Topic 4: Theory of the Firm

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Based Primarily on Frank Chapters 9 - 12

**Firms**

- **Demand:**
  - Inputs: labor, capital
  - Production
  - Buy in factor market
  - Cost

- **Supply:**
  - Output
  - Sell in product market
  - Revenue

**Objective:** firms are interested in profit = revenue - cost.
Production for a neoclassical economist is a “black box”:

- We model production as a function that turns inputs into output:
  \[ q = f(k, l) \]

- where:
  - q: output
  - k: capital input
  - l: labor input
  - f(·, ·): production function
Short Run and Long Run

- Firms may not immediately be able to change the quantity of all inputs they use.
  - Example: buildings, etc.
- The long run is defined as the shortest period of time in which a firm can change the quantity of all inputs it uses.
  - An input whose quantity can be freely adjusted is a variable input.
- The short run is the period of time during which one or more inputs cannot be varied.
  - An input whose quantity cannot be freely adjusted is a fixed input.

Production in the Short Run

When not all inputs can be varied.
Short-Run Production

- Suppose a firm produces output according to the production function \( q = f(k, l) \).
- Suppose that, in the short run, the amount of capital cannot be varied (fixed input) - assume it is fixed at \( k_0 \).
- We can then plot the amount of output produced as we vary the amount of labor (variable input).
- This gives us the *short-run production function*.

Short-Run Production Function

- The short-run production function \( f(k_0, l) \) plots the quantity of output (*total product*), as one input (labor) is varied (holding capital fixed at \( k_0 \)).
Marginal and Average Product

- The marginal product (MP) of a variable factor measures the increase in output from a small increase in the variable factor.
  - \( MP_i = \frac{\Delta q}{\Delta l} \)
  - \( MP_i \) is the slope of the short-run production function.

- The average product (AP) of a variable factor measures how much output each unit of input yields on average.
  - \( AP_i = \frac{q}{l} \)
  - \( AP_i \) is the slope of the line from the origin to the corresponding point on the production function.

Marginal Product

- The "law of diminishing returns": after some point, marginal product declines.
- The slope of the short-run production function is the marginal product (MP) of the variable input:
  - as the variable input is increased by a little, by how much does output increase?
  - \( MP_i = \frac{\Delta q}{\Delta l} \)
  - Note that this changes along the production function.
The average product (AP) of some input is the quantity of output produced, on average, with each unit of the variable input:

- \( AP = \frac{q}{l} \)
- graphically, it is the slope of the line connecting the origin and the corresponding point on the production function.

Marginal and Average Product

The marginal product (MP) is the increase in output when one additional unit of the variable input is used, while the average product (AP) is the total output divided by the number of units of the variable input.

Graphically, the marginal product is the slope of the tangent line to the production function at a given level of output, while the average product is the slope of the line connecting the origin and the corresponding point on the production function.
Buzz Group

- You own two car production sites, and you have a total workforce of 100. Each site operates a slightly different production technology, but both sites produce the same product. Currently 50 workers are employed at site A, and 50 are employed at site B.
  - If you were to add one more worker to site A, she would raise production at site A by 3 cars per day. If you were to add one more worker to site B, she would raise production at site B by 4 cars per day.
  - At site A, each worker on average produces 10 cars per day. At site B, each worker on average produces 8 cars per day.
- Should you reallocate workers between the two sites?

Maximizing Profit

- A firm’s profit is total revenue less total cost.
- In the short run, where capital is fixed at $k_0$, profit is:
  - $\pi = p \cdot q - w \cdot l - r \cdot k_0$
- A small change in labor input ($\Delta l$) changes output by $MP_l$, and profit by:
  - $\Delta \pi / \Delta l = p \cdot MP_l - w$
- If this is positive, employing more labor increases profit. If it is negative, decreasing labor input increases profit. So, at a profit maximum:
  - $p \cdot MP_l - w = 0$
  - $MP_l = w / p$
Maximizing Profit

- So we know that, in order to maximize profit, a firm employs workers until $\text{MP}_l = \text{w/p}$.
- Comparative statics:
  - as the real wage ($\text{w/p}$) increases, the firm will employ fewer workers.
- Another way of putting this:
  - each worker is paid her marginal productivity.

