Protection and the Business Cycle

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Abstract

Empirical studies have repeatedly documented the countercyclical nature of trade barriers. In this paper, we propose a simple theoretical framework that is consistent with this and other empirical regularities in the relationship between protection and the business cycle. Focusing on self-enforcing trade agreements, we find theoretical support for countercyclical movements in protection levels. The fast growth in trade volume that is associated with a boom phase facilitates the maintenance of more liberal trade policies than can be sustained during a recession phase in which growth is slow. We also find that acyclic increases in the level of trade volume give rise to protection, implying that whether rising imports are met with greater liberalization or increased protection depends on whether they are part of a cyclic upward trend in trade volume or an acyclic increase in import levels.

KEYWORDS: Protection, Business Cycle, GATT, WTO, Repeated Game

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1. Introduction

Empirical studies have repeatedly documented the countercyclical nature of trade barriers. McKeown (1984), Gallarotti (1985), Coneybeare (1987), Corden (1987), Ray (1987), Grilli (1988), Hansen (1990) and Bohara and Kaempfer (1991) all conclude that the average level of protection tends to rise in recessions and fall in booms. The Smoot-Hawley tariffs instituted during the Great Depression stand out as an example. What can account for these movements in protection over the business cycle? In this paper, we propose a simple theoretical framework that can account for these and other empirical regularities in the relationship between protection and the business cycle.

If governments turn to trade policy intervention primarily to pursue distributive goals, two logical possibilities suggest themselves as providing answers to this question. One possibility is associated with the impact of tariffs on the distribution of income among domestic residents (domestic political economy). If this approach is to deliver a theory of countercyclical protection, it must explain why governments adopt trade policies that serve the interests of import-competing sectors at the expense of export sectors during recessions but do not do so during booms. The other possibility is associated with the impact of tariffs on the distribution of income between domestic residents and the rest of the world (beggar-thy-neighbor effects). If this approach is to deliver a theory of countercyclical protection, it must explain why governments have more difficulty controlling beggar-thy-neighbor tendencies during recessions than booms.

With regard to the first of these possibilities, a common argument is that tariffs are higher in recessions, because the political pressure from import-competing firms is then most pronounced. This explanation, however, is incomplete, since it ignores the political influence of other production sectors that might press for less protection in recessions. For example, a reciprocal trade agreement that lowers import tariffs may be of particular value to exporting firms during a recession, and domestic firms that import inputs also may benefit substantially from a reduction in protection during a recession. In light of these competing political influences, the common argument for countercyclical tariffs fails to be convincing, as it does not explain why the political pressures from import-competing sectors dominate

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For further discussion of the significant political influence wielded by these other production sectors, see Milner (1993). After analyzing several industries, she argues that industries with strong export interests often oppose protectionist measures, even when the measures offer protection for their own domestic markets.
in recessions but not in booms.\footnote{A more sophisticated political theory of tariff cycles is offered by Cassing, McKeown and Ochs (1986). They draw a distinction between “old” and “new” regions, hypothesizing that old regions are experiencing secular decline and dominated by import-competing industries while new regions are experiencing secular growth and dominated by export industries. With this dichotomy in place, they argue that export (import-competing) interests drive the political process in booms (recessions), from which they conclude that domestic political economy considerations can generate countercyclical movements in protection. While the Cassing-McKeown-Ochs theory is provocative, it is clearly sensitive to the dichotomous structure that they propose.}

In this paper, we adopt the second approach noted above and develop a business cycle theory of protection that reflects cyclical variations in the effectiveness with which governments can control their beggar-thy-neighbor tendencies. Our argument builds on three essential ingredients. First, we represent the beggar-thy-neighbor aspects of trade policy with a model in which governments are tempted to exploit the terms-of-trade effects of protection. Second, we emphasize that a trade agreement designed to control beggar-thy-neighbor behavior must be self-enforcing. Finally, we analyze the role of business cycle conditions in determining the degree of liberalization that can be enforced.

If governments affect the terms of trade with their trade policy choices, a classic Prisoners’ Dilemma problem is created: Less trade occurs under noncooperative behavior than would be efficient in light of the objectives of each government. The basic idea is intuitive. When a government imposes an import tariff, its terms of trade are improved, and so part of the cost of the tariff is borne by foreign exporters, who sell at a lower price. Importing governments therefore select higher import tariffs than is efficient. Similarly, when a government imposes an export tariff, its terms of trade improve, and so part of the cost of the tariff is borne by consumers in importing countries, who buy at a higher price. As a consequence, exporting governments select higher export tariffs than is efficient. The terms-of-trade externalities associated with trade policy therefore lead to greater restrictions on trade than is efficient.

