Patience, Pensions, and Saving

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Abstract

This paper employs a stochastic life cycle model to analyze the extent to which defined benefit pension and 401(k) plan balances displace other private saving. Simulations of the model distinguish between patient consumers, who begin saving for retirement at the onset of their working lives, and impatient consumers, who engage in buffer-stock saving during the early part of their working lives before saving actively for retirement. In general, pension and 401(k) balances accumulated by impatient consumers represent new saving to a greater degree than those of patient consumers. Several tests are derived based on pension-wealth offsets and changes in the 401(k) plan during the early working life that could identify the model as one with impatient or patient consumers.

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I. Introduction

The absence of substantial personal saving for retirement by the typical household has been amply documented in the recent economics literature. Instead, most of the resources that the current generations of workers will draw upon to finance consumption in retirement are in the form of claims on the Social Security system and employer-sponsored pension plans.[1]

In addition to the academic interest in low savings as a potential refutation of the Life-Cycle model of consumption, there is considerable attention paid to low savings rates in policy circles. Concern over Social Security's importance in providing retirement income derives from its pay-as-you-go (unfunded) financing. If Social Security simply displaces personal saving that households would undertake otherwise, the long-term capital supply will be reduced (see Feldstein (1974)). Employer-provided pensions also come under scrutiny for just how much they increase household saving because associated with each dollar contributed to a pension plan (rather than paid out as wages) is a potential loss of revenue to the Treasury. If the income effect of a tax-advantaged way to accumulate retirement resources outweighs its substitution effect, then higher pension accumulations may signify lower, not higher, national savings and little additional resources to finance consumption during retirement.

The literature to date has drawn mixed conclusions on the effects of Social Security and pensions on saving. Although a wide range of estimates of the displacement of private wealth by retirement wealth does exist, the problem is more that the typical finding is in the range of 0.20-0.50 (i.e. a dollar increase in retirement wealth reduces other wealth by between 20 and 50

[1] Diamond and Hausman (1984) were among the first to document the low wealth holdings of the median household nearing retirement. Bernheim (1994) analyzed the financial preparedness of the Baby Boom generation and concluded that it was lacking. More recent work by Engen, Gale, and Uccello (1999) and Scholz, Seshadri, and Khitatakrun (2003) using the Health and Retirement Study suggests greater financial preparedness for cohorts born between 1931 and 1941. Even the latest of these studies, however, are based on data from before the current bear market in stocks began. See Gustman et al. (1999) for a descriptive analysis of pension and Social Security wealth of households nearing retirement.

cents).[2] Such a result is non-constructive in that it appears to cast doubt on the standard Life-Cycle model of consumption without offering a well-specified alternative model apart from allusions to bequest motives, imperfect annuity markets, or consumer irrationality. A more recent literature focused specifically on whether 401(k) balances are net additions to saving has reached a similar impasse.[3]

The approach in this paper differs from the existing literature by recognizing that the offset of pension wealth against other private saving is not the behavioral parameter of interest. As will be shown below, the offset is intimately related to the specific features of the economic and pension environment. The degree of offset, or the extent to which pension balances will represent new saving, under any hypothetical change to the economic or pension environment must be determined based on predictions of a behavioral model rather than a reduced form coefficient. Toward that end, the relationship between the offset and these environments is investigated using a dynamic stochastic optimization framework that includes a consumption choice, a contribution choice for a tax-advantaged savings account, an anticipated replacement rate from Social Security and DB pensions, income uncertainty in the form of permanent and transitory shocks to income, and liquidity constraints that prevent the consumer from borrowing against future income.

The model is solved under two distinct assumptions about preferences. In the first, consumers are sufficiently patient so that they begin saving for retirement at the onset of their

[2] Aaron (1982) summarizes the early literature, much of which found nearly dollar-for-dollar displacement. Later studies that found a high offset rate include Bernheim (1987) and Leimer and Richardson (1992). The estimates in Hubbard (1986) and Dicks-Mireaux and King (1984) are between 0.15 and 0.40. See Gale (1998) and Khitatrakun, Kitamura and Scholz (2000) for more recent estimates finding higher offsets and more complete summaries of the literature.

[3] The literature began with the work of Poterba, Venti, and Wise (1994, 1995) and Venti and Wise (1997), who construct and test a variety of hypothesis using repeated cross-sectional data on 401(k) participation and balances that support the proposition that 401(k)'s do increase private savings. Using a similar methodology and the same data, Engen, Gale, and Scholz (1994) claim that the data do not support such a contention. Hubbard and Skinner (1996) and Bernheim (2002) provide a review and partial reconciliation of the evidence.

working lives, as in the standard Life Cycle model. In the second, consumers are sufficiently impatient that they engage in buffer-stock saving as described in Deaton (1991) and Carroll (1992, 1997). Early in their working lives, these impatient consumers save only to maintain a target wealth-to-income ratio that serves to buffer consumption against near-term fluctuations in income. As retirement approaches, however, even these impatient consumers switch out of their buffer-stock mode and into a retirement saving mode. This transition, and how its presence and timing are affected by changes in the budget set and 401(k) plan, is at the heart of several of the model's predictions for saving.

The ultimate goal of this line of research is to estimate the distributions of the underlying preference parameters based on sample moments related to observed saving in taxable and tax-advantaged accounts.[4] With preference parameters in hand, it will then be possible to simulate the consequences for new saving based on the appropriate model (or models) of behavior. This paper provides a first step in this direction, by illustrating the pattern of new saving (the complement to the pension-wealth offset and the focus of much of the policy discussion) in 401(k) plans as a function of their characteristics. The age profile of this offset, along with age-specific responses to changes in the 401(k) plan features, can potentially serve to distinguish between the patient and impatient consumer versions of the model.

The remainder of paper is organized as follows. In Section II, the consumer's intertemporal consumption and contribution problem is specified and the solution method is discussed. Section III illustrates the optimal consumption and contribution rules, as well as the age profile of taxable and 401(k) account balances. The importance of precautionary saving is specifically investigated in Section IV. The proportion of wealth due to precautionary saving

[4] Antecedents for this type of analysis include Carroll and Samwick (1997), Samwick (1998b), Gourinchas and Parker (2002), and Scholz, Seshadri, and Khitatrakun (2003).

falls over the working life, reaching about 40 percent on the eve of retirement. The estimates from the model are broadly consistent with the empirical results in Carroll and Samwick (1998).

The proportion of DB pension wealth and 401(k) plan balances that represent new saving are investigated in Sections V and VI, respectively. For DB plans, offsets close to zero in the early working life and increasing to as much as 70 percent near retirement are consistent with several parameterizations of the model. For 401(k) plans, the pattern of offsets by age depends dramatically on whether the consumer is modeled as patient or impatient. For patient consumers, the proportion that is new saving always declines with age. For impatient consumers, the proportion that is new saving may increase with age early in the working life and may exceed 100 percent. Section VII investigates the sensitivity of 401(k) saving to key features of the plan, such as the maximum employee contribution, the penalty on pre-retirement withdrawals, and the employer match rate. The relationship between the match rate and the penalty rate is of particular importance, as it may induce a young and impatient consumer to switch from buffer stock to life cycle saving. Section VIII specifically considers the interaction of contribution and saving behavior over the life cycle, and Section IX summarizes the potential sources of identification for the preference parameters and concludes.

II. Optimal Consumption and Contributions in a Stochastic Life-Cycle Model

As a starting point, consider the following model of intertemporal consumption (C_s) that allows for contributions to a tax-advantaged account (Z_s), such as a 401(k) plan:

$$V_t(X_t, W_t) \equiv \max_{\{C_s, Z_s\}} E_t \sum_{s=t}^{T} \beta^{s-t} u(C_s)$$

such that:

(1)

$$
\begin{aligned}
&a)\ X_s = A_s + Y_s(1-\tau) \\
&b)\ W_{s+1} = (1+r)(W_s + Z_s + m_s(Z_s)) + q_{s+1}(Y_{s+1}) \\
&c)\ X_{s+1} = (1+r(1-\tau))(X_s - C_s - Z_s(1-\tau) - f_s(Z_s)) + Y_{s+1}(1-\tau) \\
&d)\ P_s = G_s P_{s-1} N_s \\
&e)\ Y_s = P_s M_s \\
&f)\ A_s \geq 0\ \ \forall s \\
&g)\ \text{-}W_s \leq Z_s \leq Z_s^{\max}\ \ \forall s
\end{aligned}
$$

The value function $V_t(X_t, W_t)$ is the expected discounted utility of consumption in each period from the current period t to the final period T, discounted by a factor of β each period. The discount factor is equal to 1/(1 + δ), where δ is the discount rate or the rate of time preference. It is similar to an interest rate in governing the utility tradeoff across periods. The within-period utility function is assumed to be additively separable in consumption and leisure, so the latter can be omitted from the maximization problem. The utility of consumption each period is assumed to take the Constant Relative Risk Aversion (CRRA) form: $u(C) = C^{1-\rho}/(1 - \rho)$.

The uncertainty in this model comes from the permanent and transitory shocks to earnings, shown in equations (1d) and (1e). Current income, Y, is defined in equation (1e) as permanent income (P) multiplied by a transitory shock (M). Permanent income evolves according to a random walk with drift (G) and permanent shock (N), as shown in equation (1d).[5] With a utility function such as CRRA that has u'''(C) > 0, there is a precautionary motive for

[5] An alternative parameterization of the stochastic component of income would be an AR(1) process as in Engen, Gale, and Scholz (1994) and Hubbard, Skinner, and Zeldes (1995) rather than the two shocks depicted here. Empirical estimates of ARMA(1,.) income processes have typically been above 0.90 and are often not significantly different from 1, yielding the random walk assumption imposed here. See Topel and Ward (1992) and Samwick (1993) for examples. More importantly, an AR(1) process implies that every shock to earnings eventually dissipates. For example, it would require that workers displaced by the collapse of the internet bubble would eventually return to their prior level of earnings. It seems more intuitively plausible to model some shocks as permanent and others as transitory.

saving, so that larger variances of shocks to income induce greater saving. Retirement can be modeled as a value of G much less than 1 in a single year in the future, where that value of G is simply the replacement rate of defined benefit programs like Social Security and pensions. In the model below, the year of retirement is taken to be the 40^{th} year out of 60 in the assumed lifetime (e.g., starting work at age 22, retiring at age 62, and living with certainty until age 82). Uncertainty and income growth are assumed to be zero after retirement.

The multiple dimensions of uncertainty (80 shocks in all) require a dynamic programming solution method in which the problem is solved sequentially from the last period of life rather than entirely as of the beginning of life. The assumption of a CRRA utility function, rather than a less realistic CARA utility function, requires numerical rather than analytical solutions for the consumption and contribution functions. In each period, the state variables are the amount of cash on hand, X, in taxable savings accounts, A, or current after-tax income, $Y(1-\tau)$, and the balance in the tax-advantaged account, W. In this setup, consumption and contribution decisions within each period occur after the income shocks are realized. For ease of exposition, the tax system has been assumed to be linear at a tax rate of τ. More complicated tax codes are certainly possible. The state variables evolve from period to period according to the equations of motion in (1b) and (1c).

The key difference between the two state variables is that taxable savings account balances accumulate at the after-tax interest rate, $r(1 - \tau)$, whereas the tax-advantaged account balances accumulate at the pre-tax interest rate, r. Over long periods of time, the tax advantage can be sizable. For example, if the interest rate is 5 percent and the tax rate is 20 percent, then after 20 years, the tax advantage results in 21 percent higher balances. After 40 years, the tax advantage is 46 percent higher balances. To keep the problem tractable, the portfolio choice problem for risky assets in both types of accounts has been suppressed.

In practice, federal income tax advantages come at the cost of liquidity. This is reflected in a penalty, $f_s(Z_s)$, applied to withdrawals in years prior to retirement. If a household's income plummeted in a given year before retirement, and the 401(k) balances were needed to support consumption, withdrawals would be subject to a penalty above and beyond the income tax due. In the current tax code, the penalty is simply a linear function of the amount of the withdrawal, i.e., $f_s Z_s$ for $Z_s < 0$, with the penalty rate set at 10 percent for withdrawals made prior to age 59 ½.[6]

Another advantage of a 401(k) plan is that employers often encourage employee contributions by offering a matched contribution, $m_s(Z_s)$, to the account. For example, some plans apply a 50 percent match on employee contributions up to 6 percent of compensation. A match rate raises the effective interest rate on the employee's contribution. For example, with 20 years left until retirement, a 50 percent match is equivalent to an increase in the pre-tax interest rate of about 2 percentage points. A dollar-for-dollar match, which is not uncommon, would raise the effective rate by over 3.5 percentage points. In the model, the employer match is assumed to be linear on all contributions, i.e., $m_s Z_s$ for any positive value of Z_s. Employers may also make contributions that do not depend on employee contributions. In general, these may be a general function income, $q_s(Y_s)$. In the model, these are specified linearly as $q_s Y_s$.

Note that the contributions to the tax advantaged account may be positive or negative. If positive, then in a frontloaded account, such as a 401(k) plan or traditional IRA plan, there is an immediate tax reduction of τZ. If negative, then when the withdrawal is made, only the after-tax

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[6] Some plans also permit hardship withdrawals. The tax rate may also be lower in a year when a pre-retirement withdrawal needs to be made. Neither of these features are incorporated into the model.

portion is available for consumption.[7] Another advantage of 401(k) plans relative to taxable accounts is the possibility that the tax rate will be lower during retirement when the money is withdrawn than during the working years when the contribution is made. Since the model here has the same linear tax system over the whole life cycle, this advantage is not incorporated.

The description of the model is completed by specifying the constraints on the control variables. Equation (1f) is the liquidity constraint on ordinary assets, which, as a simplifying assumption, requires that households cannot be net borrowers at the beginning of any period. Equation (1g) defines the contribution and withdrawal limits on the tax-advantaged accounts. Withdrawals can be no bigger than the full balance in the account, and contributions can be no larger than some specified maximum amount.

The solution method for stochastic optimization problems with multiple state and control variables is discussed in detail in Carroll (2001). Given the distributional assumptions about the shocks to income, permanent income can be divided through all of the expressions to serve as a numeraire, rather than being identified as a third state variable.[8] The solution begins in the last period of life, T, when the problem is trivial because the household simply consumes all of its assets, yielding optimal values for the control variables, C_T and Z_T, as a function of the state variables X_T and W_T. These solutions generate the value function, $V_T(X_T,W_T)$, and the partial derivatives, $V_T^X(X_T,W_T)$ and $V_T^W(X_T,W_T)$. The partial derivatives represent the marginal value of an additional dollar in either type of account at the beginning of period T. Moving back to the period T-1 problem, we can rewrite the objective function in equation (1) as:

$$(2) \qquad V_{T-1}\left(X_{T-1},W_{T-1}\right) \equiv \max_{\{C_{T-1},Z_{T-1}\}} \; u\left(C_{T-1}\right) + \beta E_{T-1}\left[V_T\left(X_T,W_T\right)\right]$$

[7] In a Roth IRA, this immediate tax reduction does not exist, but neither is the withdrawal taxable.