Production in the Long Run

When all inputs can be varied.
Long-Run Production

- In the long run, all factors of production can be varied.
- Production function $q = f(k, l)$.
  - How do we represent this graphically?
- An *isoquant* (sometimes called production isoquant) is the set of all input combinations that yield the same level of output.
  - **Example:** $q = 2kl$. What is the isoquant for $q = 16$?
    - $16 = 2kl$
    - $k = 8 / l$
    - ... and similarly for other levels of output.

Isoquants

- Isoquants for the production function $q = 2kl$. 

![Isoquant Graph](image-url)
The (absolute value of the) slope of an isoquant is the *marginal rate of technical substitution* (sometimes referred to as the *technical rate of substitution*):

• It is the rate at which, in a given production process, the firm can substitute a little more of one input for a little less of the other input.

Note the similarity with the MRS in consumer theory!

There is a relationship between MRTS and MP:

• By how much is output reduced if you reduce capital by \( \Delta k \)?
  » output reduction: \( MP_k \Delta k \) (1)

• By how much is output increased if you increase labor by \( \Delta l \)?
  » output increase: \( MP_l \Delta l \) (2)

• Along an isoquant, output is constant, so (1) and (2) are equal:
  » \( MP_k \Delta k = MP_l \Delta l \), or: \( \frac{MP_k}{MP_l} = \frac{\Delta k}{\Delta l} = \text{MRTS} \)
Returns to Scale

- In the long run, all inputs can be varied.
- Suppose all inputs were doubled. Would output:
  - double?
  - more than double?
  - less than double?
- This is a question of *returns to scale*: if we “scale up” production, does output increase more or less than proportionately?
- Returns to scale is a long-run concept.

Returns to Scale, cont’d

- **Definition**: If a proportional change in all inputs leads to a more than proportional change in output, the production process exhibits *increasing returns to scale*.
- **Definition**: If a proportional change in all inputs leads to a less than proportional change in output, the production process exhibits *decreasing returns to scale*.
- **Definition**: If a proportional change in all inputs leads to a proportional change in output, the production process exhibits *constant returns to scale*. 
Costs

Fixed, Variable, Total;
Average, Marginal.
And what to do with them.

Costs in the Short Run

When not all inputs can be varied.
Fixed and Variable Costs

- **Fixed cost (FC):** the cost of fixed inputs.
  - Example: Capital is fixed at $k_0$. The rental rate for capital (the opportunity cost, if capital is owned) is $r$.
    - What is the fixed cost?
    - $FC = k_0r$

- **Variable cost (VC):** the cost of variable inputs.
  - Example: A firm currently uses an amount $l$ of labor input (hrs). The wage rate is $w$.
    - What is the variable cost?
    - $VC = wl$

- **Total cost (TC):** the sum of FC and VC.

Short-Run Prod. & Variable Cost

![Graphs showing the relationship between input and output in the short run, with a focus on fixed and variable costs.](image)
Average and Marginal Costs

There is a simple relationship between marginal product and marginal cost:
- $MC = \frac{\Delta VC}{\Delta q}$
- $= \frac{\Delta (w \cdot l)}{\Delta q}$
- $= \frac{w \cdot \Delta l}{\Delta q}$
- $= \frac{w}{\Delta q / \Delta l}$
- $= \frac{w}{MP_i}$

Similarly, there is a simple relationship between average product and average cost:
- $AVC = \frac{VC}{q}$
- $= \frac{(w \cdot l)}{q}$
- $= \frac{w}{q / l}$
- $= \frac{w}{AP_i}$
Production and Costs, cont’d

- $MC = \frac{w}{MP_i}$, $AVC = \frac{w}{AP_i}$:

Application: Predatory Pricing

Shut-Down Condition
Areeda-Turner Rule
Predatory Pricing

- What is predatory pricing?
  - One firm lowers its price so far that it drives other firms out of the market ("dumping").
  - Once the other firms have exited from the market, the firm is then free to raise its prices, recover the losses from dumping, and make supernormal profits.
  - This is generally viewed as bad for consumers.
- Predatory pricing is anti-competitive (Sherman Act).
  - Competition authorities are responsible for antitrust legislation enforcement (Clayton Act):
    » Department of Justice (criminal action)
    » Federal Trade Commission (civil action)

Predatory Pricing, cont’d

- What constitutes predatory pricing?
  - When is a price anti-competitively low?
- A price is “too low” if, from charging such a price, the firm’s profit is so low that it would be better to shut down, even in the short run.
  - When would it be best for a firm to shut down, even in the short run?
  - The condition is known as the "shut-down condition."
Shut-Down Condition

- A firm should shut down in the short run if:
  - profit from producing < profit from shutting down (short-run)
  - revenue - TC < 0 - FC
  - revenue - (VC + FC) < - FC
  - revenue - VC < 0
  - revenue < VC
  - p · q < VC
  - p < VC / q
  - p < AVC

- This just says that the price does not even cover variable costs per unit.