As this discussion suggests, one purpose for a trade agreement between governments is to eliminate the terms-of-trade driven restrictions in trade that arise in the absence of an agreement.\footnote{The terms-of-trade rationale for trade agreements has a long history. It was first formalized by Johnson (1953-54, 1965).} In recent work (Bagwell and Staiger, 1999, 2001a, 2002), we go further and argue that this is the central purpose of such an agreement. Adopting a general representation of government preferences that allows for both national income and political motivations, we show that a trade
agreement is efficient (relative to government preferences) if and only if it succeeds in eliminating terms-of-trade driven restrictions in trade volume. This work thus provides strong support for the terms-of-trade theory of trade agreements.

We consider next the manner in which trade-facilitating agreements are enforced. In what follows, we argue that liberal trade policies become self-enforcing if the credible threat of retaliation dissuades governments from pursuing beggar-thy-neighbor trade policies. Viewed from this perspective, an enforceable trade agreement requires each government to balance the short-term incentive to deviate from the agreement and exploit the terms-of-trade benefits of protection against the long-term costs of a consequent trade war.

Our focus on enforcement difficulties at the international level reflects the perspective that international trade agreements such as the General Agreement on Tariffs and Trade (GATT) and its successor, the World Trade Organization (WTO), will be honored only if the incentives created by the agreement are compatible with the desired behavior. That is, since no external enforcement mechanism exists to punish violations, meaningful international commitments in trade policy must be self-enforcing, with violations deterred by the credible threat of subsequent retaliation. Support for this view is found in the writings of GATT legal scholars (e.g., Dam, 1970, pp. 80-81). In addition, a substantial literature in economics (e.g., Jensen and Thursby, 1984; McMillan, 1986; Dixit, 1987; Bagwell and Staiger, 1990; Maggi, 1999) and political science (e.g., Coneybeare, 1987; Rhodes, 1993; and Yarbrough and Yarbrough, 1986) emphasizes the need to view international trade agreements as necessarily self-enforcing.

Self-enforcing trade agreements involve a balance between the short-term incentive to protect and the long-term cost of a trade war, and so changes in the current or expected future trading environments can upset this balance. We explore here the way in which changes in trade volume associated with the business cycle create an initial imbalance between short- and long-term incentives. When such an imbalance occurs, adjustments in existing trade policy may be required to bring incentives back into line. In this general manner, we characterize a relationship between the state of the business cycle and the trade policies that can be agreed upon as part of a self-enforcing agreement.

Motivated by Hamilton’s (1989) description of the U.S. business cycle, we...
represent the business cycle in terms of a Markov process that switches between boom (i.e., fast-growth) and recession (i.e., slow-growth) phases. With regard to the degree of interdependence of the business cycles across the two countries of our model, we consider two extremes: at one extreme, which we refer to as the international business cycle case, countries move together between booms and recessions; at the other extreme, which we refer to as the national business cycle case, countries move between booms and recessions independently. We focus on movements in trade volume over the business cycle, and model business cycle fluctuations in trade volumes as procyclical, exhibiting fast growth during boom periods and slow growth during periods of recession. That trade volumes and trade deficits are strongly procyclical has been well-documented empirically (see, e.g., Dornbusch and Frankel, 1987; Danthine and Donaldson, 1993; and Backus, Kehoe and Kydland, 1994). We also allow for transitory fluctuations in the trade-volume level around its high-growth and slow-growth trends.

We first consider the case of an international business cycle. Here we show that the procyclical movements in import volumes lead to countercyclical movements in protection provided that trade volume growth rates are positively correlated through time, i.e., provided that the phases of the business cycle and the accompanying changes in the growth of import flows are sufficiently persistent. As positive correlation seems the natural presumption for business cycle aggregates, our theory yields a prediction of countercyclical protection, in line with the empirical studies of the cyclical properties of protection noted above. This finding derives from a simple and robust intuition. If growth rates are positively correlated through time, then the expected future loss associated with a trade war is particularly large during a boom period. Consequently, governments are able to negotiate and enforce lower tariffs during a boom phase, even though the associated short-term incentive to defect is thereby increased.