[8] Besides the linear tax system, there are two concessions in the modeling framework to achieve this normalization. The first is that the DB retirement benefits are specified only as a function of permanent income. The second is that the contribution limit on the 401(k) plan is also specified as proportional to permanent, rather than current, income.

More generally, given the function $V_{t+1}(X_{t+1}, W_{t+1})$ and the associated partial derivatives, the problem at period t is:

$$(3) \qquad V_t(X_t, W_t) \equiv \max_{\{C_t, Z_t\}} \; u(C_t) + \beta E_t\left[V_{t+1}(X_{t+1}, W_{t+1})\right]$$

Dynamic programming solves the T-period problem by converting it into a sequence of T smaller 1-period problems.

In each period, the optimal values of C_t and Z_t are the solutions to the system of two first order conditions. For consumption, the FOC is:

$$(4) \qquad u'(C_t) - \beta(1 + r(1-\tau))E_t\left[V_{t+1}^X(X_{t+1}, W_{t+1})\right] = 0$$

The first term is the marginal utility of an additional dollar of consumption in period t. The second term is the expected discounted value of saving that dollar to be used in period t+1. The dollar grows by the after-tax interest rate and has a marginal value of V^X at that time. The product of these two quantities is discounted back to period t utility at a rate β. The difference between the two is zero at the optimal level of consumption.

For the optimal contribution, the FOC depends on whether the household is contributing or withdrawing from the account, because the marginal incentives differ in the two cases. For positive contributions, the FOC is:

$$(5a) \qquad (1+r)(1+m_t)E_t\left[V_{t+1}^W(X_{t+1}, W_{t+1})\right] - (1 + r(1-\tau))(1-\tau)E_t\left[V_{t+1}^X(X_{t+1}, W_{t+1})\right] = 0$$

The optimal contribution is determined by the requirement that the household be indifferent between taking a dollar out of ordinary savings and putting it into the 401(k) plan. The first term is the increase in expected utility that would result from an additional dollar of contributions to the 401(k): the contribution would be matched at m_t, grow at (1+r), and have a marginal value of V^W next period. The second term is the decrease in expected utility that would result: there would be (1-τ) fewer dollars available for saving in the taxable account, they would grow at (1+r(1-τ)), and would be valued at V^X next period.

For withdrawals, the FOC is:

(5b) $$(1+r)E_t\left[V_{t+1}^W(X_{t+1},W_{t+1})\right]-(1+r(1-\tau))(1-\tau-f_t)E_t\left[V_{t+1}^X(X_{t+1},W_{t+1})\right]=0$$

In contrast to positive contributions, there is no match and there may be a penalty. Otherwise, the intuition for the two terms in the equation is the same. Depending on the parameters, it is possible for both equations to have internal solutions for a given value of (X_t, W_t).

One approach to finding the solution each period would be to search for the pair (C_t, Z_t) that solves the system of equations in (4) and (5). However, given the need to rely on numerical rather than closed-form analytical solutions, searching for optimal values of two control variables as a function of two state variables is slow and unreliable. A more tractable alternative is to separate the decisions into a sequence of two within-period problems. For example, the contribution choice can be assumed to occur prior to the consumption choice. Given a choice of $Z_t(X_t, W_t)$, the state variables can be updated to:

(6a) $$\hat{X}_t = X_t - Z_t(1-\tau-f_t(Z_t<0))$$

(6b) $$\hat{W}_t = W_t + Z_t(1+m_t(Z_t>0))$$

The optimal consumption choice is then a function of the new state variables. Within a period, the solution method first solves:

(7) $$\hat{V}_t(\hat{X}_t,\hat{W}_t) \equiv \max_{C_t} \; u(C_t)+\beta E_t\left[V_{t+1}(X_{t+1},W_{t+1})\right]$$

and then solves:

(8) $$V_t(X_t,W_t) \equiv \max_{Z_t} \; \hat{V}_t(\hat{X}_t,\hat{W}_t)$$

The solution method is complicated by the presence of the constraints. In the consumption problem, the constraint that A_{t+1} cannot be negative implies that the maximum amount of consumption is $C_t = \hat{X}_t$. The consumption function will have a kink in it at the value of $\hat{X}_t$ that solves the FOC to (7) with $C_t = \hat{X}_t$. This constraint can therefore be accommodated by augmenting the array of $\hat{X}_t$ points by the kink points for each value of $\hat{W}_t$. In the contribution problem, the maximum utility may obtain at the maximum withdrawal, $-W_t$, at a contribution of 0, or at the maximum contribution, Z_t^{max}, in addition to the internal solutions given by the FOCs. The solution method searches the positive and negative regions for interior optima and then compares the value function at each boundary point or interior optimum to choose the one with the highest utility. Once the optimal consumption and contribution rules have been obtained, the models can be simulated forward to generate distributions of ordinary and tax-advantaged asset balances in each year of life.

The baseline model consists of assumptions about the budget constraint, the preference parameters, and the pension environment. Prior to retirement, permanent income grows at a real rate of 1.5 percent per year (G = 1.015). At retirement, permanent income drops to 50 percent of its value (G = 0.50), and after retirement, remains constant in real terms (G = 1). The pre-tax interest rate, r, is assumed to be 5 percent and is assumed to be nonstochastic. The tax rate in the linear tax system, τ, is taken to be 20 percent, applied to both labor and investment income (in the taxable account). The after-tax interest rate is therefore 4 percent per year. The variances of the shocks to both permanent and transitory income are assumed to be 10 percent per year. In alternative models, the retirement replacement rate is set to 30 or 70 percent, and the income uncertainty is eliminated.

There are two main preference parameters. The coefficient of relative risk aversion, ρ, is assumed to be 3. In a CRRA model, this results in an intertemporal elasticity of substitution of 1/3. The rate of time preference, δ, takes either of two values. For simulations of a "patient" consumer, δ is assumed to be 4 percent. For simulations of an "impatient" consumer, δ is assumed to be 8 percent. In the absence of income uncertainty, an impatient consumer would seek to borrow against future income to finance current consumption, whereas a patient consumer would not. The difference in behavior results from the comparison of the rate of time preference to the interest rate—the patient consumer has $\delta = (1 - \tau)r < r$, whereas the impatient consumer has $r < \delta$. The two values chosen are close to the median estimates of the discount rate in Samwick (1998b).

The baseline 401(k) plan is assumed to allow employee contributions up to 20 percent of permanent income in each year, with a 50 percent employer match. Employee contributions are tax deductible, and withdrawals are subject to income tax. Pre-retirement withdrawals incur an additional penalty of 10 percent. In alternative models, the match is eliminated, the 401(k) itself is eliminated, the pre-retirement withdrawal penalty is increased to 20 percent, the employee contribution maximum is lowered to 10 percent, an unconditional 5 percent employer contribution is added, or the marginal tax rate is lowered to 10 percent.

III. Illustration of the Baseline Model

Figures 1 and 2 show the details of how the optimal contribution is chosen prior to retirement. Figure 1 graphs the expected utility (V_t) that would result from the optimal contribution or withdrawal for each value of cash on hand. The graph is for and impatient consumer in the 30^{th} period with $W_t = 1$. The (red) curve with triangles is the expected utility conditional on a contribution, and the (green) curve with squares is the expected utility conditional on a withdrawal. For low values of cash on hand, making a contribution results in very low expected utility. The reason is that without a stock of taxable assets to draw down, making a contribution results in that much lower consumption. Given the curvature of the utility function, this causes a large reduction in utility relative to forsaking the contribution.

The utility curve for contributions starts lower than that for withdrawals but rises faster. The reason is that a higher positive contribution generates a match and a smaller negative withdrawal reduces the penalty, but the match rate is higher than the penalty rate. They cross at a value of roughly $X_t = 0.70$. In Figure 2, the (red and green) dashed lines trace out the values of the contributions and withdrawals that are optimal at each value of cash on hand. The switch from optimal withdrawals to optimal contributions happens at a level of cash on hand that is lower than the one for which optimal withdrawals are zero but higher than the one for which optimal contributions are zero. Over a range of X_t that is less than 10 percent of permanent income, the consumer goes from a withdrawal of over 10 percent of permanent income to the maximal positive contribution of 10 percent of permanent income.

Figures 1 and 2 illustrate an important element of 401(k) design. When the match rate on contributions substantially exceeds the penalty rate on withdrawals, then small fluctuations in cash on hand (e.g., less than one standard deviation of the transitory shock) can generate large changes in contribution and withdrawal behavior. In the opposite case, where the penalty rate exceeds the match rate, then optimal withdrawals may cease as cash on hand rises and it may take a discrete increase in cash on hand before positive contributions begin. This is the case for this same period and value of W_t, for example, when the match rate is set to zero.

Figures 3 and 4 show the consumption functions for the baseline model with an impatient consumer, first distinguished by the level of 401(k) balances and second distinguished by the period of life. In Figure 3, consumption in the fifth period of the baseline model is graphed as a function of cash on hand, with successively higher curves reflecting values of W_t = 0, 1, 5, and 9. When $W_t = 0$, the liquidity constraint binds up to a value of $X_t = 0.7632$, after which the consumer begins to restore a buffer stock of assets, causing consumption to rise less than dollar-for-dollar with cash on hand. As taxable assets approach 0.10, the consumer switches them (abruptly, as in Figures 1 and 2) into the 401(k) account. The consumption function is monotonically increasing, because the contribution to the 401(k) never causes consumption to decline with cash on hand. The function is not concave, however, because the slope of the function over the range when the contribution is being made is lower than the slope after the maximum contribution has been made.[9]

Raising the level of W introduces several changes to the consumption function. First, the consumption function shifts upward—if the consumer has more in the 401(k), then there is less of a need to save in any form, and thus consumption at each level of cash on hand is higher. Second, the liquidity constraint defined with respect to cash on hand at the beginning of the period need not ever bind. The consumer can choose to withdraw money from the 401(k), pay the penalty and boost consumption above a low level of cash on hand. Third, at higher levels of W, the consumption function can decrease over a short range of cash on hand as additional contributions are made to the 401(k) plan. In all cases, the function is concave after the maximum contribution has been made and the buffer stock rebuilt.

[9] Note that this is for C(X,W). Monotonicity and concavity are properties of $\hat{C}(\hat{X},\hat{W})$, the consumption function once the optimal contribution choice has been made.

Figure 4 shows how the consumption function changes across different periods of life. The top curve in this figure is the same as the bottom curve from Figure 3—an impatient consumer in the fifth period with a zero balance in the 401(k) plan. The other curves are for successively later periods of life, at five-year intervals, also with a zero balance in the 401(k) plan. The amount of consumption at each level of cash on hand is lower for later periods. This is to be expected—having cash on hand equal to twice permanent income at age 27 may be very good, but at age 57 it may suggest the need for more retirement saving.

Figures 5 and 6 show the profile of average assets by age for the patient and impatient consumers, respectively. Each figure graphs six different scenarios—the DB replacement equal to 30, 50, or 70 percent, with the standard deviation of income shocks equal to either 0 or 10 percent. They are constructed by drawing 5,000 age profiles of permanent and transitory shocks to income, and then, starting in the first period of life, applying the optimal rules for consumption and contributions and generating the next period's values of taxable assets and 401(k) balances. The profiles shown in the figures reflect the average value of the assets across these 5,000 scenarios. For the purposes of these figures, the 401(k) plan is eliminated, to focus strictly on the main determinants of asset accumulation based on the budget constraint.

In Figure 5, the three curves that begin at roughly zero and show a smooth accumulation in average assets with age reflect the typical life cycle profile. These curves are for the scenarios with uncertainty, and the curves reach higher peaks as the replacement rate on Social Security and DB pensions falls from 70 to 50 to 30 percent. When uncertainty is eliminated, even the patient consumer is subject to the liquidity constraint in the first 15 to 25 years of the working life because income grows at a predictable 1.5 percent per year. In some year mid-career, the consumer no longer seeks to borrow against future income and begins to accumulate assets, with later starting years for higher DB replacement rates. Compared to the scenarios with uncertainty, these profiles are steeper and reach lower peaks.

Figure 6 presents the same six scenarios for the impatient consumer. Compared to the patient consumer, all of the profiles rise to lower peaks, as lower saving is the direct implication of having a higher discount rate. For the scenarios without uncertainty, the impatient consumer remains subject to the liquidity constraint for roughly eight additional periods compared to the patient consumer, but the profiles are otherwise quite similar. For the scenarios with uncertainty, the changes are more dramatic. Specifically, rather than beginning a steady accumulation of assets from the first period of the working life, the impatient consumer accumulates a "buffer stock" of assets that stays fixed (on average) through much of the early working life. The size of the buffer stock reflects a tradeoff between the consumer's impatience—the desire to borrow against future income in the absence of uncertainty—and the consumer's prudence—the desire to establish some assets to insure against fluctuations to income. The size of the buffer stock is obviously sensitive to the parameters of the model. The scenarios with no uncertainty are just special cases in which the required buffer stock is zero, but there is nonetheless a distinct change in behavior once the combination of the time until retirement and the income drop at retirement cross a threshold.

This change in behavior is critically important to the extent to which DB pension and 401(k) balances offset other private saving. Patient consumers may never engage in buffer stock behavior. Impatient consumers typically switch out of it at some point mid-career and behave nothing like their former selves. The timing of this transition in particular is critical to the analyses that follow.

IV. The Importance of Precautionary Motives

In a model in which consumers save for precautionary as well as life cycle reasons, the extent to which pensions and 401(k) plans allow them to accumulate precautionary balances will influence the extent to which they represent new saving. We therefore begin by considering how important precautionary saving is for wealth accumulation over the life cycle. Table 1 compares the amount of wealth in simulations with and without income uncertainty to assess the amount of wealth attributable to precautionary motives. The top panel pertains to the patient consumer, and the bottom panel pertains to the impatient consumer. Within each panel, results are presented for models with and without the baseline 401(k) plan and for DB replacement rates of 30, 50, and 70 percent.