Shut-Down Condition, cont’d

- If the firm sells output at a price above average variable cost, but below average total cost, it will still make negative profits:
  - each unit sold contributes to revenue (its price) less than it costs in total to produce that unit.

- But by shutting down, the firm would make even greater losses.
**Areeda-Turner Rule**

- So a firm should shut down, even in the short run, if the price it charges is below its AVC.
- As a rule to judge predatory pricing, this was first argued by:
- Hence: “Areeda-Turner Rule.”

**Costs in the Long Run**

When all inputs can be varied.
Long-Run Cost

- In the long run, all inputs can be varied.
- We already have a tool for representing long-run output.
  - We want cost in the same “space”.

Total Cost

- (Long-run) total cost is:
  - \( \text{LTC} = r \cdot k + w \cdot l \)
  - \( \text{LTC} - w \cdot l = r \cdot k \)
  - \( \frac{\text{LTC}}{r} - \frac{(w/r)}{l} = k \)
- The isocost line plots all combinations of inputs that have the same total cost (at given factor prices).
Minimizing Cost

- The firm wants to produce a given level of output at minimum cost.
  - This is one step to profit maximization:
    » (i) What is the minimum cost at which some output can be produced?
    » (ii) What is the optimum output?
  - This fits a “delegation” story:
    » The manager has established the quantity that some division needs to produce.
    » Now she asks the division to produce this quantity at the lowest cost.
    » How much of each factor should the division use?

Minimizing Cost, cont’d

- Implication: at the optimal choice, we have:
  \[
  \frac{MP_l}{MP_k} = \frac{w}{r}
  \]
- If the firm wants to produce the “required output level,” and it has a production technology with given marginal products for labor and capital, and faces input prices \( w \) and \( r \)...
  - … it should use \( l^* \) units of labor and \( k^* \) units of capital.
Factor Demand

- If a firm is profit maximizing (i.e. cost minimizing), it should employ inputs in its production process such that:

\[
\frac{\text{MP}_\ell}{\ell} = \frac{w}{r} \quad \text{or} \quad \frac{\text{MP}_k}{k} = \frac{w}{r}
\]

... that is, it should use inputs such that the marginal product per dollar spent on each input is the same.

The “law of the equal bang for the buck”.

Long-Run Output Expansion Path

- As required output rises, the firm chooses its optimal input combination. This gives us the long-run output expansion path.

- Each point on the output path is a
  - specific quantity of output (which isoquant is it on?)
  - specific cost (which combination of inputs, at given factor prices?)

- The output expansion path contains the long-run total cost curve, which plots output against (minimum) cost.
Long-Run Total Cost

- The long-run total cost curve traces the least cost of producing given output levels.
- It always passes through the origin: In the long run, a firm can avoid all costs by not producing.

LMC, LAC

- From long-run total cost (LTC), we can derive long-run marginal cost (LMC) and long-run average cost (LAC) in the usual way:

\[
\text{LMC} = \frac{\Delta \text{LTC}}{\Delta q}
\]

\[
\text{LAC} = \frac{\text{LTC}}{q}
\]
LMC, LAC, cont’d

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**Buzz Group**

- You are in charge of Bell Atlantic’s directory assistance.
  - Currently every call is taken by an operator, who looks up the name of the person whose number is requested and then tells the caller that number.
  - Voice recognition software has become much cheaper in recent years, and the price of computers has fallen. Wages have remained the same.
- What long term decisions should you make to ensure Bell Atlantic’s survival in the new millennium?
There is a simple connection between long-run total cost and returns to scale:

- If a production function exhibits constant returns to scale, a doubling of all inputs results in a doubling of output.
- If you double all inputs, long-run total cost doubles:
  - \( \text{LTC} = r \cdot k + w \cdot l; \)
  - \( r \cdot 2k + w \cdot 2l = 2 \text{LTC} \)
- So: a production process exhibits constant returns to scale if a doubling of output results in a doubling of cost, that is, if the LTC curve is a straight line.