We also show that transitory increases in import levels lead to increased protection regardless of the phase of the business cycle. The intuition that underlies this conclusion may be understood as follows. Within any given phase of the business cycle, if a transitory surge in the level of imports occurs, then each country experiences a heightened incentive to defect from the agreement and increase its

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5The empirical evidence suggests that output is positively correlated across countries, but with a few exceptions the correlations are not particularly strong (see, e.g., Danthine and Donaldson, 1993, and Kose, Prasad and Terrones, 2003).

6On estimates of transition probabilities for business cycle phases, see Hamilton (1989), who finds positive correlation in growth rates of quarterly GDP for the United States.
level of protection, since the short-term benefits of greater protection are then secured on a larger volume of trade. The agreement can be maintained, however, if governments allow special protection when transitory surges in import volumes occur. The theory developed here therefore predicts that low “baseline” levels of protection will be interrupted by occasional episodes of temporary protection that accompany transitory surges in import volumes. Accordingly, our results provide an equilibrium interpretation of the GATT escape clause provision (Article XIX) as well as the practice of “managed trade”.7

While our theory is consistent with evidence of countercyclical protection, it also provides insight into more subtle empirical features of trade policy. In particular, Trefler (1993) reports evidence that protection rises with rising import levels or import penetration. Given that trade volumes are procyclical, Trefler’s evidence appears contradictory with the fact that protection is countercyclical. Our theory points to a possible resolution. If rising imports are part of a cyclical upward trend in trade volume, then protection will decline as import volumes grow. On the other hand, if rising imports reflect a transitory increase in import volumes, then protection will rise with import penetration.

We next consider the case of national business cycles. This is a more complicated setting, but we are able to establish results for a particular representation. Our approach is to represent the national business cycle case in terms of three growth states for trade volume: high growth when both countries are in expansion, medium growth when one country is in a boom and the other is in a recession, and low growth when both countries are in a recession. We establish that the international business cycle results extend to the case of national business cycles. That is, with sufficient persistence in the phases of each country’s business cycle, protection will be countercyclical, rising when either country moves from boom to recession and falling when either country recovers. An interesting implication is that each country’s protection level depends countercyclically on the state of its own business cycle and the state of the business cycle in the rest of the world.

Finally, we remark on the relation of our model to those investigated in the literature on collusion behavior and business cycle conditions. The pioneering paper in this literature is by Rotemberg and Saloner (1986), who model the business cycle in terms of transitory shocks to demand levels. In a subsequent effort, Haltiwanger and Harrington (1991) model the business cycle as a deterministic

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7We first examined this interpretation in an earlier paper (Bagwell and Staiger, 1990). The present paper generalizes the association between transitory shocks and special protection to a model that includes persistent business-cycle fluctuations.
process with predictable turning points and explore the implications of persistent demand movements. In the present paper and our related paper on collusion (Bagwell and Staiger, 1997), we extend and generalize previous theories of cooperation by adopting a business cycle model that includes persistent movements, transitory shocks, and stochastic turning points.

The remainder of the paper proceeds as follows. The static model of trade and protection is developed in Section 2, and it is here that the Prisoners’ Dilemma problem confronting countries is presented. In Section 3, we develop and analyze the model of international business cycles. The national business cycle model is presented in Section 4. Section 5 concludes. Remaining proofs are contained in an Appendix section.

2. Static Model

In this section, we develop the basic static model of trade between two countries. We also present the Prisoners’ Dilemma problem that arises when governments affect the terms of trade with their trade policy choices.

2.1. The Static Tariff Game

We consider a world comprised of two countries, with foreign country variables distinguished by an “*.” Each country is endowed with a large number of locally abundant goods, where each locally abundant good in one country is distinct from every locally abundant good in the other country. A country is endowed with 3/2 units of each of its locally abundant goods. The domestic country’s demand for each of its locally abundant goods is given by $D(P) = \frac{3}{2} - P$, where $P$ is the local price of the good in the domestic economy. Similarly, the foreign country’s demand for each of its locally abundant goods is represented as $D(P^*) = \frac{3}{2} - P^*$, where $P^*$ is the local price of the good in the foreign economy.