At each age, the fraction of wealth due to uncertainty is equal to:

(9) $$\frac{A_t^1 - A_t^0 + \left(W_t^1 - W_t^0\right)(1-\tau)}{A_t^1 + W_t^1(1-\tau)}$$

In this expression, the superscripts 1 and 0 on the taxable account and 401(k) balances distinguish the models with and without uncertainty, respectively. The 401(k) balances are multiplied by $(1 - \tau)$ to indicate that before these balances can be used to directly support consumption, they would have to be withdrawn and subjected to the income tax.[10]

In all models, the proportion due to precautionary motives starts at 100 percent and falls over the life cycle. The reason is clear from Figures 5 and 6. The liquidity constraint binds in the early periods of life in the models with no uncertainty. When uncertainty is added, the patient consumer begins saving for retirement immediately, and the impatient consumer saves

[10] In a more realistic model with a nonlinear income tax, this calculation would be considerably more complicated, since the marginal tax rate would depend on when the money was withdrawn and the other income received in that year. In this table and the next, the tax adjustment has little effect on the numbers presented.

for a buffer stock of assets. Any wealth that is observed must be due to precautionary motives.[11] The proportion due to precautionary motives is lower at each age when the 401(k) is present, since the 401(k) plan favors retirement relative to precautionary saving. The proportion due to precautionary motives is higher at higher DB replacement rates, because the need for retirement saving is lower when the replacement rate is higher.

The comparison of whether the patient or impatient consumer has more wealth due to precautionary motives changes over the life cycle. Early in the life cycle, the impatient consumer has a greater fraction attributable to precautionary motives, largely because very little saving for retirement has been done. However, in the ten years prior to retirement, the patient consumer has more wealth attributable to precautionary motives. Over the working life, income uncertainty generates higher wealth on average, and after the income shocks have largely passed, the patient consumer looks to smooth this wealth by maintaining it in retirement. The impatient consumer looks to consume it, and so the effect of income uncertainty at later ages is lower for the impatient consumer.

Carroll and Samwick (1998) provide empirical estimates of the sensitivity of wealth to income uncertainty. Their results suggest that between 32 and 50 percent of the wealth of the typical household is attributable to precautionary motives. Given the simplifications in the model relative to the data (i.e., only 2 possible motives for saving), it is not surprising that the model overpredicts this proportion to some extent.

[11] If the models contained other motives for saving, such as college educations or the purchase of a home, then the fractions due to precautionary motives would naturally fall, particularly in these early years. See Samwick (1998a) for a discussion of tax policy and the interaction between different saving objectives.

V. Net New Saving from Defined Benefit Pensions

The early literature on the offset between pension and other wealth focused on Social Security and pensions that were available prior to the 1990s, which were almost entirely DB plans. The model in Section II allows the offset to be considered in a model that also incorporates precautionary saving and the presence of a 401(k) plan.

Table 2 presents the proportion of the present value of DB pension benefits at each that represent new wealth. The experiment is to consider a change in the DB replacement rate from 30 to 50 or 50 to 70 percent and then to compare the resulting reductions in other assets, both taxable and 401(k) balances, to the increase in the present value of future DB payments. The present value is calculated based on the after-tax pension payments using the after-tax interest rate of 4 percent. Mathematically, the proportion of the DB wealth that is new is:

$$(10) \qquad \frac{\left(B_t^1 - B_t^0\right)\left(1-\tau\right) + A_t^1 - A_t^0 + \left(W_t^1 - W_t^0\right)\left(1-\tau\right)}{\left(B_t^1 - B_t^0\right)\left(1-\tau\right)} = 1 + \frac{A_t^1 - A_t^0 + \left(W_t^1 - W_t^0\right)\left(1-\tau\right)}{\left(B_t^1 - B_t^0\right)\left(1-\tau\right)}$$

In this expression, B denotes the present value of pre-tax DB pension payments, and the superscripts 0 and 1 indicate the models with the lower and higher replacement rates, respectively.

Comparisons are presented for patient consumers in the top panel and for impatient consumers in the bottom panel. For each type, there are separate comparisons controlling for whether there is uncertainty, for whether a 401(k) plan is also available, and for whether the change in the replacement rate is 30 to 50 or 50 to 70 percent. In all cases, the proportion that is new wealth declines with age. When confronted with a higher expected replacement rate from the beginning of life, households will save less in each period. That reduction accumulates over the working life, causing the proportion of the DB benefit that will actually serve to raise retirement income to shrink as the consumer nears retirement. Similarly, the proportion that is new saving is higher in all cases for the impatient consumer compared to the patient consumer, because the patient consumer spends a greater fraction of the working life saving for retirement when the effect of the change in the DB replacement rate can be undone.

In general, the proportion that is new saving is higher when the replacement rate goes from 50 to 70 percent than when it goes from 30 to 50 percent. The reason is that the liquidity constraint binds in more cases when the DB replacement rate is higher, preventing the household from dissaving as much as it otherwise would. The exception to this rule is the case with both uncertainty and a 401(k) plan for the patient consumer, in which the comparison of the two shows slightly more of an offset for the 50 – 70 percent increment.

The effects of uncertainty and the availability of the 401(k) on the proportion of DB balances that are new depend on the parameters. For the impatient consumer, the proportion that is new saving is higher without uncertainty at all ages and without the 401(k) at most ages. These two factors predict lower saving for retirement and thus less of an opportunity to offset the change in DB pension wealth. For the patient consumer, however, the proportion of the DB that is new saving can be lower without uncertainty if a 401(k) is present. The proportion is also higher with the 401(k) rather than without it at all ages when there is income uncertainty and at older ages when there is no uncertainty.

VI. Net New Saving from the Baseline 401(k) Plan

We next consider the proportion of 401(k) plan balances that represents new saving. The analysis differs from the preceding one for DB plans for two reasons. First, the DB wealth is presumed to exist when the worker first enters the labor market. This makes it essentially impossible to have anything but a high proportion that is "new" in the early years, since the consumer cannot reduce assets below zero. That stock of DB wealth may also not represent net additions to the capital stock—Social Security is largely unfunded and private pensions are funded only based on the proportion of the eventual benefit that has been earned to date. Comparisons at ages nearer retirement are likely to be more informative. Second, the liquidity of the 401(k) plan is greater than that of the DB plan though less than the taxable account. It may therefore serve as a vehicle for precautionary saving as well as retirement saving.

The proportion of 401(k) balances that represents new saving can be expressed as:

$$\text{(11)}\quad \begin{aligned} Gross: &\quad \frac{W_t^1 + A_t^1 - A_t^0}{W_t^1} = 1 + \frac{A_t^1 - A_t^0}{W_t^1} \\ Net: &\quad \frac{W_t^1(1-\tau) + A_t^1 - A_t^0}{W_t^1(1-\tau)} = 1 + \frac{A_t^1 - A_t^0}{W_t^1(1-\tau)} \end{aligned}$$

In these expressions, the superscripts 1 and 0 refer to the models with and without the 401(k) plan, respectively. The difference between the gross and net measures is that the latter accounts for the deferred income tax liability on 401(k) withdrawals. Balances in the 401(k) plan can represent new saving only to the extent that taxable assets do not fall relative to the model without the 401(k) plan. As equation (11) makes clear, the net share will deviate from 1 by more than the gross share, because the amounts the in the 401(k) that will eventually be paid in taxes are excluded from the comparison.

Table 3 shows the proportions of new saving in 401(k) plans by patient consumers for each of the three DB replacement rates. The first column is the amount of taxable assets in the model without the 401(k) plan. The next two columns are the taxable assets and 401(k) balances in the baseline model. Note that in all cases, taxable assets fall when the 401(k) is available, but by less than the amount of the 401(k) plan balance. The final two columns are the proportion of the 401(k) balances, gross and net of taxes, which are not offset by the reduction in taxable assets. Two important patterns are evident. First, the proportion representing new saving in the

401(k) is higher at higher replacement rates. The reason is that the consumer in the model without the 401(k) accumulates more taxable assets at lower DB replacement rates. Second, the proportion representing new saving in the 401(k) declines over the working life, by about 17 percentage points (net) for each of the DB replacement rates. The decline is attributable to the presence of precautionary motives in the early part of the working life that limit the amount of the more liquid, taxable assets that can be reduced in favor of the less liquid 401(k) accumulations.

Table 4 presents the analogous results for the impatient consumer in the baseline model. The distinguishing feature of the results is that the proportion of the 401(k) balance that represents new saving is increasing in the first part of the working life and then decreases closer to retirement. Similarly to the patient consumer in Table 3, the decline closer to retirement coincides with the period when the impatient consumer without the 401(k) begins to save for retirement rather than buffer stock reasons. Toward the end of the working life, the comparisons across models based on the DB replacement rate are similar to those in Table 3. Higher DB replacement rates are associated with higher proportions of new saving in the 401(k) plan when measured close to retirement.

The increases in the proportion of new saving in the 401(k) in the early years of the working life require a different explanation. For a consumer who would exhibit buffer stock behavior in the absence of a 401(k) plan, there is a greater urgency to accumulate the buffer stock when the 401(k) is introduced. It is only after the buffer stock is accumulated that the consumer can contribute to the 401(k) without the fear of having to pay the penalty if subsequent income draws turn out to be low. Acquiring assets earlier raises the marginal value of the 401(k) to an impatient consumer. In the table, it is possible for the proportion of the 401(k) that represents new saving to increase so much that it exceeds 100 percent. This outcome requires the taxable assets to actually be higher once the 401(k) is present. As discussed below, when the match rate on 401(k) contributions exceeds the penalty rate on early withdrawals, an impatient consumer has an incentive to "churn" his contributions. He can contribute in one year, receive the match, and then withdraw money from the account in the following year, paying the penalty but still having more in taxable assets because of the disparity in the match and penalty rates.[12] There is evidence in Table 4 of this occurring in years prior to the switch from buffer stock to retirement saving for the impatient consumer.

On the eve of retirement, the proportion of the 401(k) plan that is new saving is 31 percent for the impatient consumer, compared to 26 percent for the patient consumer. Both of these numbers are slightly lower than for the DB plans in the same models shown in the first columns of Table 2. For the DB plan, 35 and 28 percent of the present value of after-tax benefits were new saving for the impatient and patient consumer, respectively. The mandatory aspect of the DB plan dominates the marginal incentive to save provided by the 401(k) plan in these scenarios. In simulations with the employee maximum at 20 percent rather than 10 percent (not shown), fewer consumers contribute at the maximum and thus face a marginal incentive to save. In these simulations, it was possible to obtain higher fractions of 401(k) plan balances representing new savings compared to the DB plans.

[12] In simulations (not shown) in which the match rate is equal to the penalty rate at 10 percent, this increase in the non-401(k) saving rate even as 401(k) contributions are being made is barely evident. When the match rate is below the penalty rate (e.g., no match with a 10 percent penalty or a 10 percent match and a 20 percent penalty), the effect disappears entirely.

VII. The Effect of 401(k) Plan Features on Saving

There are three critical features of the 401(k) plan in the baseline model. First, employee contributions are capped at a maximum percentage of (permanent) income. In practice, and perhaps not surprisingly, this constraint is binding in many of the simulations, particularly in years when the consumer is engaged in life cycle saving. Second, the consumer faces a penalty if 401(k) balances are withdrawn before retirement. The size of this penalty governs the extent to which the 401(k) plan can be used as a vehicle for precautionary as well as retirement saving. Third, the employer matches employee contributions, providing a strong incentive to contribute. In this section, the importance of each of these features to the amount of 401(k) and total asset accumulation is investigated.

In each of Tables 5 – 7, the age profiles of both accounts are shown under the baseline model and a model with one of these features changed. The difference in 401(k) balances, $(W^1 - W^0)/W^0$, is shown, along with the difference in total asset accumulation (net of taxes):

(12) $$\frac{A_t^1 - A_t^0 + \left(W_t^1 - W_t^0\right)\left(1-\tau\right)}{A_t^0 + W_t^0\left(1-\tau\right)} = \frac{A_t^1 + W_t^1\left(1-\tau\right)}{A_t^0 + W_t^0\left(1-\tau\right)} - 1$$

Table 5 examines the effect of the employee maximum contribution (and, by extension, the maximum match) by raising the cap from 10 to 20 percent of permanent income. Over the full working career, this change must result in higher balances. This is shown clearly in the last two columns for period 40. For the patient consumer, 401(k) balances are 3 percent higher and total assets are 5 percent higher when the cap is increased. For the impatient consumer, the corresponding reductions are 11 and 9 percent.

In the early years, however, the effect of a higher cap is to *lower* the amount that appears in 401(k) balances. This counterintuitive result is due to the effect of the higher cap in future, rather than current, years. In the model with the higher cap, the consumer in the early years knows that there will be greater opportunities to make contributions in later years, when the probability of being constrained by the higher cap is lower. This reduces the premium on unused room under the cap in the earlier years. Recall that the figures in the table reflect the mean contribution behavior over a sample of income draws.

In some of the scenarios where low income draws would lead to lower consumption or taxable account accumulation rather than 401(k) contributions when the cap in future years is 10 percent, they lead to lower 401(k) contributions when the cap in future years is 20 percent. This effect is so strong that, in the early years, it outweighs the higher contributions from limit contributors who received higher income draws.

For the patient consumer, 401(k) balances are 15 percent lower after 5 years, and that disparity continues to grow through period 15, when it peaks at 30 percent. After that point, the direct impact of the higher cap when maximum contributions (and therefore employer matches) would be the norm begins to dominate, and the difference shrinks before eventually turning positive. In all periods, the impact on total asset accumulation is positive (or zero), indicating that much of the lower 401(k) contributions have come from higher saving in the taxable account, rather than higher current consumption. For the impatient consumer, the effect of the higher cap is even more pronounced in the early working years. Further, the effect on total accumulation is negative, largely because the baseline consumer is saving in taxable accounts only to maintain a buffer stock. Total accumulations are 21 percent lower at year 5, falling to only 1 percent fifteen years later.

Table 6 provides an analogous comparison of the baseline to a model in which the early withdrawal penalty has been increased from 10 to 20 percent. In the short term, the effect of this change is to make the 401(k) less attractive to both the patient and impatient consumers alike. A higher penalty discourages contributions by consumers who are saving for precautionary motives. In this particular simulation, the effect is more pronounced for the patient consumer than the impatient consumer. In the long-term effect, measured by the accumulations in year 40, the higher penalty results in higher 401(k) balances and total wealth accumulations for both consumers. On average, retirement savings are positive, and the penalty on withdrawals serves to make pre-retirement consumption more expensive relative to post-retirement consumption. Fewer withdrawals are made, as the penalty traps the money in the 401(k) and the offset from the taxable account is less than perfect. The effect is smaller for the patient consumer, who is more amenable to shifting consumption later in life. Neither of these findings are robust across parameterizations, however. In other simulations (e.g., with the employer match at 10 percent rather than 50 percent), the negative effect on balances in the early years is more severe for the impatient consumer and the tendency for the higher penalty rate to trap contributions does not overcome the negative effect of penalizing withdrawals.

Table 7 shows the comparisons when the employer match rate is lowered from 50 to 0 percent. In the years just prior to retirement, the absence of the match results in much lower 401(k) balances and total assets: 27 and 20 percent lower, respectively, for the impatient consumer and 16 percent lower in both cases for the patient consumer. Note that the match was previously 50 percent, so that employer contributions were responsible for one third of the total 401(k) balances. For both consumers, the long-term effect of a reduction in the match rate is therefore to *increase* the employee contribution rate over the course of the whole working lifetime, as the income effect of the lower total compensation outweighs the lower substitution effect of the 50 percent marginal incentive to save.

