Solutions to Additional Problems III # 1-9.

**III.1** A beam of negatively charged particles ("cathode rays") can be deflected by either an electric field or by a magnetic field. By studying the path followed by such cathode rays in the simultaneous presence of applied uniform electric and magnetic fields of known magnitude, J. J. Thomson was able to measure the charge to mass ratio \( (e/m) \) for such particles. He found that:

\[
(e/m) = -1.7588 \times 10^{11} \text{ C kg}^{-1} ;
\]

The negative sign is consistent with the observation that cathode rays are negatively charged particles -- they emanate from the negative electrode and are attracted to the positive electrode.

**III.2** By measuring the charges that oil drops could acquire by colliding with gaseous ions produced by the action of ionising radiation (e.g. X-rays) on air, Robert Millikan found that although the drop could acquire charges of different magnitude, all the values observed were INTEGRAL multiples of the quantity \( 1.602 \times 10^{-19} \text{ C} \). He therefore deduced that \( 1.602 \times 10^{-19} \text{ C} \) was a fundamental quantity of charge in nature, and that \( -1.602 \times 10^{-19} \text{ C} \) was the charge on the electron.

**III.3** The result for \( e \) from Millikan's "oil drop" experiment taken together with the value of \( (e/m) \) from J. J. Thomson's experiments allowed one to calculate a value for the mass, \( m \), of the electron. Using the above values we obtain:

\[
m = \frac{\{ e \}}{\{ (e/m) \}} = \{ -1.602 \times 10^{-19} \text{ C} \} / \{ -1.7588 \times 10^{11} \text{ C kg}^{-1} \}
\]

\[
= 9.11 \times 10^{-31} \text{ kg}
\]

This was an important result, since it showed that the electron has a mass which is very much less than the mass of the lightest atom (i.e. hydrogen). In fact:

\[
m \approx (m_\text{H} / 2000)
\]

Thus, since atoms are electrically neutral, an atom of atomic number \( Z \) must contain a source of positive charge \( (+Ze) \) of magnitude equal to the total charge associated with the electrons. In addition, since electrons are much lighter than the lightest atom, the source of positive charge must be responsible for essentially all the atomic mass.

**III.4.** The approach in this problem is to use the Density (= Weight / Volume) together with the molecular weight to calculate the molar volume from which we may calculate the molecular volume. If we then assume that the atom is spherical with radius, \( r \), then we can calculate a value for \( r \).

For Au : Density = 19.3 g/cm\(^3\) \hspace{1cm} \text{Atomic Weight} = 197 g/mol
Density = \{(Weight of one mole) / (Volume of one mole)\}

\[= \text{(Atomic Weight)} / V_m\]  \hspace{1cm} (1)

where \(V_m\) is the molar volume

Rearranging equation (1) we have:

\[V_m = \text{(Atomic Weight)} / \text{Density} = (197 \text{ g/mol}) / (19.3 \text{ g/cm}^3) = 10.2 \text{ cm}^3 / \text{ mol}\]

Even in a closest-packed structure of spherical atoms, not all the space will be occupied by the atoms. For this structure the atoms occupy 74\% of the available space.

\[\therefore \text{ Actual volume occupied by one mole of atoms} = (74/100) \times (10.2 \text{ cm}^3 / \text{ mol})\]

\[= 7.5 \text{ cm}^3 / \text{ mol}\]

Thus, Volume of one atom = \((7.5 \text{ cm}^3 / \text{ mol}) / (6.0 \times 10^{23} \text{ atoms / mol})\)

\[= 1.3 \times 10^{-23} \text{ cm}^3 / \text{ atom}\]  \hspace{1cm} (2)

Assuming spherical atoms: Volume of one Au atom = \((4/3) \pi \text{ } r_{\text{Au}}^3\)  \hspace{1cm} (3)

where \(r_{\text{Au}}\) is the radius of a spherical Au atom

From equations (2) and (3): \((4/3) \pi \text{ } r_{\text{Au}}^3 = 1.3 \times 10^{-23} \text{ cm}^3 / \text{ atom}\)  \hspace{1cm} (4)

and \(r_{\text{Au}} = \{(3)(1.3 \times 10^{-23} \text{ cm}^3) / (4 \times 3.14)\}^{1/3} = 1.5 \times 10^{-8} \text{ cm}\)

Alternatively, we could express \(r_{\text{Au}}\) in Angstroms (Å) (=1 \times 10^{-8} \text{ cm});

i.e. \(r_{\text{Au}} = 1.5 \text{ Å}\)

\textbf{III.5} On the basis of the observations described in problems 1-3 and the fact that the radii of all atoms are approximately 1 Å (i.e. \(10^{-8} \text{ cm} = 10^{-10} \text{ m}\)), J. J. Thomson had proposed a model of the atom in which both the charge and the mass of the positively charged component of the atom were distributed uniformly throughout a spherical volume of radius \(\approx 1\text{Å}\).

In Rutherford's experiments, \(\alpha\)-particles (He\(^{2+}\) ions) were collimated into a beam which was allowed to strike a thin piece of metallic foil and the angles through which the \(\alpha\)-particles were deflected were measured. Most of the \(\alpha\)-particles remained undeflected, a few were deflected through small angles, but the amazing result was that a very small number were deflected through \textbf{LARGE} angles, even 180°!
From these observations, Rutherford concluded that the positive charge (and hence the mass) must be confined in an extremely SMALL VOLUME of radius \( \approx 10^{-13} \text{ cm} \) (i.e. \( 10^{-15} \text{ m} \)). This was the largest value the spherical volume of positive charge could take and still allow Rutherford to explain the very large (i.e. \( 180^\circ \)) deflections of \( \alpha \)-particles observed.