Each country also has symmetric demand for and a small endowment of a number of the goods that are available abundantly in the other country, and this forms the basis for trade between the two countries. In particular, the domestic country has demand $D(P) = \frac{3}{2} - P$ for $G$ of the goods available abundantly in the foreign country, and the domestic country is also endowed with 1/2 unit of each of these $G$ goods. Each of the $G$ goods is thus a potential import (export) good for the domestic (foreign) country. Similarly, the foreign country has demand $D(P^*) = \frac{3}{2} - P^*$ for $G^*$ of the goods available abundantly in the domestic...
economy, and the foreign country is also endowed with 1/2 unit of each of these $G^*$ goods. Accordingly, each of the $G^*$ goods is a potential import (export) good for the foreign (domestic) country. Notice further that $G$ ($G^*$) gives the number of import-competing sectors in the domestic (foreign) economy. For now, we fix $G$ and $G^*$.

The government in each country can restrict or promote trade volume through the choice of specific import and export taxes or subsidies. Let $\tau_m$ and $\tau_x$ represent the domestic country’s tariff policy, where $\tau_m$ denotes the import policy (tax if positive, subsidy if negative) applied to each of its $G$ import goods and $\tau_x$ designates the export policy (tax if positive, subsidy if negative) applied to each of its $G^*$ export goods. Similarly, the foreign country chooses an import tariff, $\tau^*_m$, and an export tariff, $\tau^*_x$, on the $G^*$ goods that it imports and the $G$ goods that it exports, respectively.8

For each of the $G^*$ goods that the foreign country imports, let $P^*_m$ and $P^*_x$ represent the price of the good in the foreign and domestic markets, respectively. Likewise, for each of the $G$ goods that are imported by the domestic country, we denote the domestic and foreign prices as $P_m$ and $P^*_x$, respectively. We have now that

\[ P^*_m = P_x + \tau_x + \tau^*_m \]

\[ P_m = P^*_x + \tau^*_x + \tau_m. \]  \hspace{1cm} (2.1)

(2.2)

The structure of the basic model is completed with the further requirement of market clearing for each product. This requirement may be expressed as

\[ 2 = \left[ \frac{3}{2} - P^*_x \right] + \left[ \frac{3}{2} - P^*_m \right] \]  \hspace{1cm} (2.3)

\[ 2 = \left[ \frac{3}{2} - P^*_m \right] + \left[ \frac{3}{2} - P^*_x \right]. \]  \hspace{1cm} (2.4)

Using a “$^\sim$” to denote market-clearing values, we solve (2.1)-(2.4) for market-clearing prices and import volumes, $M(P^*_m) \equiv D(P^*_m) - 1/2$ and $M^*(P^*_m) \equiv D^*(P^*_m) - 1/2$, which are:

\[ \hat{P}_x = \left[ 1 - (\tau_x + \tau^*_m) \right]/2; \quad \hat{P}^*_m = \left[ 1 + (\tau_x + \tau^*_m) \right]/2 \]  \hspace{1cm} (2.5)

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8Given the symmetry across each of the $G$ domestic import goods and across each of the $G^*$ export goods, we consider a single import (export) policy applied symmetrically to all goods imported (exported) by the domestic country, and similarly for the foreign-country trade policy.
\[ \hat{P}_x^* = \frac{1 - (\tau^*_x + \tau_m)}{2}; \quad \hat{P}_m = \frac{1 + (\tau^*_x + \tau_m)}{2}. \]  

(2.6)

\[ M(\hat{P}_m) = \frac{1 - (\tau^*_x + \tau_m)}{2}; \quad M^*(\hat{P}_m^*) = \frac{1 - (\tau^*_x + \tau^*_m)}{2} \]  

(2.7)

Thus, under free trade, each good is sold at the price of 1/2 in both countries, and the per-good import volume is also 1/2, so that consumption is identical across countries. When taxes are imposed, however, the volume of trade is reduced, and consumers in the importing (exporting) country pay a price above (below) 1/2. Observe that trade is prohibited for the \( G \) (\( G^* \)) goods potentially imported (exported) by the domestic country when \( \tau^*_x + \tau_m \geq 1 \) (\( \tau_x + \tau^*_m \geq 1 \)).