For the patient consumer, the higher 401(k) contributions begin immediately, with 401(k) balances never falling by as much as the full one third of the balances in the baseline scenario. The added contributions come largely at the expense of saving in the taxable account. The added contributions necessarily occur in scenarios in which there was room left under the cap, i.e., when income draws had been low rather than high.[13]

For the impatient consumer, the pattern over the working life is quite different. The loss of the match causes the 401(k) balance to be 99 percent lower after 5 years, 90 percent lower after 10 years, and 62 percent lower after 15 years. All of these reductions go far beyond the simple loss of the employer's contributions, suggesting an important role for the marginal incentives of the match. For the impatient consumer, who is shown in Figure 5 to be engaged only in buffer stock saving over the first 15 years, the ability to obtain the match appears to be the only reason why contributions were made to the 401(k) at all. After that time, the consumer would have transitioned into a retirement saving mode even without the 401(k), and so the absence of the match has a less dramatic effect on the total amount in the 401(k). As in the case of the patient consumer, who never had a phase of buffer stock saving at the beginning of the working life, the impatient consumer then begins to fund the 401(k) at an even greater rate than when the match was present.

Figures 7 and 8 illustrate just how important the match is for the saving behavior of the impatient and patient consumers, respectively. In Figure 7, the bottom profile is the baseline model without a 401(k) plan, and the top profile is the same model with the DB replacement rate lowered from 50 to 30 percent. These two curves were also shown in Figure 6. The next curve up from the bottom is the age profile when the 401(k) is added without the employer match. The

[13] In an analogous comparison (not shown) with the employee maximum contribution equal to 20 percent rather than 10 percent of permanent income, the added employee contributions were sufficient to increase total 401(k) balances in the early working years, i.e., to more than compensate for the loss of the employer matched contribution.

addition of this 401(k) shortens but does not eliminate the time spent in buffer stock saving mode and increases the total accumulation for retirement. The remaining curve is the age profile when the 401(k) is added with the employer match. The increase in total pre-retirement accumulation is about two-thirds of the increase in the accumulation when the DB replacement rate is lowered to 30 percent (though it is worth remembering that the employer is contributing a third of that amount). More importantly, the presence of the employer match completely eliminates the period of buffer stock saving for the impatient consumer. For comparison, Figure 8 shows the analogous models for the patient consumer. Judged relative to the effect of dropping the DB replacement rate to 30 percent, adding a 50 percent employer match has the same impact on accumulations for the patient consumer. The rate of accumulation continues to be more rapid than for the impatient consumer. However, the basic shape of the age profile remains the same in all cases.

A given change in saving behavior may be consistent with increases or decreases in consumer welfare. Table 8 compares the equivalent variation of implementing several changes in the budget set or 401(k) plan relative to the baseline model. The equivalent variation is expressed in percentage points of permanent income, which is a convenient numeraire since the entire model is homothetic in permanent income. For example, the first row shows that eliminating the baseline 401(k) plan entirely would have the same effect on expected utility as reductions of 4.87 and 1.97 percent of permanent income for the patient and impatient consumers, respectively. Removing just the employer match is equivalent to reductions of 3.07 and 1.65 percentage points, or 63 and 84 percent of the respective totals. Removing the employer match, particularly the match in excess of the early withdrawal penalty, removes most of the utility gain from the 401(k) plan. Doubling the early withdrawal penalty is equivalent to reductions of 1.40 and 0.67 percentage points. The next two rows show that increasing the penalty has only a minor effect if the match rate is not above the penalty rate. Doubling the employee maximum contribution to 20 percent of permanent income increases welfare by 1.74 and 0.45 percentage points of permanent income. The effect is naturally larger in all of these cases for the patient consumer, who is more willing to use the 401(k) plan for retirement saving.

Adding an unmatched employer contribution of 5 percent of permanent income is equivalent to an increase of about 3.6 percentage points of permanent income for both types of consumer. This is roughly equal to the 5 percent increase multiplied by the two-thirds of the lifetime over which it is received. Replacing the employer match, which could be at most 5 percent of permanent income, with an unconditional employer contribution of 5 percent is equivalent to increases of 1.19 and 2.34 percent of permanent income. The effect is larger for the impatient consumer, who was less willing to shift consumption to retirement to obtain the match in the baseline scenario.

To put these pension changes in perspective, the remainder of the table considers changes to the budget constraint apart from the pension. Changes to the DB replacement rate of 20 percentage points in either direction are equivalent to small reductions in permanent income of between 0.67 and 1.53 percentage points. Since these reductions in income are fully anticipated and occur at the end of the life cycle, there are many opportunities to pre-fund them during the working life when marginal utility of consumption is fairly small. Reducing the marginal tax rate from 20 to 10 percent is equivalent to increases in permanent income of about 12.7 percentage points. The increase is greater than the ten-point tax rate reduction because the marginal tax rate is applied to saving outside the 401(k), and this distortion also reduces utility. Finally, eliminating the permanent and transitory shocks to income would have a large effect on welfare—equivalent to increases in permanent income of 17.97 and 22.07 percent for the impatient and patient consumers, respectively. Cutting income uncertainty by half generates much more than half of these gains.

VIII. Contribution and Saving Behavior over the Life Cycle

The previous section highlighted the importance of 401(k) plan features in determining 401(k) and total asset accumulation. In particular, with a generous match, even an impatient consumer can forsake the buffer stock saving period altogether. To better understand the way that consumers respond to incentives, this section examines the distribution of contribution and saving behavior across scenarios, as a complement to the analysis of the mean profile by age.

The analysis considers three different categories for contributions to the 401(k) plan: positive, negative, and zero. Figure 9 graphs the percentage of the simulated population of impatient consumers that are in each category at each age for the baseline model. The figure shows the proportion of the 5,000 randomly drawn profiles that, conditional on all past realizations of uncertainty and optimal contribution and saving decisions, find it optimal to contribute, withdraw, or do nothing with their 401(k) in the given period.

In the first period, the impatient consumer finds it optimal to contribute for only (the highest) 10 percent of the income draws. There is no money in the 401(k) at this point, so withdrawals are not feasible. In the second period, about half of those who contributed in the first period withdraw some of the balance, and about a quarter of the consumers are now contributing. Over the first half of the working life, the proportion of the population that makes no contribution to the 401(k) vanishes. The proportion making a contribution increases steadily over the whole working life, and the proportion making a withdrawal at a given age grows for the first half and then recedes in the second half of the working life.

The groups of contributors can be further disaggregated by how they are saving outside the taxable account as well. Consumption can be compared to after-tax income, with three outcomes. $C = Y(1 - \tau)$ indicates in most cases that the consumer is liquidity constrained. $C < Y(1 - \tau)$ indicates that the consumer is actively increasing the balance in the taxable account, and $C > Y(1 - \tau)$ indicates that the consumer is dissaving out of the taxable account. In practice, consumers are not observed to be liquidity constrained in this sense if they are actively contributing or withdrawing from the 401(k). This generates seven possible categories of contribution and saving behavior.

Figure 10 graphs these seven categories for the impatient consumers in the baseline model. Effectively, the contributors have been split into two groups, one that is also saving in the taxable account and one that is dissaving in the taxable account. This latter group ($Z > 0$, $C > Y$) does not exist in the first period but emerges to represent about a third of all consumers around the 25th period before falling back toward zero. Public policy concern regarding the efficacy of tax-advantaged saving accounts focuses some attention on this group, which "reshuffles" its assets to take advantage of the tax advantage while still consuming more than its after-tax income.

The group that is neither contributing nor withdrawing has been split into three groups, one for each savings category. About a third of all consumers are liquidity constrained and non-contributors in the first period, and then this share drops rapidly over the next few periods. As would be expected in a buffer stock model, the shares saving and dissaving are comparable in each period, and in both cases falling in absolute magnitude as more consumers begin participating in their 401(k) plans.

Finally, the group of withdrawers has been split into two groups, corresponding to those saving and dissaving in the taxable account. Most of those who withdraw funds from the 401(k) are also drawing down their taxable accounts ($Z < 0$, $C > Y$). However, another group, comprising 2 – 4 percent of all consumers, is saving in their taxable accounts while withdrawing from their 401(k) plans ($Z < 0$, $C < Y$). Given that funds withdrawn will grow at the after-tax rather than the pre-tax interest rate, it may seem counterintuitive that the consumer would withdraw any funds that would not be immediately consumed. The resolution of the issue comes from the potential for an employer match, not in the current period but in subsequent periods. Making a withdrawal in a future period precludes making a contribution in a future period that would be matched by the employer. Given this constraint, when taxable assets are low, so that a withdrawal in the current period is necessary, the consumer may withdraw more funds than he will currently consume, so that the likelihood of needing to make another withdrawal in the next year is reduced. Public policy concern also focuses on this group, which "churns" its 401(k) assets through alternating periods of contribution and withdrawal to maximize the employer match and tax advantage while minimizing the reduction in current consumption.[14]

That the desire to preserve future matches is what drives this result is demonstrated in Figure 11, which presents the analogous graph when the employer match is eliminated. Almost all instances of churning have been eliminated. Without the match, there are many more

[14] The scope for churning is higher in simulations with a higher maximum employee contribution, since a higher cap allows for more bunching of contributions in a single year. For example, when the cap is 20 percent, the proportion of the impatient consumers withdrawing from the 401(k) but not consuming the entire withdrawal hovers around 10 percent.

observations with Z = 0. There is also less reshuffling—instances of positive contributions financed by reductions in taxable assets (Z > 0, C > Y).

For comparison, Figures 12 and 13 present the analogous results for the patient consumer. In Figure 12, when the match is present, there are no instances of Z = 0. There are many instances of reshuffling (nearly 50 percent of all consumers at the peak), with positive contributions being financed by dissaving from the taxable account. Instances of churning are present in most periods but are not very large after the first few years. Figure 13 shows the same absence of churning (Z < 0, C < Y) without the match but also a nontrivial amount of Z = 0 activity in the mid-career. Without the match, the fraction of patient consumers making positive contributions starts out very high and generally declines over the working career.

IX. Conclusion

Any analysis of the extent to which tax-advantaged savings vehicles such as 401(k) plans actually serve to promote retirement savings must recognize that households save for reasons other than retirement. In the model in this paper, households could save for precautionary as well as retirement reasons, and they could choose to do so in a highly liquid but taxable account or in a less liquid but tax-advantaged 401(k) plan. Wealth could also be promised to them in the form of DB pensions and Social Security.

Two parameterizations of the model were considered: one in which consumers were "patient" enough to begin saving for retirement early in the life cycle and another in which consumers were so "impatient" that, absent income uncertainty, they would seek to borrow against future income to support current consumption. Under either set of parameters, the model's predictions for the proportion of wealth attributable to precautionary motives and for the proportion of DB pension wealth that is offset by lower personal saving were broadly consistent with estimates in the literature. With a low employee maximum contribution like 10 percent of permanent income, the fraction of 401(k) plans that represent new saving was lower than for DB plans, in large part because the 401(k) is voluntary rather than mandatory, even though it provides a marginal incentive to save for retirement. Higher maximum contributions lead to fewer consumers being limit contributors and thus having no marginal incentive to save.

There are two main conclusions of the analysis presented here. First, the age profile of 401(k) participation and contributions can help identify whether the patient or impatient parameterization better characterizes the data. The proportion of the 401(k) plan that is new saving depends on how the 401(k) plan changes the duration of the impatient consumer's period of buffer-stock saving. Observing such changes could help identify which parameterization is appropriate. There were two useful examples:

1) From Tables 3 and 4: The offset of 401(k) balances against other wealth increases with age at all ages for the patient consumer. During the early working period, the offset could decline with age for the impatient consumer. An empirical finding that the offset declines with age is therefore evidence in favor of an impatient consumer in the model.
2) From Table 7: If the match rate is lowered and employee contributions to 401(k) balances increase during the early part of the working life, this indicates a patient consumer, who is responding more to the income effect of lower retirement wealth rather than the substitution effect of lower returns to saving. In contrast, the income effect is non-existent for the impatient consumer during the early working life, when the objective in the absence of the match is solely to maintain a buffer stock.

The employer match rate, in particular, is critical to the behavior of the impatient consumer. For example, an employer match of only 50 percent was sufficient to induce the impatient consumer modeled here to completely forsake the period of buffer stock saving. The effect of the employer match is even greater in a more general model of hyperbolic discounting, as in Laibson, Repetto, and Tobacman (1998).

The second conclusion is more a cautionary tale. Short-term and long-term effects of changes to the 401(k) plan were often in opposite directions. For example, increasing the employee maximum contribution raises 401(k) balances on the eve of retirement, as expected, but decreases them in the near term. With the greater availability of room under the cap in mid-career—when the consumer expects to be desire contributions that exceed the lower cap—the young consumers can now afford to lower early contributions to boost consumption or taxable account balances. A single framework is capable of generating varied and sometimes opposite behavior depending on the details of the 401(k) plan in question and the preferences of the consumer. To test any model of saving using data on 401(k) contributions or account balances, it is necessary to model the full, dynamic opportunity set facing the consumer, as in the careful modeling of pension incentives in studies of the timing of retirement or any other nonlinear budget set application.