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Constant returns to scale:
LTC and Returns to Scale, cont’d

- If a production function exhibits *increasing returns to scale*, a proportional change in *all* inputs results in *more than* a proportional change in output.
- If you change all inputs by a factor of $t$, long-run total cost changes by a factor of $t$:
  - LTC = $r \cdot k + w \cdot l$
  - $r \cdot tk + w \cdot tl = tLTC$
- So: a production process exhibits *increasing returns to scale* if a change in output (by a factor of $t$) results in a change in long-run total cost of *less* than a factor $t$; that is, the LTC curve is concave.

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LTC and Returns to Scale, cont’d

- Increasing returns to scale:

![Graph showing increasing returns to scale](image_url)
LTC and Returns to Scale, cont’d

- If a production function exhibits *decreasing returns to scale*, a proportional change in *all* inputs results in *less* than a proportional change in output.
- If you change all inputs by a factor of $t$, long-run total cost changes by a factor of $t$:
  \[
  \text{LTC} = r \cdot k + w \cdot l;
  \]
  \[
  r \cdot tk + w \cdot tl = t\text{LTC}
  \]
- So: a production process exhibits *decreasing returns to scale* if a change in output (by a factor of $t$) results in a change in long-run total cost of *more* than a factor $t$; that is, the LTC curve is convex.

LTC and Returns to Scale, cont’d

- Decreasing returns to scale:
Buzz Group

- You are a management consultant working for a company that hopes to offer telephone service on cable in Hanover. Your job is to find the company’s Long Run Average Cost curve.
  - The way the company operates is this: First, it builds a cable network that passes every house in Hanover. Then, every time a consumer makes a call, the company incurs a very low cost related to the wear in its main switching facility.
  - Hint: it may help if you draw the Long Run Total Cost curve first.

LTC and Returns to Scale, cont’d

- In almost all industries, decreasing returns to scale set in eventually (i.e. for high enough \( q \)).
  - This explains the shape of the long-run total cost curves we have drawn so far.
- If an industry exhibits increasing returns to scale throughout, we refer to it as a natural monopoly:
  - it is socially better to have one firm exploit the returns to scale, than to have more than one firm produce (at a higher cost).
Natural Monopolies

- Every additional unit produced in the firm lowers long-run average cost: it is better (less costly) to have one firm than two (or more).

Natural Monopolies, cont’d

- Examples of natural monopolies (?):
  - electricity distribution (though not generation)
  - water supply
  - railroads: tracks, signals
  - mail delivery
  - telephone network (but: cellular networks, cable)
  - Dartmouth Bookstore (but: amazon.com)
Minimum Efficient Scale

- $q^*$ is the minimum efficient scale: if a firm produces below $q^*$, it could lower its per-unit cost by producing more.
- If $q^*$ is large (relative to industry output), we should expect the market to be dominated by a few firms.
- (conversely for $q^*$ small)

Costs in the Long and the Short Run

The Envelope Theorem
**Long-Run and Short-Run Costs**

- In the short run, a firm’s capital input is fixed:
  - If it wants to produce more or less, it can only vary labor (the variable input): (ATC is short-run ATC)

**L-R and S-R Costs, cont’d**

- In the long run, the firm can choose capital input.
  - Suppose it can choose between levels $k'$, $k''$, $k'''$. 

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L-R and S-R Costs, cont’d

- In the long run, the firm can choose capital input.
  - Suppose it can choose any level of $k$.

Each average cost curve has a marginal cost curve, that intersects it at its lowest point:
L-R and S-R Costs, cont’d

- In the short run, not all factors are variable.
  - Suppose capital is fixed at \( k \).
  - How does short run cost compare to long run cost?

Production, Cost & Mkt. Structure

- The term "market structure" refers to the environment a firm operates in:
  - Does the firm operate in a competitive market?
  - ... in a market where it is the only supplier (monopoly)?
- What we have so far covered in this topic (production, costs) does not depend on market structure:
  - Production function and (by implication) costs are independent of market structure.
- But a firm’s behavior does depend on market structure: monopolists act differently from competitive firms.
  - We now turn to our study of market structure and firm behavior.
Perfect Competition

ex pluribus unum

Four Conditions

- Perfectly competitive markets have four properties:
  - homogeneous product;
    - all goods sold in this market are “the same” (standardized)
  - firms are price takers;
    - firms treat the market price as given: each firm is small relative to the size of the market
    - free entry or exit
    - (excludes monopoly, etc.)
  - perfect factor mobility;
    - firms can expand/contract/cease production
  - perfect information.
Firm Objective: Profit

- What is a firm’s objective?
  - Firms aim to maximize profit.
- Profit is revenue minus cost:
  \[ \pi = p \cdot q - TC \]
- Distinguish economic profit and accounting profit:
  - Economic profit includes opportunity cost (e.g. for capital that is owned rather than rented)
  - Accounting profit does not include opportunity cost.