With (2.5)-(2.7) in place, we are now ready to define the welfare functions that governments maximize. For simplicity, we ignore political economy considerations and assume that each government seeks to maximize national income. Thus, each government sets its trade policy so as to maximize the sum of producer surplus, consumer surplus and net tariff revenue on traded goods for its country. Formally, letting \( W_x(\tau_x, \tau^*_m) \) and \( W_m(\tau_m, \tau^*_x) \) represent the domestic-country welfare received on each of its \( G^* \) export and \( G \) import goods, respectively, we have that

\[ W_x(\tau_x, \tau^*_m) = \int_{\hat{P}_x}^{3/2} D(P)dP + (3/2)\hat{P}_x + \tau_x M^*(\hat{P}_m^*) \]  

(2.8)

\[ W_m(\tau_m, \tau^*_x) = \int_{\hat{P}_m}^{3/2} D(P)dP + (1/2)\hat{P}_m + \tau_m M(\hat{P}_m) \]  

(2.9)

so that total domestic-country welfare, \( W(\tau_m, \tau_x, \tau^*_m, \tau^*_x; G, G^*) \), is given by

\[ W(\tau_m, \tau_x, \tau^*_m, \tau^*_x; G, G^*) = G^* W_x(\tau_x, \tau^*_m) + GW_m(\tau_m, \tau^*_x). \]

In an exactly analogous manner, we may define the foreign-country welfare received on each export and import good as \( W_x^*(\tau^*_x, \tau_m) \) and \( W_m^*(\tau^*_m, \tau_x) \), respectively, with total foreign-country welfare expressed as \( W^*(\tau^*_m, \tau^*_x, \tau_m, \tau_x; G, G^*) = GW_x^*(\tau^*_x, \tau_m) + GW_m^*(\tau^*_m, \tau_x) \).

We now define the static tariff game as the game in which both governments simultaneously select import and export tariffs, where the domestic government chooses its tariff policy \( (\tau_m, \tau_x) \) to maximize \( W(\tau_m, \tau_x, \tau^*_m, \tau^*_x; G, G^*) \), and the foreign government selects its tariff policy \( (\tau^*_m, \tau^*_x) \) to maximize \( W^*(\tau^*_m, \tau^*_x, \tau_m, \tau_x; G, G^*) \).
2.2. Nash Equilibria of the Static Tariff Game

Before characterizing the Nash equilibria of the static tariff game, it is instructive to identify the two key effects of trade policy for this game. First, a country’s trade policy affects the terms-of-trade, and it is through this *terms-of-trade effect* that a country can redistribute surplus from its trading partner to itself. Second, taxes on trade have an *efficiency effect*, as they restrict the volume of trade and thereby reduce welfare. We argue below that the terms-of-trade effect leads governments to restrict trade. This restriction in turn causes efficiency losses, implying that countries face a Prisoners’ Dilemma problem when trade policy is the outcome of a noncooperative process.

To develop this argument formally, we maximize $W$ with respect to $\tau_x$ and $\tau_m$, finding that the best-response tariffs for the domestic government are defined implicitly by

$$M(\hat{P}_m) = \tau_m$$  \hspace{1cm} (2.10)

$$M^*(\hat{P}_m^*) = \tau_x.$$  \hspace{1cm} (2.11)

The LHS of (2.10) captures the benefits to the domestic country from a slight increase in its import tariff, holding fixed the level of import volume. In particular, $M(\hat{P}_m)$ corresponds to the net effect on tariff revenue and consumer surplus for the $M(\hat{P}_m)$ units of traded goods following a slight increase in the import tariff; this is the terms-of-trade effect, and it reflects a redistribution of surplus from the foreign exporters to the domestic country. The RHS of (2.10) gives the cost to the domestic country when its import tariff is raised slightly. A higher import tariff results in lower import volume, and this efficiency effect in turn diminishes the tariff revenue earned by the domestic government.

The export tariff condition (2.11) admits a similar interpretation. For fixed export volume, a higher export tariff redistributes a portion of foreign consumer surplus into domestic tariff revenue; this terms-of-trade benefit is represented in the LHS of (2.11) by the term $M^*(\hat{P}_m^*)$. A higher export tariff also has a cost, however, and this is captured in the RHS of (2.11). A higher export tariff reduces export volume, and this efficiency loss results in less tariff revenue.

Solving (2.10) and (2.11), we find that the best-response tariffs for the domestic government take the explicit forms:

$$\tau_{xr}(\tau_m^*) = \frac{1 - \tau_m^*}{3}$$  \hspace{1cm} (2.12)
Thus, the domestic country’s optimal tariffs are positive, provided that the foreign-country tariffs do not already prohibit all trade.\(^9\) The first order terms-of-trade benefits of a slight tariff dominate the second order efficiency losses. Foreign-country welfare-maximizing tariff responses may be derived analogously.