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Table 1
Proportion of Wealth Attributable to Precautionary Saving

A) Patient Consumers (Net of Taxes)

	Has 401(k)			No 401(k)		
	DB Replacement Rate =			DB Replacement Rate =		
Age	30%	50%	70%	30%	50%	70%
5	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
10	98.79%	100.00%	100.00%	100.00%	100.00%	100.00%
15	91.71%	98.82%	100.00%	100.00%	100.00%	100.00%
20	78.80%	93.08%	99.63%	94.34%	100.00%	100.00%
25	65.54%	81.31%	93.19%	80.63%	94.85%	100.00%
30	54.59%	67.57%	83.23%	64.79%	81.21%	97.02%
35	44.98%	54.99%	68.47%	49.20%	64.12%	83.51%
40	36.12%	44.41%	54.32%	36.07%	47.22%	64.07%

B) Impatient Consumers (Net of Taxes)

	Has 401(k)			No 401(k)		
	DB Replacement Rate =			DB Replacement Rate =		
Age	30%	50%	70%	30%	50%	70%
5	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
10	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
15	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
20	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
25	86.47%	99.55%	100.00%	99.74%	100.00%	100.00%
30	62.11%	81.83%	99.50%	75.36%	97.84%	100.00%
35	45.42%	56.25%	73.02%	47.42%	67.83%	96.80%
40	31.09%	38.69%	44.21%	28.49%	38.87%	55.32%

Table 2
Proportion of DB Balances that Represent New Saving

A) Patient Consumers (Net of Taxes)

	Std Dev of Income Shocks = 10%				Std Dev of Income Shocks = 0%			
	Has 401(k)		No 401(k)		Has 401(k)		No 401(k)	
Age	30 - 50	50 - 70	30 - 50	50 - 70	30 - 50	50 - 70	30 - 50	50 - 70
5	95.67%	95.73%	93.78%	94.36%	100.00%	100.00%	100.00%	100.00%
10	89.43%	88.36%	85.78%	86.81%	98.46%	100.00%	100.00%	100.00%
15	81.95%	80.93%	77.70%	79.16%	87.26%	98.13%	100.00%	100.00%
20	73.81%	72.73%	69.09%	70.97%	67.39%	87.48%	90.86%	100.00%
25	64.34%	63.84%	59.90%	62.15%	53.82%	72.01%	70.70%	92.20%
30	52.96%	53.68%	49.59%	52.15%	47.73%	54.93%	54.05%	71.43%
35	42.56%	41.35%	38.62%	41.28%	42.37%	46.45%	40.37%	53.35%
40	30.11%	28.60%	26.26%	28.64%	31.71%	39.08%	29.13%	38.49%

B) Impatient Consumers (Net of Taxes)

	Std Dev of Income Shocks = 10%				Std Dev of Income Shocks = 0%			
	Has 401(k)		No 401(k)		Has 401(k)		No 401(k)	
Age	30 - 50	50 - 70	30 - 50	50 - 70	30 - 50	50 - 70	30 - 50	50 - 70
5	95.82%	97.07%	99.69%	99.97%	100.00%	100.00%	100.00%	100.00%
10	93.10%	93.12%	98.46%	99.59%	100.00%	100.00%	100.00%	100.00%
15	89.69%	90.59%	94.65%	98.42%	100.00%	100.00%	100.00%	100.00%
20	81.12%	87.01%	86.14%	95.01%	100.00%	100.00%	100.00%	100.00%
25	69.11%	77.62%	73.69%	86.57%	88.94%	99.76%	99.87%	100.00%
30	57.24%	63.32%	60.05%	72.52%	68.58%	86.18%	80.73%	99.09%
35	45.97%	49.59%	46.17%	56.46%	58.75%	68.17%	57.71%	78.75%
40	33.20%	35.51%	31.12%	39.37%	42.69%	55.83%	39.93%	55.75%

Table 3

Proportion of 401(k) Balances that Represent New Saving, Patient Consumer

A) Baseline Pension and Assumptions

Age	Liquid Assets No 401(k)	Liquid Assets Has 401(k)	401(k) Balance	New Share Gross	New Share Net
5	0.3666	0.1247	0.5482	56%	45%
10	0.8109	0.1656	1.3227	51%	39%
15	1.2628	0.2063	2.0825	49%	37%
20	1.7441	0.2623	2.8386	48%	35%
25	2.2859	0.3101	3.6668	46%	33%
30	2.9277	0.3852	4.5930	45%	31%
35	3.6709	0.2841	5.9037	43%	28%
40	4.6272	0.1458	7.5777	41%	26%

B) DB Replacement Rate is 30%

Age	Liquid Assets No 401(k)	Liquid Assets Has 401(k)	401(k) Balance	New Share Gross	New Share Net
5	0.4242	0.1216	0.6023	50%	37%
10	0.9599	0.1581	1.4706	45%	32%
15	1.5267	0.1827	2.3791	44%	29%
20	2.1572	0.2489	3.2929	42%	28%
25	2.8912	0.2771	4.3809	40%	25%
30	3.7871	0.2609	5.7509	39%	23%
35	4.8527	0.2728	7.3002	37%	22%
40	6.2307	0.2026	9.4065	36%	20%

C) DB Replacement Rate is 70%

Age	Liquid Assets No 401(k)	Liquid Assets Has 401(k)	401(k) Balance	New Share Gross	New Share Net
5	0.3142	0.1229	0.5010	62%	52%
10	0.6727	0.1871	1.1434	58%	47%
15	1.0162	0.2251	1.7769	55%	44%
20	1.3561	0.2849	2.3548	55%	43%
25	1.7146	0.3425	2.9440	53%	42%
30	2.1120	0.3770	3.6163	52%	40%
35	2.5404	0.3368	4.4262	50%	38%
40	3.0754	0.1251	5.6629	48%	35%

Table 4
Proportion of 401(k) Balances that Represent New Saving, Impatient Consumer

A) Baseline Pension and Assumptions

Age	Liquid Assets No 401(k)	Liquid Assets Has 401(k)	401(k) Balance	New Share Gross	New Share Net
5	0.0818	0.0597	0.0713	69%	61%
10	0.1087	0.0874	0.1792	88%	85%
15	0.1317	0.1355	0.2641	101%	102%
20	0.1854	0.1794	0.3970	98%	98%
25	0.3535	0.1557	0.7969	75%	69%
30	0.7146	0.1067	1.5119	60%	50%
35	1.3197	0.1005	2.4904	51%	39%
40	2.2681	0.1489	3.8449	45%	31%

B) DB Replacement Rate is 30%

Age	Liquid Assets No 401(k)	Liquid Assets Has 401(k)	401(k) Balance	New Share Gross	New Share Net
5	0.0847	0.0534	0.1278	76%	69%
10	0.1249	0.1146	0.2356	96%	95%
15	0.1951	0.1688	0.3749	93%	91%
20	0.3706	0.1560	0.7416	71%	64%
25	0.7506	0.1098	1.4371	55%	44%
30	1.3957	0.1038	2.4268	47%	33%
35	2.3562	0.1521	3.7262	41%	26%
40	3.7659	0.3356	5.4273	37%	21%

C) DB Replacement Rate is 70%

Age	Liquid Assets No 401(k)	Liquid Assets Has 401(k)	401(k) Balance	New Share Gross	New Share Net
5	0.0815	0.0713	0.0229	55%	44%
10	0.1044	0.0750	0.1045	72%	65%
15	0.1130	0.0939	0.1768	89%	86%
20	0.1187	0.1271	0.2452	103%	104%
25	0.1508	0.1704	0.3562	106%	107%
30	0.2461	0.1702	0.6508	88%	85%
35	0.4814	0.1093	1.2662	71%	63%
40	0.9498	0.0756	2.1838	60%	50%

Table 5
Effect of Employee Maximum Contribution Level on 401(k) and Total Net Assets

A) Patient Consumer

	Baseline 401(k)		Increase Max to 20%		Impact	Impact on
Age	Taxable	401(k)	Taxable	401(k)	on 401(k)	Total (Net)
5	0.1247	0.5482	0.2176	0.4672	-15%	5%
10	0.1656	1.3227	0.4751	0.9486	-28%	1%
15	0.2063	2.0826	0.7056	1.4513	-30%	0%
20	0.2623	2.8386	0.8553	2.0885	-26%	0%
25	0.3101	3.6668	0.8500	3.0251	-18%	1%
30	0.3852	4.5930	0.7300	4.2861	-7%	2%
35	0.2841	5.9037	0.6946	5.5673	-6%	3%
40	0.1458	7.5777	0.2721	7.7768	3%	5%

B) Impatient Consumer

	Baseline 401(k)		Increase Max to 20%		Impact	Impact on
Age	Taxable	401(k)	Taxable	401(k)	on 401(k)	Total (Net)
5	0.0597	0.0713	0.0666	0.0320	-55%	-21%
10	0.0874	0.1792	0.0882	0.1111	-38%	-23%
15	0.1355	0.2641	0.1334	0.2101	-20%	-13%
20	0.1794	0.3970	0.2064	0.3572	-10%	-1%
25	0.1557	0.7969	0.3339	0.5994	-25%	3%
30	0.1067	1.5119	0.4014	1.1789	-22%	2%
35	0.1005	2.4904	0.2661	2.4136	-3%	5%
40	0.1489	3.8449	0.0992	4.2697	11%	9%

Table 6
Effect of Early Withdrawal Penalty on 401(k) and Total Net Assets

A) Patient Consumer

	Baseline 401(k)		Raise Penalty to 20%		Impact	Impact on
Age	Taxable	401(k)	Taxable	401(k)	on 401(k)	Total (Net)
5	0.1247	0.5482	0.3341	0.0000	-100%	-41%
10	0.1656	1.3227	0.6416	0.1568	-88%	-37%
15	0.2063	2.0826	0.5743	1.0289	-51%	-25%
20	0.2623	2.8386	0.4220	2.1240	-25%	-16%
25	0.3101	3.6668	0.2494	3.3921	-7%	-9%
30	0.3852	4.5930	0.1773	4.6667	2%	-4%
35	0.2841	5.9037	0.1364	6.0313	2%	-1%
40	0.1458	7.5777	0.0818	7.7275	2%	1%

B) Impatient Consumer

	Baseline 401(k)		Raise Penalty to 20%		Impact	Impact on
Age	Taxable	401(k)	Taxable	401(k)	on 401(k)	Total (Net)
5	0.0597	0.0713	0.0687	0.0351	-51%	-17%
10	0.0874	0.1792	0.0754	0.1258	-30%	-24%
15	0.1355	0.2641	0.0905	0.2275	-14%	-21%
20	0.1794	0.3970	0.1134	0.4027	1%	-12%
25	0.1557	0.7969	0.0947	0.8774	10%	0%
30	0.1067	1.5119	0.0778	1.6555	9%	7%
35	0.1005	2.4904	0.0815	2.7079	9%	7%
40	0.1489	3.8449	0.1193	4.1218	7%	6%

Table 7
Effect of Employer Match Rate on 401(k) and Total Net Assets

A) Patient Consumer

	Baseline 401(k)		Remove Employer Match		Impact	Impact on
Age	Taxable	401(k)	Taxable	401(k)	on 401(k)	Total (Net)
5	0.1247	0.5482	0.1059	0.3882	-29%	-26%
10	0.1656	1.3227	0.1309	0.9827	-26%	-25%
15	0.2063	2.0826	0.1503	1.5974	-23%	-24%
20	0.2623	2.8386	0.1489	2.2776	-20%	-22%
25	0.3101	3.6668	0.1493	3.0432	-17%	-20%
30	0.3852	4.5930	0.1399	3.9594	-14%	-19%
35	0.2841	5.9037	0.1288	5.0168	-15%	-17%
40	0.1458	7.5777	0.1320	6.3456	-16%	-16%

B) Impatient Consumer

	Baseline 401(k)		Remove Employer Match		Impact	Impact on
Age	Taxable	401(k)	Taxable	401(k)	on 401(k)	Total (Net)
5	0.0597	0.0713	0.0841	0.0005	-99%	-28%
10	0.0874	0.1792	0.1076	0.0182	-90%	-47%
15	0.1355	0.2641	0.0994	0.0995	-62%	-48%
20	0.1794	0.3970	0.0741	0.2841	-28%	-39%
25	0.1557	0.7969	0.0607	0.6153	-23%	-30%
30	0.1067	1.5119	0.0688	1.1316	-25%	-26%
35	0.1005	2.4904	0.1230	1.8445	-26%	-24%
40	0.1489	3.8449	0.3059	2.8226	-27%	-20%

Table 8
Equivalent Variation of Changes in Parameters

Modification	Patient Consumer	Impatient Consumer
Eliminate 401(k) Entirely	-4.8646	-1.9707
Eliminate Employer Match	-3.0665	-1.6487
Reduce Employer Match to 10%	-2.5219	-1.4667
Raise Penalty to 20%	-1.3970	-0.6652
Also Reduce Match to 10%	-2.6691	-1.4893
Also Eliminate Match	-3.1557	-1.6552
Raise Employee Max to 20%	1.7392	0.4505
Add Employer 5% Unmatched	3.6336	3.6032
Also Eliminate Match	1.1855	2.3432
Lower DB Rep Rate to 30%	-1.5259	-0.9803
Raise DB Rep Rate to 70%	1.3749	0.6743
Reduce MTR to 10%	12.7810	12.6265
Reduce Income Uncertainty to 5%	16.2002	13.2568
Eliminate Income Uncertainty	22.0726	17.9733

Note: The entries in the table show the percentage change in permanent income that would be equivalent to implementing each change in the budget constraint or 401(k) plan.