Profit Maximization in the Short Run, under Perfect Competition

… when firms cannot enter or leave the market
The slope of the total revenue (TR) curve is \( \text{marginal revenue (MR)} \), or just the price \( p \).

At the profit-maximizing output level, the slopes of TR and TC are the same, or …

\[ \text{MR} = p = \text{MC} \]

**Profit-Maximization (S-R)**

- Profit-maximizing condition: \( p = \text{MC} \)

- \( \text{MC} = \text{MR} \) is the profit-maximizing condition if it is above AVC (recall the shut-down condition!)
Short-Run Firm Supply

- The MC curve (above AVC) tells us how much a firm produces for each given price.
  - It is the firm’s supply curve.

Short-Run Industry Supply

- The supply curve of a competitive industry is just the horizontal sum of the supply curves of the firms in the market.
  - Recall how market demand was made up of the (horizontal) sum of all individual demand curves.
Short-Run Competitive Equilibrium

- Remember one of the conditions for competitive equilibrium: firms are price-takers:
  - Each firm is “small”: it cannot raise price, and there is no point in lowering price. Demand facing a firm is horizontal.

Producer Surplus

- How much better off is a firm as a result of producing some quantity $q^*$?
  - In the short run, if the firm produces nothing, it still has to pay fixed cost. So producer surplus is the difference between revenue and variable cost.
Producer Surplus, cont’d

- Producer surplus is the area between MC and the market price.
  - The MC curve is the firm’s supply curve.
  - So: aggregate producer surplus in a market is the area between the supply curve and the market price.

![Diagram showing producer surplus](image)

Profit Maximization in the Long Run, under Perfect Competition

… when entry and exit is possible
Not a Long-Run Equilibrium

- Suppose $p^*$ were the market price.
  - In the long run, firms can vary all inputs, so the firm would choose to produce where its $\text{LMC} = p^*$ (with the given short-run curves, $\text{MC}$ and $\text{ATC}$). This firm makes positive profits.

Not a Long-Run Equilibrium, cont’d

- This cannot be a long-run equilibrium:
  - A firm makes positive profits
  - Since other firms are perfectly informed about profit opportunities (perfect information assumption!), other firms would enter this market.
  - The supply curve shifts to the right.
**Toward Long-Run Equilibrium**

- Entry shifts the market supply curve to the right.
  - Entry into the market reduces profits.

\[
\begin{align*}
D & \quad S \\
p^* = \text{MR} \\
\text{market} & \quad q \\
D' & \quad S' \\
p^* = \text{MR} \\
\text{firm} & \quad q^*_f \\
L & \quad C
\end{align*}
\]

**Long-Run Equilibrium**

- In long-run equilibrium, all profit has been eliminated through entry into the market:
  - each firm in this industry produces at the lowest point of its long-run average cost curve.
Long-Run Equilibrium, cont’d

- What is good about equilibrium?
  - All firms earn normal (zero) profits.
  - Goods are produced at the lowest possible cost (production at the minimum point on long-run AC).
  - Price is equal to marginal cost:
    » This is allocatively efficient: all gains from trade are realized (no room for private side-trades):
      • Consumers would buy more if the price were lower; but at \( p = MC \), the cost required to produce one more unit is just \( p \), and at that price, consumers do not want to consume more.
      • Producer would supply more at a higher price; but consumers do not wish to buy more at that price.

Long-Run Industry Supply

- The long-run industry supply curve is horizontal, at minimum average cost.
Long-Run Industry Supply, cont’d

- There is an exception to horizontal long-run supply:
  - We have assumed that input prices (and therefore long-run cost) are constant.
  - If input prices rise with expanding production, long-run supply is upward-sloping ("pecuniary diseconomy")

Monopoly

The power to set price.
Perfect Competition and Monopoly

- We have just studied firm behavior in perfectly competitive markets.
  - In competitive markets, firms are price takers.
- We will now study firm behavior in a market in which the firm is the only supplier.
  - This means the firm can choose the price it charges.