We turn next to a characterization of the Nash equilibria of the static tariff game. We begin with the interior Nash equilibrium in which positive trade takes place. Solving for the interior Nash equilibrium trade tariffs yields

\[
\tau^*_x = \frac{1}{4}; \quad \tau^*_m = \frac{1}{4},
\]

where \(\tau^*_x\) and \(\tau^*_m\) are the Nash export and import tariffs, respectively, imposed by the domestic and foreign governments. Note also that \(\tau^*_x + \tau^*_m = 1/2\). Thus, interior Nash tariffs do not prohibit trade.

There also exists a set of autarky Nash equilibria. In any such equilibrium, all export tariffs are set at or higher than \(\tau^a_x \equiv 1\) and all import tariffs are set at or higher than \(\tau^a_m \equiv 1\). In this case, no unilateral incentive to reduce tariffs exists, as the tariff rates in the other country ensure that a trade subsidy sufficient to induce trade would lead to a lower welfare level than that achieved under autarky.

### 2.3. Efficient Trade Policies and the Prisoners’ Dilemma Problem

We next characterize the efficient trade policies, which are the trade policies that maximize joint welfare, \(W + W^*\). Efficient export and import policies, \((\tau^e_x, \tau^e_m)\), satisfy

\[
\tau^e_x + \tau^e_m = 0.
\]

\(^9\)Export tariffs are of course rare in practice. It is important to stress that this feature of the model is not central to our argument; in particular, it does not arise as a consequence of the terms-of-trade motivations that we emphasize as forming the basis for trade agreements. Rather, the central implication of the terms-of-trade approach is that Nash export tariffs (subsidies) will be higher (lower) than is efficient. The positive value that the Nash export tariff happens to take is a consequence of our simplifying assumption that political economy effects are absent. Admittedly, the lower-than-efficient export subsidy implication of the terms-of-trade approach seems itself at odds with observed international efforts to reduce export subsidies. But as we have argued elsewhere (Bagwell and Staiger, 2001b), this implication is shared broadly by the existing economic models of trade policy, and upon closer inspection the international efforts to reduce export subsidies can be given an interpretation within the existing models.
A policy of free trade \((\tau_x = \tau_m = 0)\) is thus efficient. Equivalently, efficiency would be achieved if each government ignored the terms-of-trade effects of its trade policy choices (see (2.10) and (2.11) above). Using (2.14) and (2.15), we have that

\[
\tau_x^e + \tau_m^e = 0 < 1/2 = \hat{\tau}_x^n + \hat{\tau}_m^n
\]

Thus, Nash trade policies result in too little trade relative to efficient trade policies.

The static tariff game illustrates the Prisoners’ Dilemma problem that confronts countries. Joint welfare is maximized when countries ignore their ability to alter the terms-of-trade. But the efficient trade policy does not constitute a Nash equilibrium: each country does even better when it unilaterally exploits the terms-of-trade consequences of its policy choices, as in this way it redistributes surplus from its trading partner to itself. In the static tariff game, both countries are tempted by such “beggar-thy-neighbor” policies, and as a consequence joint welfare in the interior Nash equilibrium is inefficient. The autarky equilibrium is even worse in this respect, as welfare is reduced further.

We may now summarize our findings for the static tariff game as follows:

**Theorem 1:** In the static tariff game,

(i). there exists a unique interior Nash equilibrium with positive trade volume, and there also exist a continuum of autarky Nash equilibria.

(ii). in the interior Nash equilibrium, the import and export tariffs are positive.

(iii). the volumes of trade in the interior and autarky Nash equilibria are inefficiently low given the objectives of each government.

While the static tariff game identifies the sources of potential gain from an international trade agreement, it also provides a useful starting point for understanding why protection might be countercyclical. In particular, suppose that the degree to which efficient trade policies can be maintained varies with the business cycle. Since the efficient trade agreement involves lower trade barriers and greater trade volume than the Nash equilibrium outcomes, if the effectiveness with which countries can implement more efficient trade policies is procyclical, then protection will be countercyclical. We develop this argument next.

### 3. Protection and International Business Cycles

We now present a dynamic model of tariff determination and develop our theory of countercyclical protection. A dynamic model provides scope for more efficient
trade agreements, since a country then encounters a tradeoff when considering a tariff increase: on the one hand, a higher tariff continues to enhance the country’s welfare in the short term, but, on the other hand, opportunistic behavior of this kind could trigger a painful tariff war in the long term. Clearly, this tradeoff is influenced by the rate at which the country discounts the future as well as the rate at which each country’s demand for products of the other is expected to grow. This suggests that the level of tariff-policy cooperation may vary through time, along with the underlying business-cycle conditions that determine the expected growth rates for import demand.