Figure 1
Expected Utility, by Contribution Status

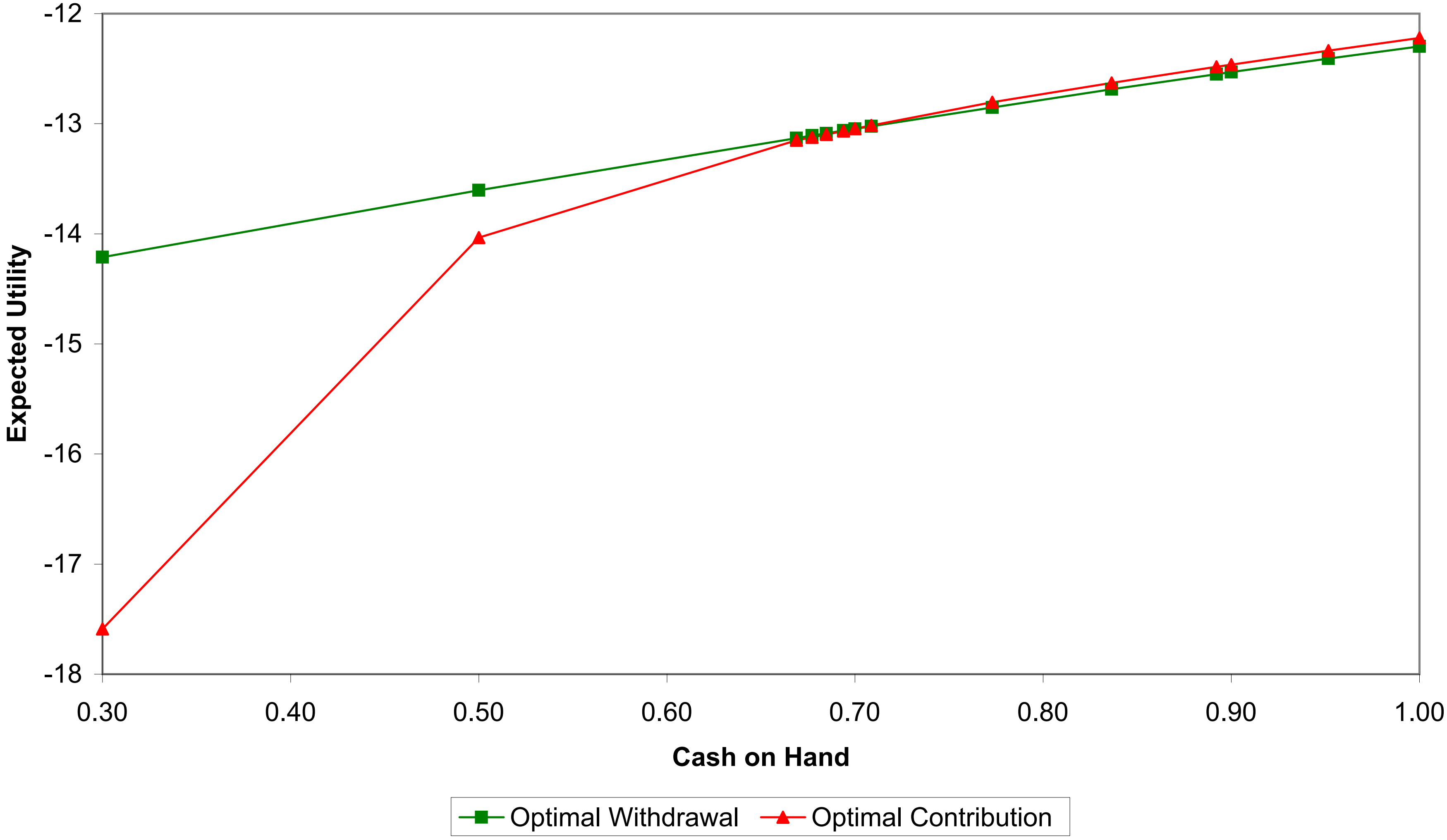

Figure 2
Optimal Contributions, by Contribution Status

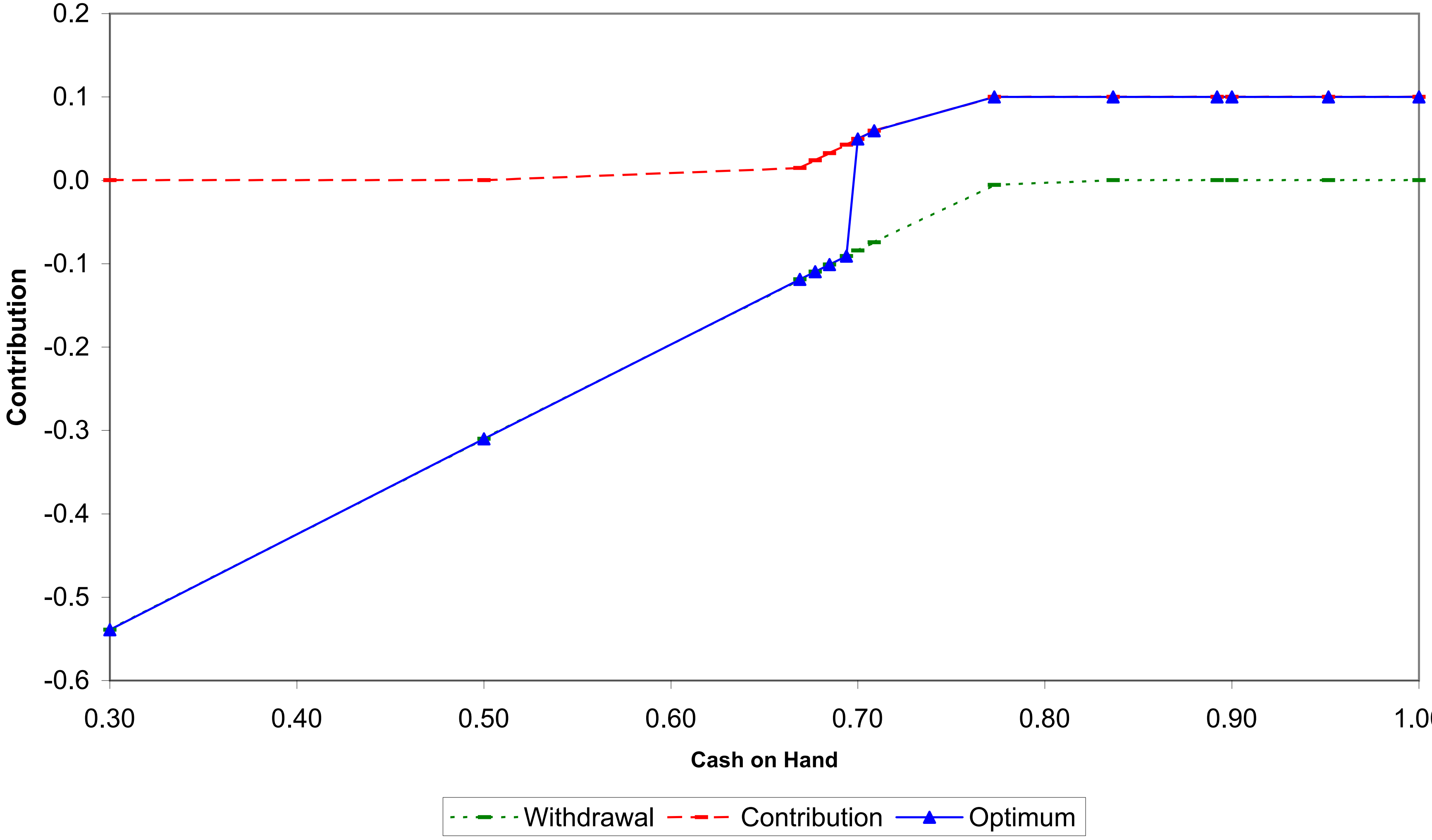

Figure 3
Consumption Functions by 401(k) Balances

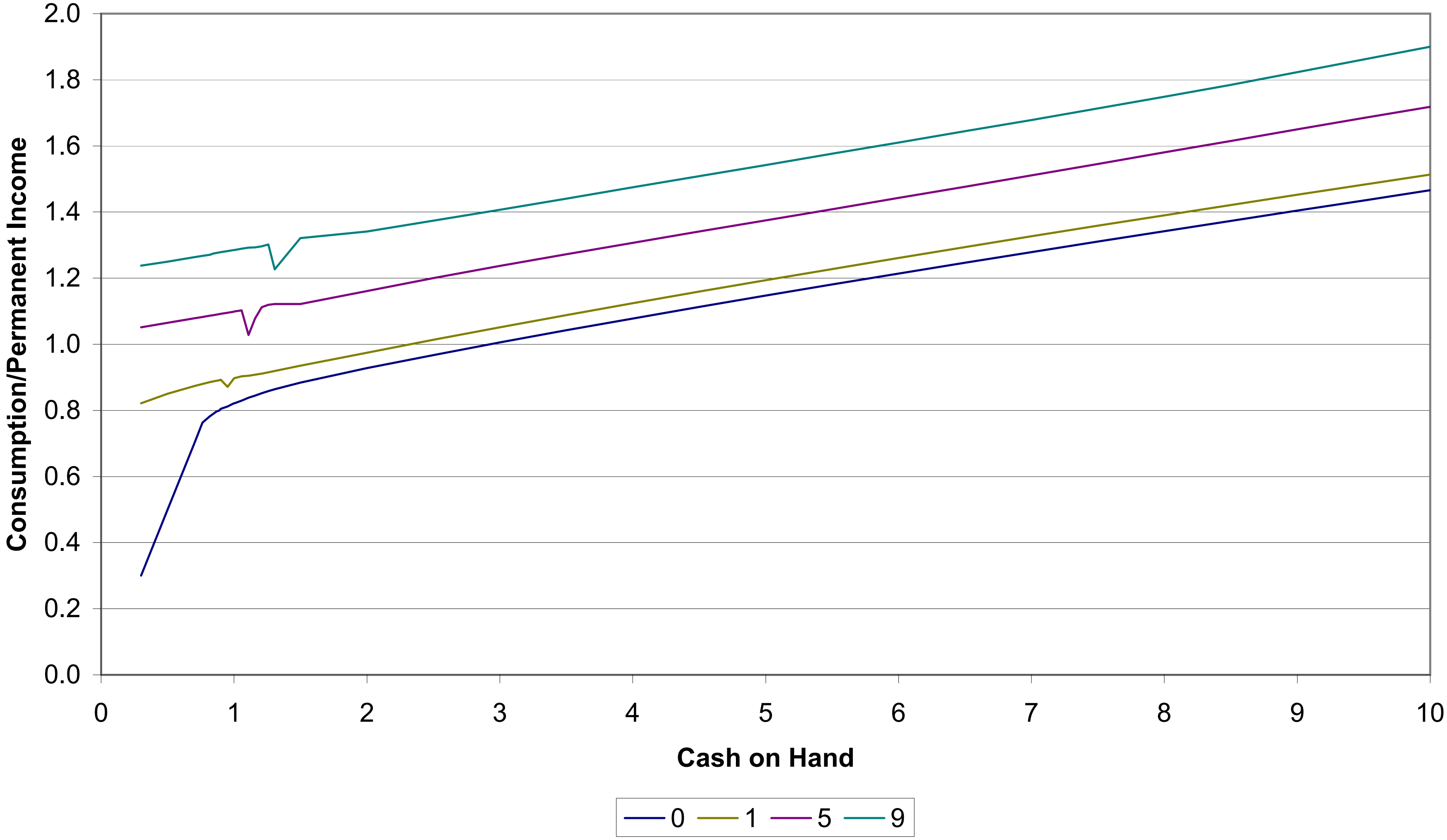

Figure 4
Consumption Functions by Period

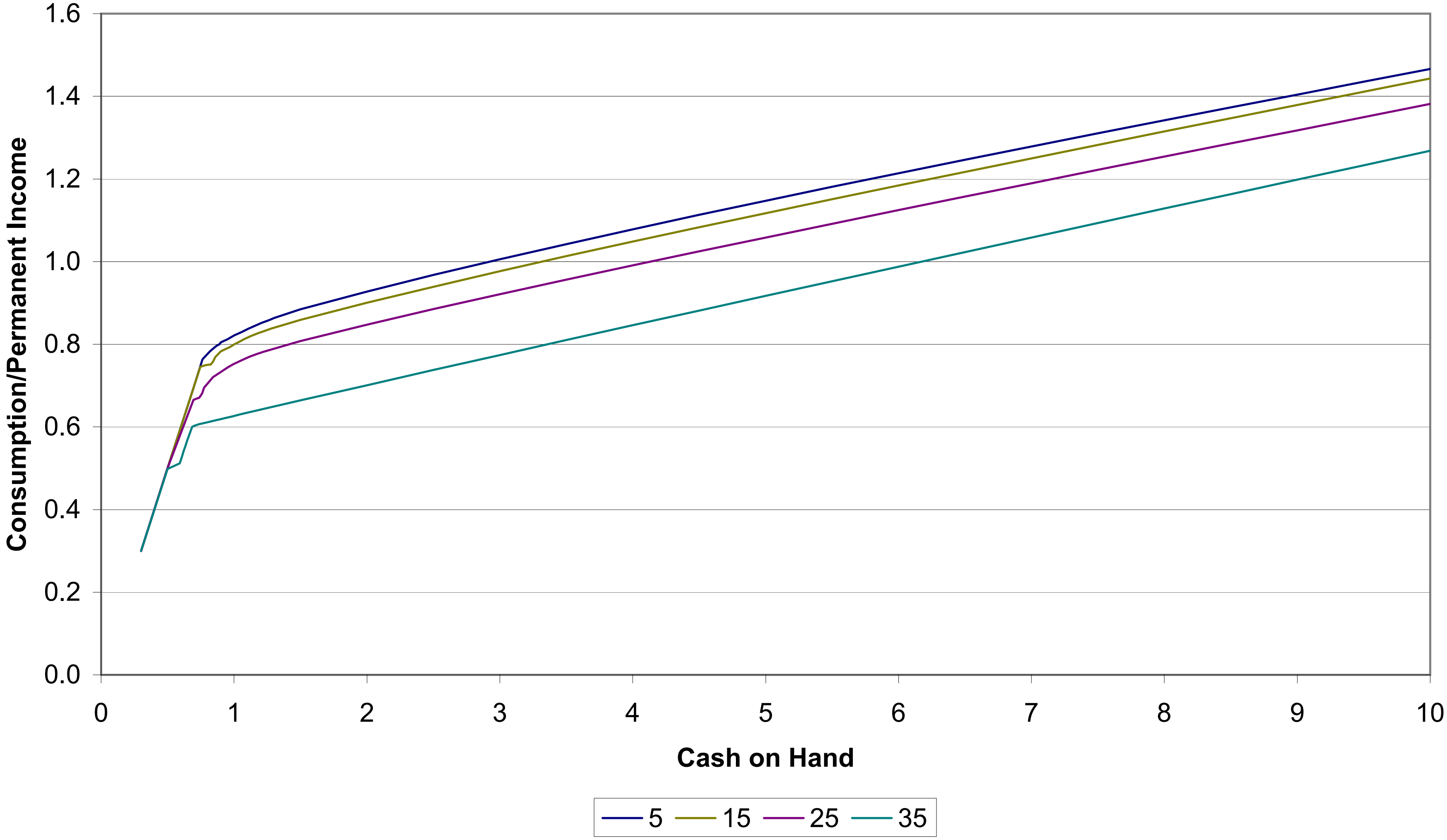

Figure 5
Age Asset Profiles for Patient Consumer
by DB Replacement Rate and Income Uncertainty