### III.6

The condition for maximum diffracted intensity (i.e. constructive interference) derived in lecture is:

\[
d \sin \theta = m \lambda \tag{1}
\]

where

- \( \theta \) = the angle of diffraction
- \( d \) = the spacing of the rulings or lines on the diffraction grating or the spacing between the slits
- \( m \) = the order of the diffraction
- \( \lambda \) = the wavelength of the incident radiation

Since we are given \( \lambda \) and the diffraction orders that we should consider, we only need a value of \( d \) to complete the calculations requested.

From the data given:

\[
d = \left\{ \frac{2.54 \text{ cm/inch}}{13,400 \text{ lines/inch}} \right\} = 1.90 \times 10^{-4} \text{ cm}
\]

i.e.

\[
d = 1.90 \times 10^{-6} \text{ m}
\]

In such problems we must take care to put all the quantities in a set of consistent units. We must express the value for \( \lambda \) in \( \text{cm} \) (if we express \( d \) in \( \text{cm} \)) or in \( \text{m} \) (if we express \( d \) in \( \text{m} \)).

Thus,

\[
\lambda = 6328 \text{Å} = 6328 \times 10^{-8} \text{ cm} \text{ or } 6328 \times 10^{-10} \text{ m}
\]

i.e.

\[
\lambda = 6.328 \times 10^{-7} \text{ m}
\]

For first-order diffraction, \( m = 1 \), and thus,

\[
\sin \theta = \frac{\lambda}{d} = \left\{ \frac{6.328 \times 10^{-7} \text{ m}}{1.90 \times 10^{-6} \text{ m}} \right\} = 0.334
\]

\[
\therefore \quad \theta \text{ first-order} = 19.5^\circ
\]

For second-order diffraction, \( m = 2 \), and thus,

\[
\sin \theta = \frac{(2\lambda)}{d} = \left\{ \frac{(2)(6.328 \times 10^{-7} \text{ m})}{1.90 \times 10^{-6} \text{ m}} \right\} = 0.666
\]

\[
\therefore \quad \theta \text{ second-order} = 41.9^\circ
\]
Below, I've given a version of Planck's explanation (which is the way the book describes it) although, as I mentioned in class, a modern discussion only involves the quantization of the changes in the energies of the light, and does not involve the oscillations of the matter in the black body, which really is only there to allow the light to come to equilibrium.

In the "classical physics" model not only is the average energy of a collection of oscillators (in the wall of the black body cavity) at some absolute temperature $T$ equal to $RT$ (per mole), but, in addition, the allowed energies of an oscillator was believed to depend only on the amplitude of the vibration, and thus independent of its oscillation frequency, $\nu$. Radiation is emitted by oscillating charges at the frequency of the oscillator. Thus, it was believed that:

\[
\text{the intensity of radiation emitted (i.e. the energy given out) in a particular small frequency range } dv \text{ at a particular absolute temp } T = \\
\text{the number of oscillators with frequency between } \nu \text{ and } \nu + dv \\
\times \\
\text{the average energy of an oscillator (independent of } \nu) \text{ at temp } T
\]

It is possible to show (we didn't do this) the number of oscillators, $dn$, with frequency between $\nu$ and $\nu + dv$ is given by:

\[
dn \propto \nu^2 dv
\]

Thus, the number of oscillators in a small frequency range $dv$ will increase without limit as the frequency $\nu$ increases. Since the average energy per oscillator was believed to be independent of $\nu$, it meant that the intensity of the radiation also increases without limit as the frequency increases. Since the ultra-violet region of the Electromagnetic (EM) Spectrum lies on the high frequency side of the visible region of the spectrum, this model suggested that the intensity of radiation emitted should increase as we go from the red (lower frequency) end to the blue (higher frequency) end of the visible region of the EM spectrum and that it should be even greater in the UV region -- hence the phrase "the ultra-violet catastrophe"

Max Planck suggested that:

**the energy, $\varepsilon$, of an oscillator is directly proportional to its frequency, $\nu$, i.e. $\varepsilon = n \ h \ \nu$.**

where $n = 1, 2, 3, 4, \ldots \ldots \ldots $. i.e. $n$ is an INTEGER

$h = 6.626 \times 10^{-34} \text{ J s} \quad \text{ Planck's constant}$

This postulates that the energy of an oscillator can take ONLY CERTAIN DISCRETE VALUES i.e. the energy of an oscillator is QUANTIZED.

Clearly, this is a radical departure from the ideas of classical physics which asserted that the energy of an oscillator was independent of its frequency and could take
ANY value consistent with a particular absolute temperature T. The energy given off (as radiated light) or accepted by the oscillator could thus only be in multiples of $h\nu$.

III.8 To explain the photoelectric effect Einstein stated the following revolutionary postulate:

1. Light consists of discrete particles (later called PHOTONS by G. N. Lewis) with energy $\varepsilon = h\nu$ (Note: Oxtoby uses $E$ for the energy of a photon)

In addition, he asserted that:

2. Electrons are bound to the metal with a well-defined binding energy, $\phi$, called the work function of the metal

3. Electrons are ejected by interacting with a single photon

It is postulate 1. which introduces the revolutionary idea that the energy of a PHOTON is directly proportional to its FREQUENCY -- this is the idea of discreteness or quantization applied to light.

III.9. In the particle theory of light proposed by Einstein to explain the photoelectric effect, light consists of a collection of discrete particles called PHOTONS. In this theory

$$\text{Intensity of light at a detector} = \text{the Number of Photons per unit area arriving per second at the detector}$$