To explore this possibility, we construct dynamic tariff models in which import demand fluctuates through time. Our approach is to model growth in aggregate demand as evolving cyclically and to highlight the implications of these cyclical movements for import volume. Specifically, we model cyclical movements in aggregate demand in terms of growth in the number of new goods demanded. In other words, we let \( G_t \) give the number of foreign export goods demanded by the domestic country at date \( t \), while \( G_t^* \) denotes the number of domestic export goods demanded by the foreign country at date \( t \). With this, the business-cycle conditions transpiring in the domestic (foreign) country can be interpreted in terms of the evolution of \( G_t \) (\( G_t^* \)), and the evolution of the number of goods traded in total can be determined as \( G_t^{uw} \equiv G_t + G_t^* \). Business cycles are then “international” in nature if \( G_t \) and \( G_t^* \) are perfectly correlated through time, while domestic- and foreign-country business cycles are “national” and sometimes “out of sync” with one another when these variables are imperfectly correlated. We consider the case of international business cycles in the present section, leaving the analysis of

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10 We let growth in the number of new goods supplied vary procyclically as well, but the endowment of each new good is small relative to the demand. This ensures that cyclical movements in import volume are driven by cyclical movements in demand and delivers procyclical trade deficits. In focusing on new goods, we recognize that growth in trade for existing goods is also an important ingredient in accounting for overall growth in import demand. Our focus on growth of trade in new goods is not without empirical support, however. For example, Bernard and Jensen (2001) document the importance of goods transitioning between non-traded and traded status through time, and our model can be interpreted along these lines. In any case, our association between growth in the number of traded goods and growth in import demand seems a plausible abstraction, and particularly so in light of the technical simplifications that this approach affords. In the symmetric model of trade presented here, the number of traded goods enters welfare in a proportional fashion, and so simple characterizations of expected discounted welfare over the business cycle can be derived. Consequently, incentive constraints for the dynamic tariff games can be captured in a tractable form. For simplicity, we also treat \( G_t \) and \( G_t^* \) as continuous variables.
national business cycles for the next section.

3.1. The Incentive to Cheat

Before developing any particular model of the business cycle, we first characterize
the domestic country’s short-term or single-period incentive to cheat on a proposed
tariff policy agreement. To this end, suppose that the agreement calls for a set of
tariffs \( \{ \tau_m, \tau_x, \tau_m^*, \tau_x^* \} \) at date \( t \), and consider the gain to the domestic country
from violating the agreement and defecting to its optimal response tariffs, \( \tau_{mr}(\tau_x^*) \)
and \( \tau_{xr}(\tau_m^*) \). On each of its \( G_t^* \) export goods, the domestic country gains welfare
in amount \( \Omega_x(\tau_x, \tau_m^*) \), while for each of its \( G_t \) import goods the domestic-country
welfare gain is \( \Omega_m(\tau_m, \tau_x^*) \), where

\[
\Omega_x(\tau_x, \tau_m^*) = W_x(\tau_{xr}(\tau_m^*), \tau_x^*) - W_x(\tau_x, \tau_m^*) \tag{3.1}
\]

\[
\Omega_m(\tau_m, \tau_x^*) = W_m(\tau_{mr}(\tau_x^*), \tau_x^*) - W_m(\tau_m, \tau_x^*) \tag{3.2}
\]

so that the domestic country’s total incentive to cheat is defined by \( G_t^* \Omega_x(\tau_x, \tau_m^*) +\)
\( G_t \Omega_m(\tau_m, \tau_x^*) \). The incentive to cheat for the foreign country can be defined similarly.

To better understand the incentive-to-cheat function, we next exploit the symmetry
present in the model. As (2.12) and (2.13) suggest, a country’s welfare function is symmetric across import and export sectors. In fact, it is easy to confirm that
\( W_x(\tau, \tau^*) = W_m(\tau, \tau^*) + 1/2 \), with the difference corresponding to the
different autarky payoffs for export and import markets. For any fixed good and
foreign-country tariff, it follows that the domestic country’s incentives associated
with a particular domestic tariff level are independent of whether the given good
is imported or exported. It is thus natural to model a country as selecting a single
tariff that applies to both exports and imports. Furthermore, given that the
countries are also symmetric, it is natural as well to consider the case in which
the domestic and foreign countries select the same tariff. Let us therefore set
\( \tau_m = \tau_x = \tau_m^* = \tau_x^* \equiv \tau \), and evaluate the incentive that a country has to cheat
at date \( t \) on an agreement that calls for all tariffs to be set at level \( \tau \).

