Introduction to Systems

Contents:

System Types and Subtypes and a General Framework for Their Consideration

System types and subtypes considered in this course.

- Reacting
  - Chemical
  - Biological
  - Environmental & resource
- Electrical
- Mechanical
  - Linear
  - Rotational
- Fluid
- Thermal

General framework

For each system type and subtype considered, the following topics will be considered:

1. System variables.
2. System components and the equations (element laws) that describe them.
3. Interconnection laws (accounting equations or interconnection laws) and their use to formulate system models.
4. Mathematical solution of system models.

System variables are entirely different for each system type and subtype. Equations describing the behavior of system component bear resemblance to each other in some cases. The accounting relationships or interconnection laws are very similar between different systems. Finally, the mathematics used to solve the equations resulting from incorporating state variables and component equations into an accounting equation are *identical* for all the systems analyzed in this course and for lumped systems in general. Thus systems analysis becomes progressively more generic as we proceed down the list of topics presented above.

Benefits and Applications of Systems Analysis

1. **Predict System Behavior.** The most direct application of systems analysis is to calculate, either analytically (by hand, on paper) or numerically (in a computer simulation) what a system will do, given a description of the system, its initial conditions, and its inputs. There are a wide variety of ways in which this can be useful. A quick, rough calculation written on a napkin can be used to evaluate whether a new idea that comes up at lunch is potentially viable. A detailed model of a complex system can be used to perform a computer simulation that very accurately predicts the behavior of the real system. This can be less expensive and faster than an experimental test, and can also be used in situations where the test would be problematic, for example in the case of global warming or dosage determination for a new drug.

2. **Design.** In engineering, we typically do not simply want to know "what will system X do?" More often, we know what we want it to do, and the question is to find a system that will do that. We know the inputs and outputs of the system, and the objective is to find the systems, or some parameters of the system. Sometimes this is a simply matter of using the same equations used to predict system behavior, but solving them for a different variable. Other time it requires or benefits from creative problem solving or mathematical optimization. In any case, being able to predict system behavior is essential to doing anything other than trial and error.

3. **Develop Models.** Systems analysis can be used to predict behavior, given a model for the system or a model for the components from which a system model can be built. Sometimes the relevant details of the components, and the underlying physics (or chemistry or other science) of the components are known, thus providing a model for the components. But when this is not the case, systems analysis can help us develop models. A hypothesized model can be used to develop quantitative predictions of behavior that may be compared to the system—the classic scientific method. Systems analysis also includes methods for deriving models of systems from their behavior without relying on an initial hypothesis. These *system identification* techniques won't be discussed in detail this course, but the techniques we develop provide the basis for system identification.

4. **Structure understanding.** Quantitative models are a means to structure and develop understanding of a system. By using quantitative models, we can develop understanding of far more complex systems than can otherwise be understood.

   In many cases in engineering and in systems analysis, the most detailed, accurate model is not necessarily the best to use. A simpler model may, for example, lend more insight or facilitate a more comprehensive design optimization. Comparing predicted behavior using the simple model to predicted behavior with a detailed model, or to measured behavior of the actual system, allows evaluating whether the simple model is sufficient.

5. **Control.** Systems analysis is the basis for designing *control systems* that modify system inputs in response to system outputs to achieve *control* of system performance. Examples include a thermostat, insulin release by the pancreas, and the tracking circuits that keep the optical system in a CD player focused on a track.

System Classification

System – A collection of interacting elements for which there are cause and effect relationships among the variables.

State variables - A set of variables such that a knowledge of their values at any reference time t_0 and a knowledge of the system inputs for all $t \geq t_0$ is sufficient to determine the system output and state variable values for all $t \geq t_0$.

Systems can be usefully categorized into three general classes. These can be differentiated based on their inputs and outputs, state variables, their elements, and the mathematics useful for their analysis.

Lumped systems

Input-output: System input(s) may be continuous functions of time or constants. System behavior results in system output(s) that are continuous functions of time. Continuous here means that the function may take on any real-number value, as contrasted with discrete functions that can take on only integer values (or equivalent discrete values). It does proscribe discontinuities—abrupt changes in function values.

State variables: The system state can be defined by a finite number of continuous state variables.

Elements: A lumped system element is modeled without recognizing the reality that the physical properties of real system elements are distributed over some area or volume. In addition, a lumped system element is idealized as having one important physical property. For example, a lumped representation of a spring has elasticity and no mass whereas a real spring has both mass and elasticity. If the mass of a real spring is unimportant in a system of interest (e.g. if the other masses in the system a far larger than that of the spring), then it may be neglected entirely in lumped system analysis. If the mass of the spring cannot be assumed to be unimportant, then a lumped representation of the real spring would involve an *ideal* spring (with no mass) connected to an *ideal* mass (with no elasticity).

Mathematics. Lumped systems are represented and analyzed using ordinary differential equations.

The "payoff" for the idealizations inherent in the definition of lumped elements is that ordinary differential equations can be used instead of partial differential equations. Lumped systems are typically easier to analyze and to understand than distributed systems, and the assumption of lumped system elements is often defensible in the sense that the differences between a lumped and distributed system description are unimportant for the purpose at hand.

Distributed systems

Input-output: System input(s) may be functions of time or functions of time and of one or more spatial coordinate. System behavior results in system output(s) that are functions of time or functions of time and one or more spatial coordinate.

State variables: The system does not have a finite number of points at which state variables can be defined.

Elements: Distributed systems involve some spatially-continuous medium (which may have discontinuous properties) rather than discrete elements as defined for lumped systems. From a mathematical perspective, analysis of distributed systems involves differential elements (either infinitely small in the case of analytical solutions or finite in the case of numerical solutions) that are used to formulate the equations representing the system.

Mathematics: Distributed systems are represented and analyzed using partial differential equations.

Discrete systems

Input-output: The system input and/or output take on discrete values rather than being continuously variable.

State variables: Are often also discrete; may not be physical.

Elements: A distributed system element is described in an input-output fashion, as is the case for lumped system elements. Discrete system elements differ from lumped system elements because either the input or output variables (and more typically both) of discrete systems are discrete.

Mathematics: Discrete systems are represented and analyzed using discrete mathematics, which depending on the problem may be based on difference equations, finite automata theory, finite state machines, and formal grammars.

Systems involving a very broad range of phenomena exhibit lumped, distributed, or discrete system behavior, and can be analyzed using mathematical tools appropriate to that system class.

System Type	System Class		
(phenomenological)	Continuous		Discontinuous
	Lumped	**Distributed**	**Discrete**
Electrical	√	√	√ (digital)
Mechanical	√	√	
Chemical	√	√	
Thermal	√	√	
Biological	√	√	√ *
Environmental & resource	√	√	√
Industrial			√
Transportation			√
Communication	√	√	√
Social	√	√	√

* Some brain function and some molecular phenomena are discrete.
Note that systems comprising multiple sub-systems from different system types and classes are both common and important.

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