P. C. and Monopoly, cont’d

- Why these two limiting cases?
  - They are “easy”:
    - In perfectly competitive markets, each firm is so small that it has a negligible effect on total output. If a firm reduces its output, this has no effect on the price in the market, and therefore no effect on other firms.
    - A monopolist is the only firm in the market, so if it reduces output, it will raise price; but it is the only player in the market, so we need not consider issues of interaction.
  - Interaction is difficult to model:
    - If there are few firms in a market, the decision of each influences the decision of all the others which influences the decision of all the others which … (and so on).
The difference between perfect competition and monopoly is this:

- Each firm in a perfectly competitive market faces a demand curve that is horizontal: the price elasticity of demand is infinite.
  - If a firm chooses to raise price, it will sell nothing.
- The demand curve the monopolist faces is the market demand curve. For the reasons we have discussed in Topic 3, the market demand curve is downward sloping: the price elasticity of demand is finite.
  - If the monopolist raises price, it will sell less, but will not lose all its customers.

Recall: \( TR = p \cdot q \)

- For a firm in a perfectly competitive market, price is fixed:
- If a monopolist wants to increases output, she has to lower price:
The Monopolist’s Short-Run Profit Maximization

When not all inputs can be varied.

- The slope of the total revenue (TR) curve is *marginal revenue* (MR).
- At the profit-maximizing output level, the slopes of TR and TC are the same, or …
- MR = MC
Optimality and Marginal Revenue

- The profit-maximizing condition for a monopolist is, as for a perfectly competitive firm, MR = MC.
  - For a competitive firm, MR = p:
    - if it expands output by one unit, revenue increases by p.
  - But for a monopolist, MR is not equal to price:
    - if a monopolist wants to increase output, she has to lower price (because she faces a downward sloping demand curve), and she has to lower the price for all (not just the last) units she sells; therefore:
    - if monopolist expands output by one unit, revenue increases by less than p: MR < p.

Marginal Revenue

- For small changes in output, this approximates the solid-line MR curve.
- Suppose the monopolist currently charges price $p'$ (so that she sells nothing), and considers selling one more unit of output.
  - To sell one more unit, she has to lower the price to $p''$.
- Starting from $p''$:
  - To sell one more unit, she has to lower the price to $p'''$ for the second and the first unit sold. So her marginal revenue is not $p'''$, but $p''' - (p'' - p''') \ldots$ (etc.)
**Marginal Revenue and Elasticity**

- A producer currently charges price $p$ and sells $q$ units of output:
  - her total revenue is $p \cdot q$
- To expand output by $\Delta q$ (to $q + \Delta q$), she has to lower price to $p + \Delta p$ (where $\Delta p$ is negative and small):
  - her total revenue is $(p + \Delta p) \cdot (q + \Delta q) =
  - = p \cdot q + p \cdot \Delta q + \Delta p \cdot q + \Delta p \Delta q$

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**MR and Elasticity, cont’d**

- Her marginal revenue is the change in total revenue, divided by $\Delta q$ (i.e. for a small change $\Delta q$ in output):
  
  $$\text{MR} = \frac{(p \cdot q + p \cdot \Delta q + \Delta p \cdot q + \Delta p \Delta q - p \cdot q) \cdot \Delta q}{\Delta q} = p + \frac{\Delta p}{\Delta q} \cdot q + \Delta p \Delta q$$

  - (This is clearly less than price since demand is downward-sloping, i.e. $\Delta p/\Delta q < 0$.)
Buzz Group

- What is the definition of price elasticity of demand?
- Draw a linear (straight line) demand curve.
  - What is the price elasticity of demand at the point where \( q=0 \)?
  - What is the price elasticity of demand at the point where \( p=0 \)?
  - Where is the price elasticity of demand -1?

MR and Elasticity, cont’d

- Recall the definition of price elasticity of demand:
  \[
  \eta = \frac{\Delta q}{\Delta p} \frac{p}{q}
  \]
- And we have just derived:
  \[
  MR = p + \frac{\Delta p}{\Delta q} q = p \left( 1 + \frac{\Delta p}{\Delta q} q \right)
  \]
- So we have
  \[
  MR = p \left[ 1 + \frac{1}{\eta} \right]
  \]
  (Recall that \( \eta \) is negative: \( MR < p \))
The elasticity relationship we have derived helps us graph the MR curve:

Recall that
\[ MR = p \left( 1 + \frac{1}{\eta} \right) \]

And: for a linear demand curve \((p = a - bq)\) the elasticity at \(q\) is: \(1 - (a/bq)\).