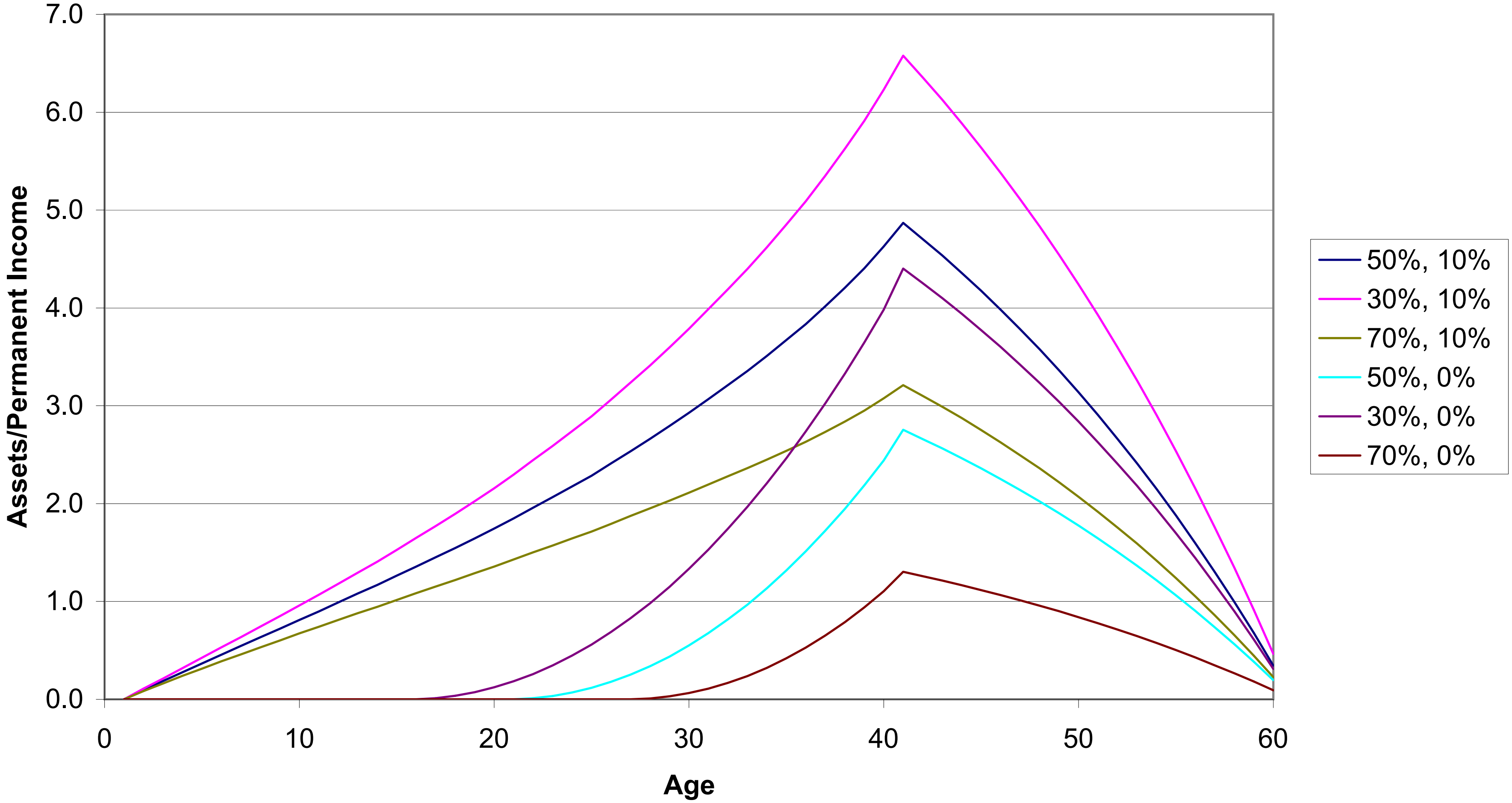

Figure 6
Age Asset Profiles for Impatient Consumer
by DB Replacement Rate and Income Uncertainty

