The first-order conditions are:

(i) \( 0.2/c_b = \lambda \cdot 0.5 \)
(ii) \( 0.8/c_g = \lambda \cdot 0.5 \)
(iii) \( 55 = 0.5 \cdot c_b + 0.5 \cdot c_g \), or: \( 110 = c_b + c_g \)

From (i) and (ii) combined you get \( 0.2 \cdot c_g = 0.8 \cdot c_b \), or \( c_b = (0.2/0.8)c_g \). Put this into (iii) to get \( 110 = (0.2/0.8)c_g + c_g \), or \( 110 = (5/4)c_g \), or \( c_g = 88 \). From that, you get \( c_b = (0.2/0.8)88 = 22 \).
7. (20 points) “When individuals can lend and borrow at the same rate of interest, the consumption decisions of individual shareholders are completely separate from the investment decisions of the firm.” Explain, using a diagram.

answer: I would have expected an explanation of: what the investment opportunity schedule represents (i.e. it is the constraint according to which the firm can choose how to make its investments, i.e. how to make payouts to shareholders over time); about how being able to lend and borrow at the same rate of interest introduces an intertemporal budget constraint for shareholder along which they can lend and borrow away from the allocation that the firm gives them; how this implies that the firm should choose to structure its payouts such that the budget constraint is as far out as possible (i.e. maximizing the present value of the shareholders’ endowments, i.e. maximize the present value of the firm); and how that implies that the shareholders’ preferences are best satisfied by this strategy, and therefore independent of what their tastes for current versus future consumption are. The relevant diagrams are on slides 32 - 33 of Topic 1a.