So elasticity is -1 just where \(q = (1/2)(a/b)\), i.e. halfway along the demand curve.

A monopolist would therefore never produce on the inelastic part of the demand curve:
- on that portion, each additional unit contributes negatively to revenue and increases total cost.
Maximization

- To maximize profit, the monopolist produces output $q^*$ such that $MR=MC$.
- In order to sell $q^*$ units of output, she needs to price output at $p^*$.
- Her profit is $\pi$, the difference between total revenue and total cost.

Maximization and Elasticity

- We know that: $MR = p \left(1 + \frac{1}{\eta}\right)$
- We also know that a monopolist optimally chooses output so that $MC = MR$:
  $$MC = p \left(1 + \frac{1}{\eta}\right)$$

The fraction $\left(1/(1+1/\eta)\right)$ is the monopolist's markup:
- It is the fraction by which a monopolist "marks up" price over marginal cost.
Shutdown Condition

- A monopolist should shut down, even in the short run, when there is no quantity she could sell at which revenue covers variable cost:
  - She should shut down if average variable cost is everywhere above the demand curve.
- If she shuts down, she will still incur fixed costs, but that loss is less than producing any positive amount.

Monopoly Supply

- There is no well-defined (unique) relationship between price and MR when demand shifts:
  - A monopolist may produce different quantities at the same price when demand shifts.
- A monopolist has no supply curve.
  - Instead, she has a supply rule, viz. to set MR=MC.
The Monopolist’s Long-Run Profit Maximization

When all inputs can be varied.

Long-Run Profits

- This monopolist has increasing returns to scale (declining LAC curve throughout), i.e. it is a natural monopoly. This means that long-run profits may persist.
  - The optimal level of capital (or, the fixed factor) is such that it gives rise to the short-run curves MC and ATC.)
Monopoly and Efficiency

Dead-Weight Loss

In the long run, competitive markets operate efficiently: production takes place at minimum average cost, and there are no unexploited gains from trade.

Monopolistic markets (even in the long-run) will not generally be efficient:
- The monopolist restricts output so as to charge a higher price. This should make you suspicious:
- If the monopolist could make one more trade (at a slightly lower price, but without having to reduce the price on the output she already sells), she would want to do it; similarly, there are consumers who would be willing to buy at a slightly lower price.
Inefficiency of Monopoly

The efficiency loss is the loss in surplus: area D (deadweight loss).

In the long-run, this monopolist would make positive profits.
- Her producer surplus is area A, and consumer surplus is area B.
- In a competitive market, profits would encourage entry, to the point where every firm only makes normal profits: output would rise until \( P = LMC \): Consumer surplus is area C.

Inefficiency of Monopoly, cont’d

We have just compared a monopolist to a perfectly competitive market ...
- ... because the cost-structure would have allowed a competitive market.
  - Why is there monopoly in this case? Maybe because the firm has a patent. Even in this case, does the deadweight loss measure the loss in welfare accurately? Not necessarily: without the promise of monopoly profits the patented product might never have been developed.
- This comparison does not always make sense: when the industry is a natural monopoly, what is the alternative to monopoly, for purposes of welfare comparisons?
Natural Monopoly

- If an industry has the cost structure of a natural monopoly, it is socially efficient (least costly) to have one firm produce all the output.
  - The problem is a pricing issue: A monopolist always has the incentive to restrict output and increase the price, below the competitive level.

Policy toward Natural Monopoly

- Do nothing ("laissez-faire").
  - Efficiency? Fairness?
- State ownership.
  - Example: Postal Service
- Private ownership, state regulation of prices.
  - Example: Telecommunications
- Competitive tendering/bidding.
  - Example: PCS auction
Price Discrimination

... when a monopolist can charge different prices

Single-Price Monopoly

- Why does the inefficiency of monopoly arise?
  - The monopolist restricts output so that she can charge a high price:
  - The single-price monopolist (i.e. a monopolist who has to charge the same price for all units she sells) does not increase output because she would have to reduce price on all output she sells.
    - This is just the old story that MR < P.
  - She would like to increase output if she could lower the price on just the additional unit sold, i.e. if she could price-discriminate.
**Price Discrimination**

- **First-degree (perfect) price discrimination:**
  - different prices for different units of output, and
  - different prices for different consumers.

