Review of Complex Numbers

1. "Rectangular form"
   $z = x + jy$, where $j = \sqrt{-1}$. (We use j so that we can use i for electric current.)

   $x = \text{Re}\, z$ is the *real part* of z, and $y = \text{Im}\, z$ is the imaginary part.

   $z^* = \bar{z} = x - jy$ is the *complex conjugate* of z

2. "Polar form"

   $z = A\, e^{j\phi}$

   $A = |z| = \sqrt{x^2 + y^2}$ = *magnitude* of z

   $\phi = \angle z = \arg z = \text{atan2(y,x)}$

   $z^* = A\, e^{-j\phi}, \quad z\, z^* = |z|^2 = A^2$

   $x = A \cos \phi, \; y = A \sin \phi$

3. Properties of complex numbers
   $z_1 + z_2 = (x_1 + x_2) + j\,(y_1 + y_2)$

   $z_1\, z_2 = (x_1\, x_2 - y_1\, y_2) + j\,(x_1\, y_2 + x_2\, y_1) = A_1\, A_2 \exp\left[j\,(\phi_1 + \phi_2)\right]$

   $|z_1\, z_2| = |z_1|\;|z_2|; \quad \angle(z_1\, z_2) = \angle z_1 + \angle z_2;$

   $\frac{1}{z} = \frac{z^*}{|z|^2}$; $\left|\frac{1}{z}\right| = \frac{1}{|z|}$; $\angle\frac{1}{z} = -\angle z$.

4. Euler's formula

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$e^{j\theta} = \cos\theta + j\sin\theta, \quad e^{-j\theta} = \cos\theta - j\sin\theta$$

5. Sum of a sine wave and cosine wave

$$A\cos\omega t + B\sin\omega t = C\cos\left(\omega t - \phi\right)$$

$$C = \sqrt{A^2 + B^2}, \quad \phi = \arctan\left(\frac{B}{A}\right)$$

ω is angular frequency (radians/sec).