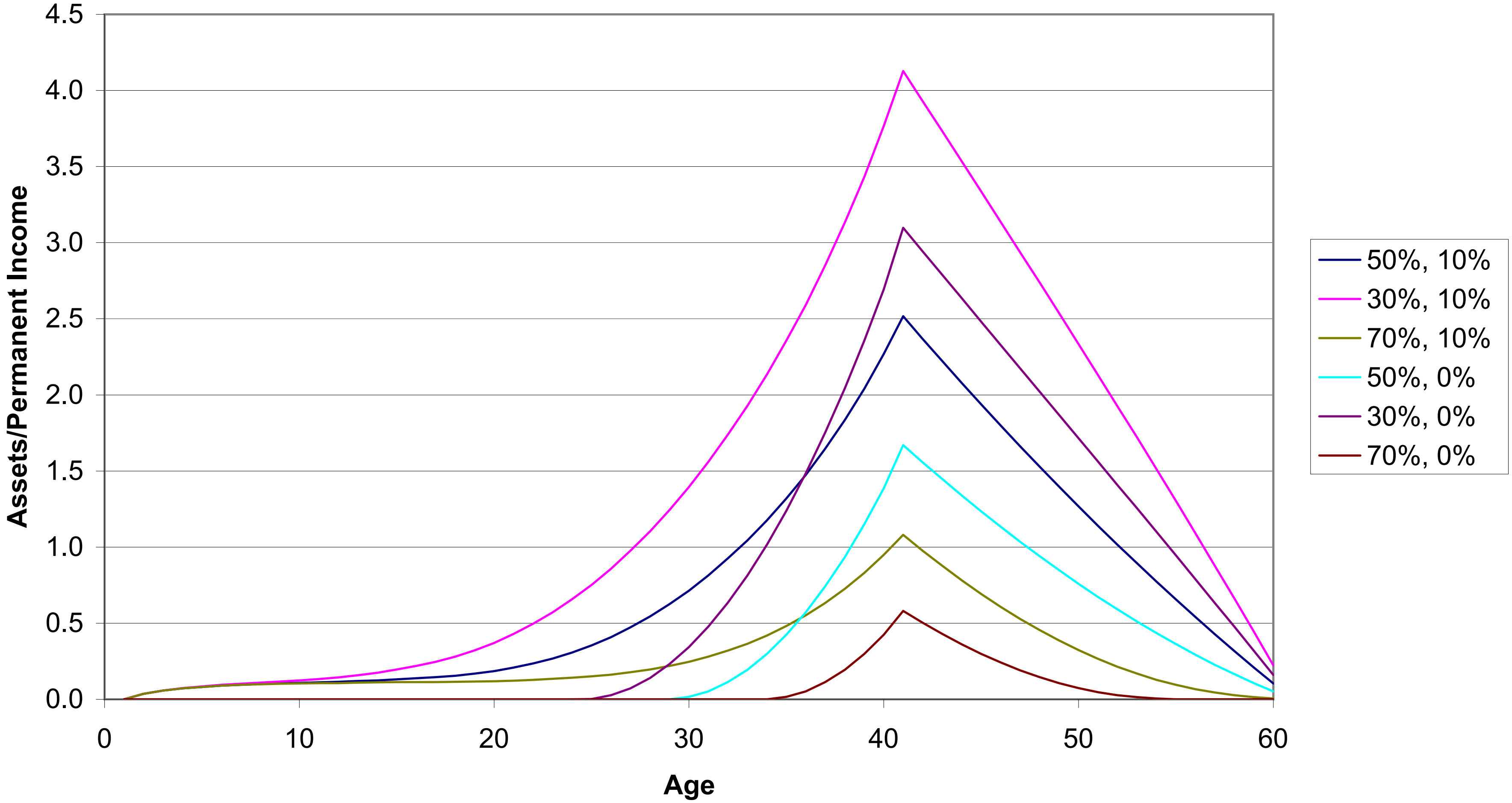

Figure 7
Effect of Employer Match on Total Assets, Impatient Consumer

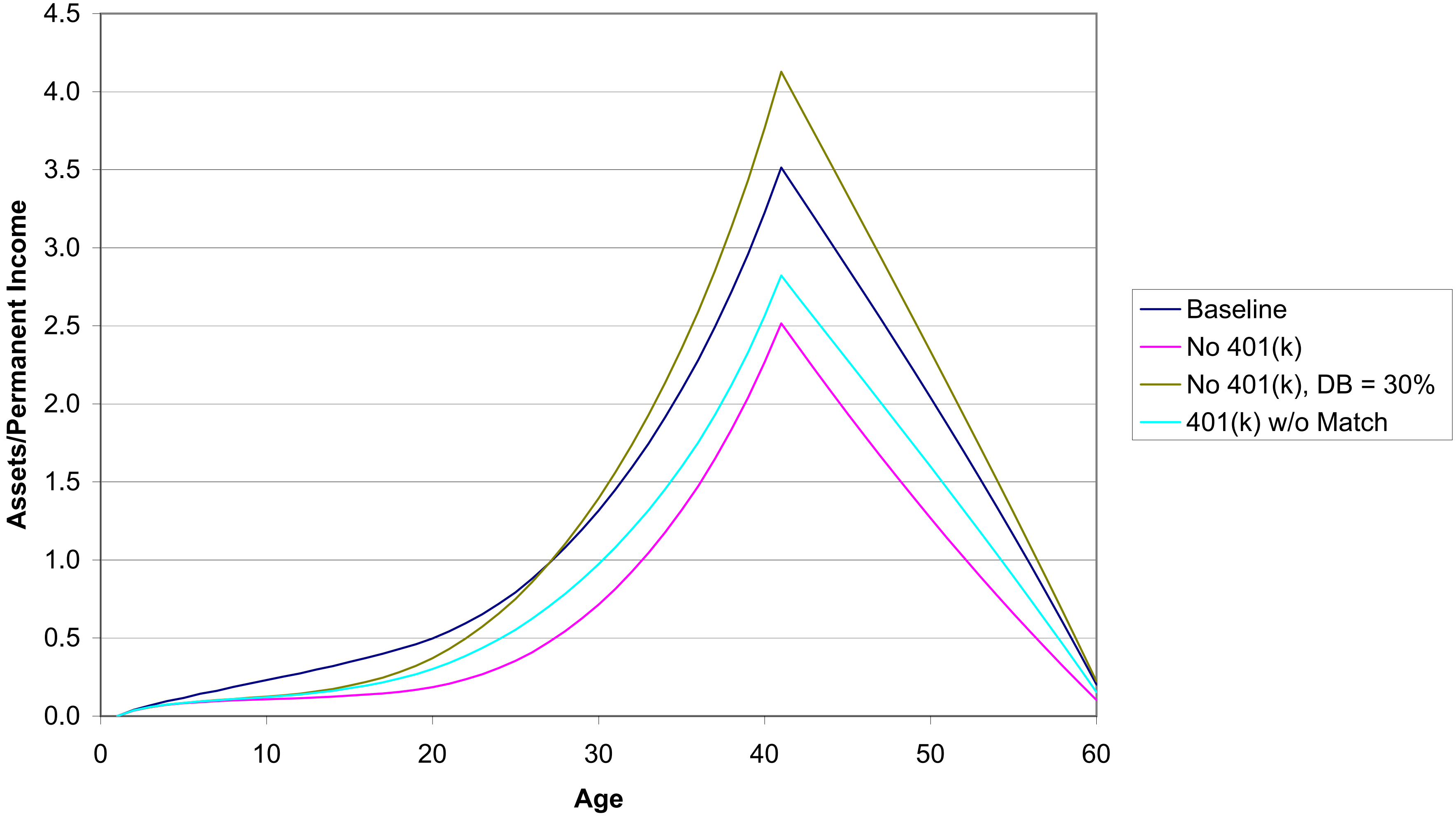

Figure 8
Effect of Employer Match on Total Assets, Patient Consumer

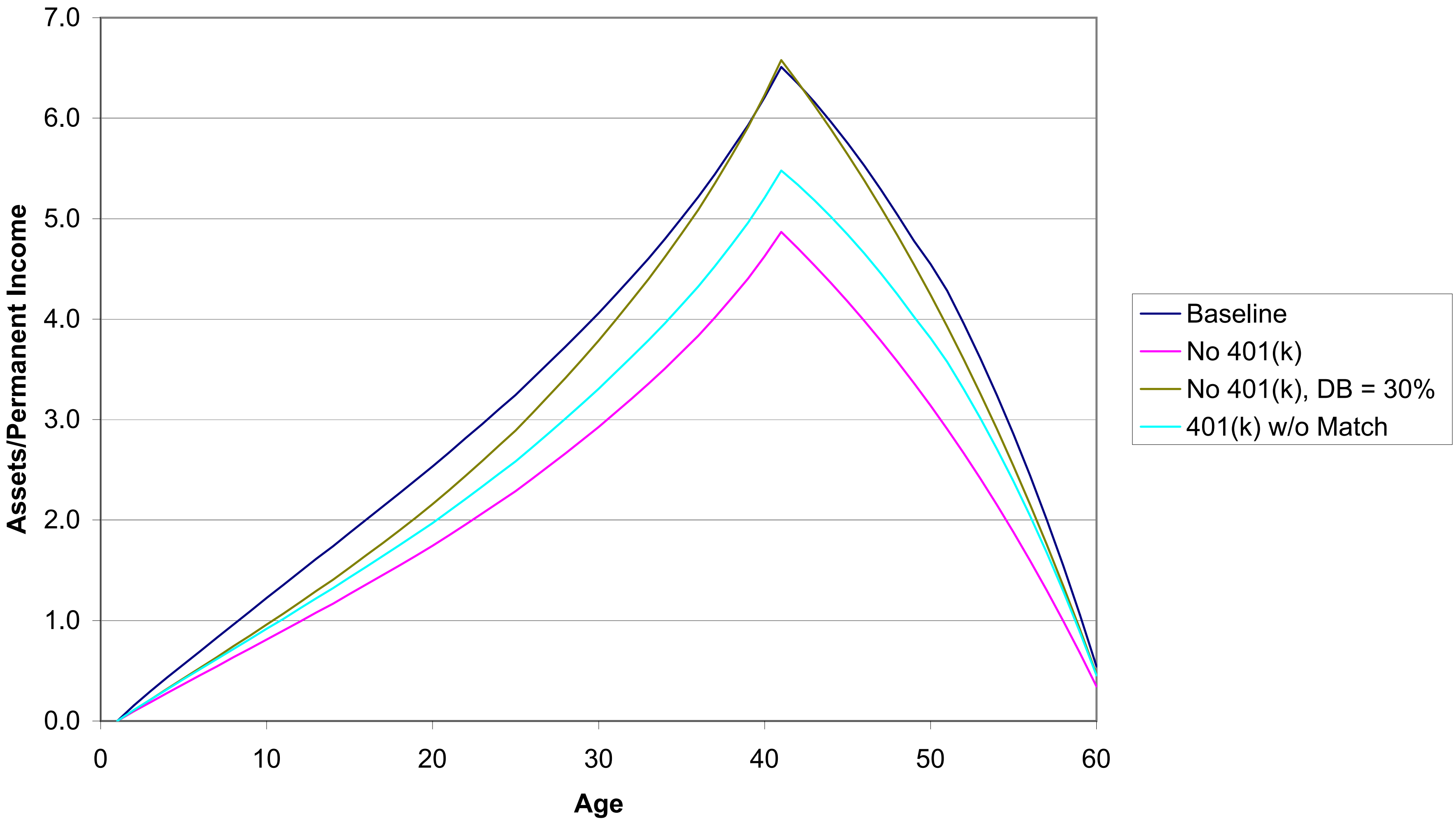

Figure 9
Pre-Retirement 401(k) Contributions
Impatient Consumer

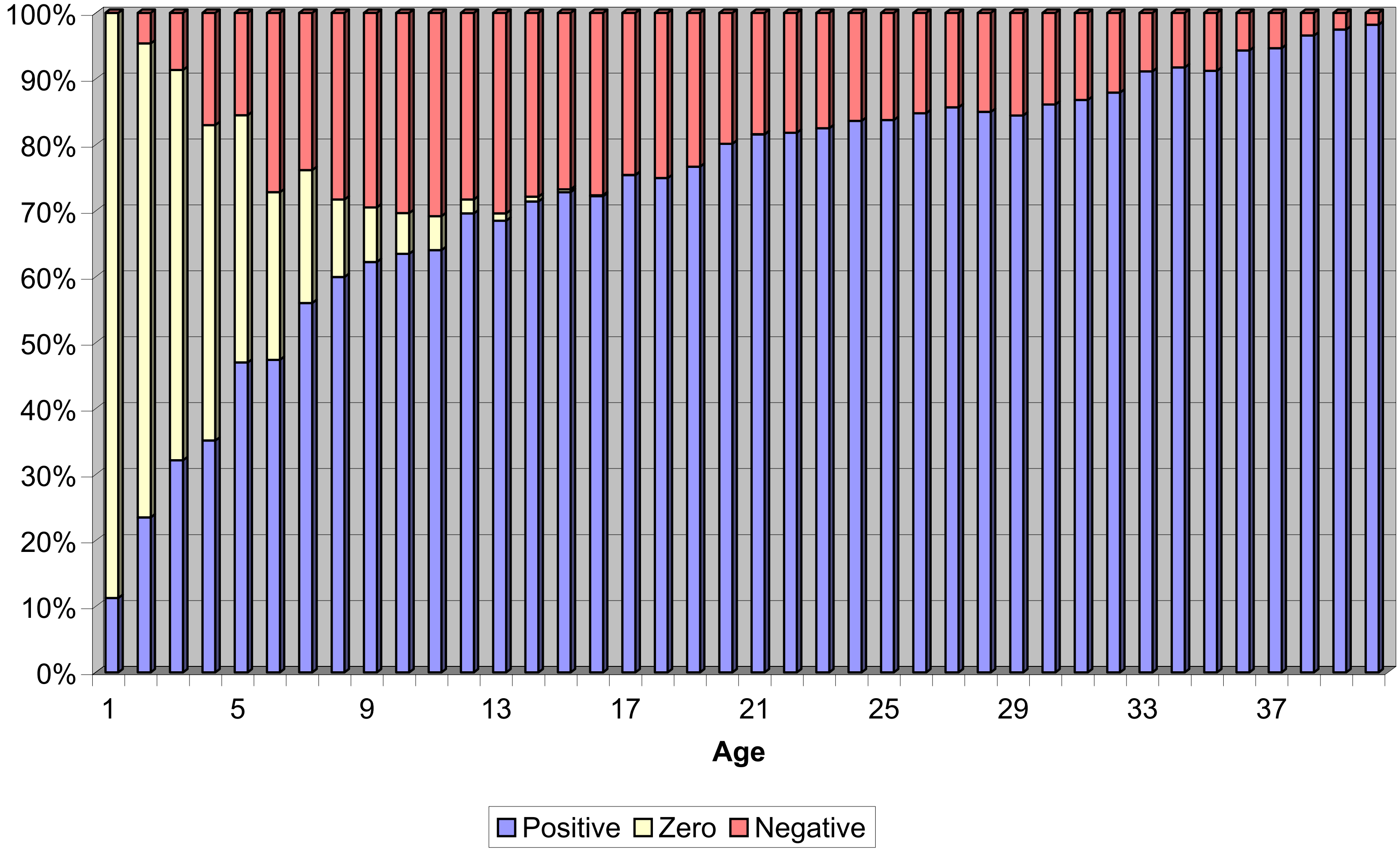

Figure 10
Pre-Retirement Contributions and Consumption
Impatient Consumer: Baseline 401(k)

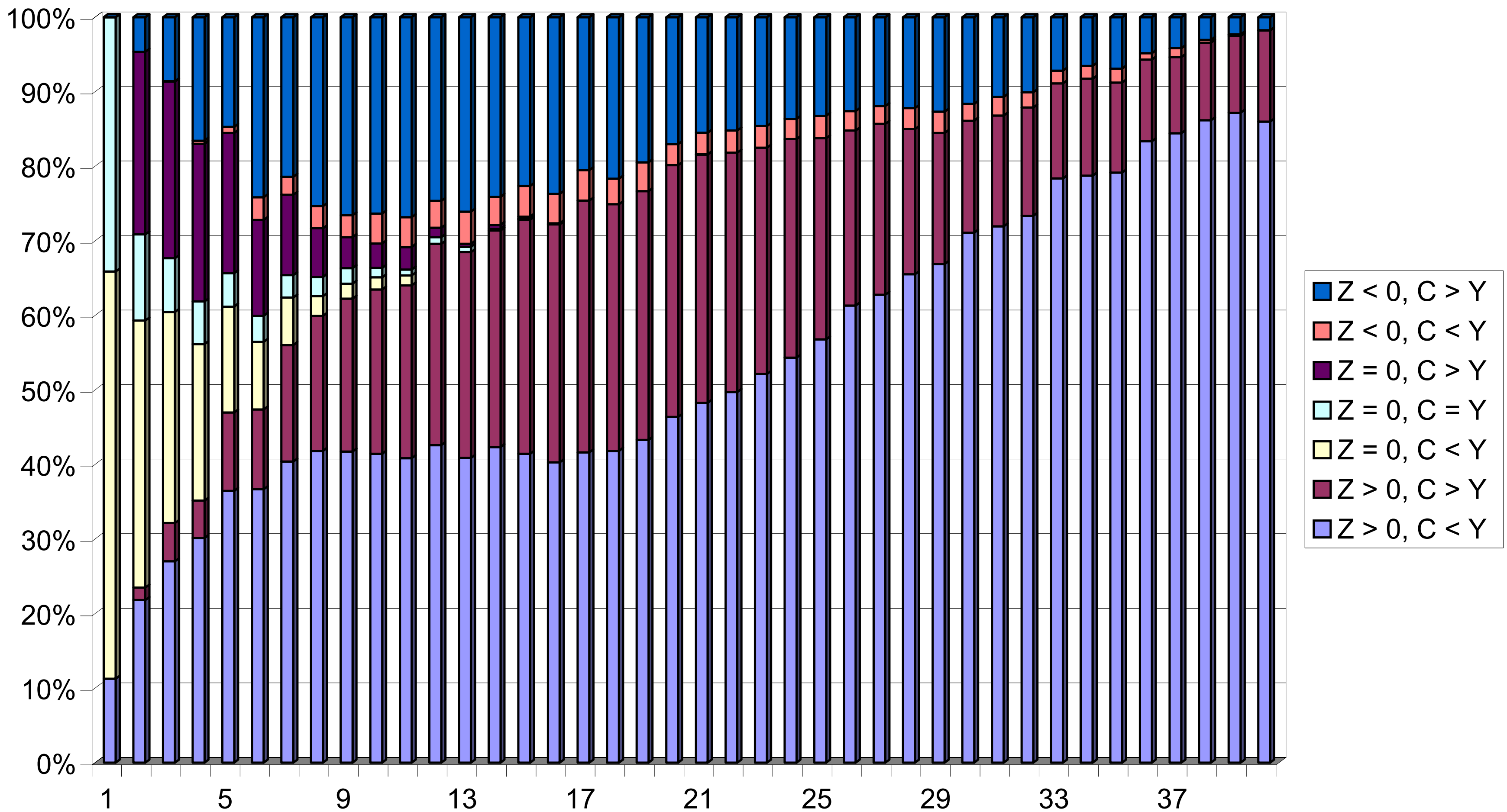

Figure 11
Pre-Retirement Contributions and Consumption
Impatient Consumer: 401(k) Has No Match

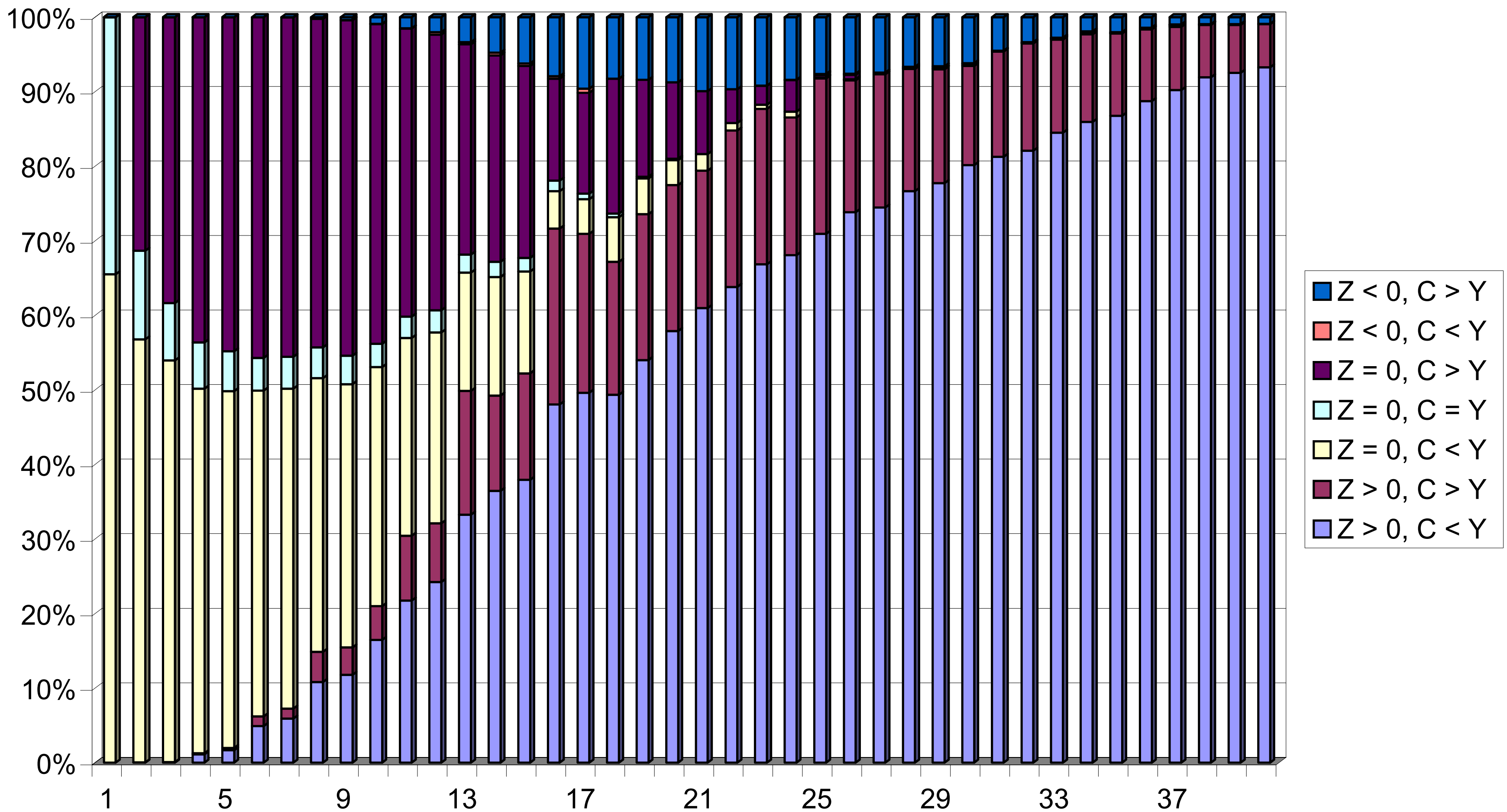

Figure 12
Pre-Retirement Contributions and Consumption
Patient Consumer: Baseline 401(k)

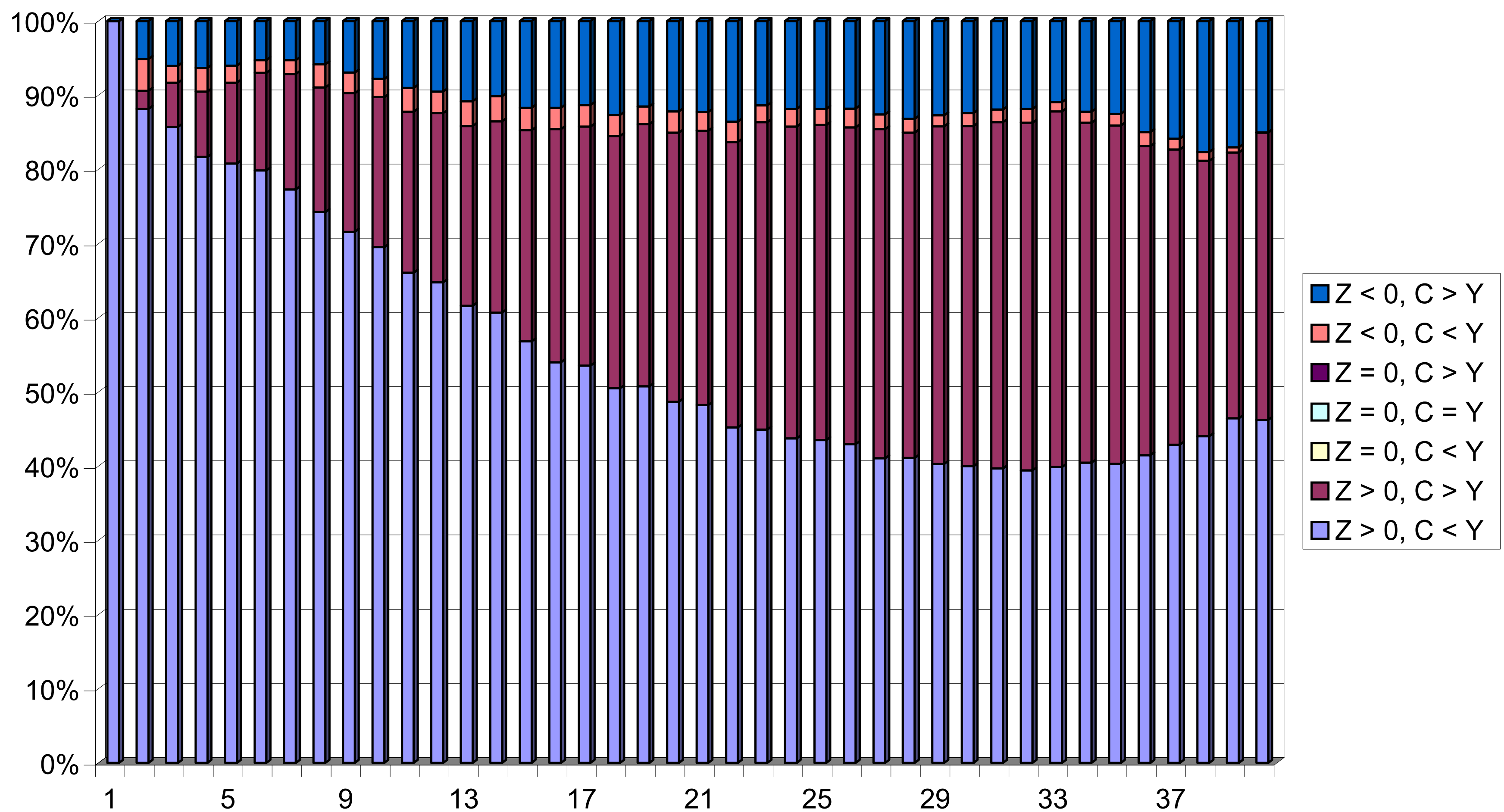

Figure 13
Pre-Retirement Contributions and Consumption
Patient Consumer: 401(k) Has No Match