- **Second-degree price discrimination (non-linear pricing):**
  - different prices for different units of output, and
  - same prices for similar customers.

- **Third-degree price discrimination:**
  - same prices for different units of output, but
  - different prices for different customers.

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**First-Degree Price Discrimination**

- Assumption: The perfectly discriminating monopolist knows each consumer’s demand curve.
- The monopolist prices each unit of output at each consumer’s marginal willingness to pay.
First-Degree Price Disc., cont’d

- Perfectly discriminating monopolist would like to sell:
  - to consumer 1: $q_1$ at price $A + A'$; to consumer 2: $q_2$ at $B + B'$
- All consumer surplus is extracted by the monopolist.
- First-degree price discrimination is efficient.
- But: informationally demanding.

First-Degree Price Disc., cont’d

- What limits first-degree price discrimination:
  - unobservable preferences:
    - “informationally demanding”;
  - competition (or the threat of entry)
  - arbitrage (resale):
    - Example: Suppose my marginal willingness to pay is low (i.e. I pay a low price for the quantity I buy). Since my consumer surplus is zero, there are gains from trade if I sell to you (your willingness to pay is high);
  - administrative costs.
Two-Part Tariff

- The monopolist could achieve the same outcome by charging a two-part tariff:
  - charge a one-off fee of $A$ (consumer surplus), and
  - charge each unit bought at marginal cost.
- The consumer will then buy $x_1$ units (i.e. up to where price = willingness to pay), and all consumer surplus is extracted.
- As before, this is efficient, but informationally demanding.

Two-Part Tariff: Examples

- How does economic theory (the theory of two-part tariffs) explain features of the real world?
- Amusement parks:
  - admission fee + marginal cost per ride.
- Telephone line:
  - connection charge + marginal cost per call.
- Xerox photocopiers:
  - rental fee + marginal cost per copy.
Second-Degree PriceDisc. 

- Suppose a monopolist cannot observe each customer’s marginal willingness to pay.
  - But: she can observe the quantity demanded by customers.
  - She could sell different price-quantity “packages”, aimed at customers with different marginal willingness to pay: customers will self-select into buying the “package” designed for them.
- This explains “nonlinear” pricing schedules, e.g. different per-unit prices for large and small users of electricity.

Third-Degree Price Discrimination

- The monopolist charges different prices to different customers (i.e. in different elasticity markets).
  - Examples: private/business telephony, student discounts, business/economy class air travel, …
  - (The monopolist must be able to observe a customer’s demand elasticity.)
- Marginal cost equals marginal revenue in each market. (Monopoly pricing in each market.)
  - (argument by contradiction)
Third-Degree Price Disc., cont’d

Example:
- Low elasticity market: demand $D_1$ (e.g. private telephony)
- High elasticity market: demand $D_2$ (e.g. business telephony)
- Price where marginal cost = marginal revenue

Price is high in the low elasticity market, and low in the high elasticity market.

Third-Degree Price Disc.: Welfare

The welfare effects of third-degree price discrimination (compared with standard monopoly pricing) are ambiguous:

Two inefficiencies:
- Output is too low:
  - The monopolist charges the monopoly price in each market.
  - (She restricts output below the efficient level.)
- Misallocation of goods:
  - Goods are allocated to the wrong individuals.
  - Example: I value a theatre ticket at $40, you value it at $20. You get a student discount (ticket for $15) and buy the ticket. I have to pay the normal price ($50) so I don’t buy the ticket. But my valuation is higher than yours, so I should get the ticket!
Monopolistic Competition

Differentiated Products and the Hotelling Model

Product Differentiation

- Monopolistic Competition: every firm faces a downward-sloping demand curve (i.e. has some degree of monopoly power).
- In an industry with non-homogeneous products, how do firms choose their products’ characteristics?
  - Example: cars, economics courses, …
- Imagine one product characteristic that can be chosen continuously: e.g. location of two ice-cream vendors along a beachfront.
Hotelling’s “principle of minimum differentiation”: both ice-cream vendors locate in the middle of the beach.

- This is not welfare maximising (the location choice in the left-hand panel in the diagram is).
- More examples: political parties, radio stations, ...