For this symmetric environment, straightforward calculations reveal that

\[
\Omega_x(\tau, \tau) = \Omega_m(\tau, \tau) = (2/3)[\hat{\tau}^n - \tau]^2. \tag{3.3}
\]

where \( \hat{\tau}^n = 1/4 \) is the symmetric tariff in the interior Nash equilibrium. Using
(3.3), it is apparent that a country’s total incentive to cheat may now be written
simply as

\[ G^w_t \Omega(\tau) \equiv G^w_t\Omega_x(\tau, \tau) + G_t \Omega_m(\tau, \tau) = G^w_t(2/3)[\hat{\tau}^n - \tau]^2, \tag{3.4} \]

where \( \Omega(\tau) \) measures the incentive to cheat on any one export or import good. It is now easy to verify that \( G^w_t \Omega(\tau) \) is positive, decreasing and convex in \( \tau \), and increasing in \( G^w_t \) for \( \tau \leq 1/4 \).

Intuitively, the incentive to cheat depends only upon the total number of goods traded, as opposed to the distribution of those goods across countries, since export and import sectors are symmetric. The incentive to cheat is thus high when the total number of traded goods is large, since the optimal tariff then can be applied to a larger volume of trade. On the other hand, a higher agreed-upon tariff, \( \tau \), acts to reduce the incentive to cheat, because the tariff is then already close to its optimal level. Indeed, when \( \tau = \hat{\tau}^n = 1/4 \), the incentive to cheat is zero. Figure 1 illustrates.

### 3.2. The Dynamic Tariff Game with International Business Cycles

With the short-term benefits from cheating now characterized, let us next specify a model of the business cycle, so that the long-term welfare costs of a trade war can be evaluated. Motivated by the empirical analysis performed by Hamilton (1989), we assume that the business cycle within any given country is described by fast- and slow-growth demand phases, where the transition between phases is determined by a Markov process. We assume further that the business cycle is international, in that a single unifying business cycle operates on the economies of both the domestic and foreign countries.

Given the symmetry between export and import sectors, the possible consequences of business-cycle fluctuations for tariff cooperation are completely summarized by the manner in which the total number of traded goods, \( G^w_t \), fluctuates through time. We therefore describe the business-cycle model in terms of this variable. In particular, we assume \( G^w_t \) obeys the following nonstationary process:

\[ G^w_t = g_t(G^w_{t-1}/\varepsilon_{t-1})\varepsilon_t, \tag{3.5} \]

where \( g_t \in \{b, r\} \) is the period-\( t \) growth rate, which is stochastic and determined by a Markov process, as described below. Letting \( b > r > 0 \), we say that period \( t \) is a boom (recession) period when \( g_t = b \) (\( g_t = r \)). With regard to \( \varepsilon_t \), we assume that it is iid through time with full support over \( [\varepsilon, \bar{\varepsilon}] \) where \( E\{\varepsilon_t\} = 1 \in (\varepsilon, \bar{\varepsilon}) \) and \( \varepsilon > 0 \).
Intuitively, the total number of traded goods fluctuates between fast- and slow-growth periods, with $b$ indicating the growth rate in boom periods and $r$ representing the growth rate in recession periods. In addition, the number of traded goods in period $t$ is affected by a period-$t$ transitory shock, which alters the number of traded goods in period $t$ but leaves unaffected the number of traded goods in future periods. The period $t$ transitory shock is represented in (3.5) with the variable $\varepsilon_t$, and notice there that past shocks are indeed transitory as the period $t-1$ shock is eliminated from the base from which all future growth occurs. Thus, $\varepsilon_t$ may be appropriately interpreted in terms of the transitory shocks to trade volume that occur within broader business cycle phases. Given the iid manner in which $\varepsilon_t$ is distributed, we will sometimes drop the time subscript when no confusion is created.

The transition between boom and recession periods is assumed to be governed by a Markov process, in which

$$
\rho \equiv \text{Prob}(g_t = r \mid g_{t-1} = b) \in [0, 1] \\
\lambda \equiv \text{Prob}(g_t = b \mid g_{t-1} = r) \in [0, 1] \\
\mu \equiv \text{Prob}(g_1 = b) \in [0, 1]
$$

Thus, $\rho$ is the transition probability associated with moving from a boom to a recession, while $\lambda$ is the transition probability corresponding to moves from recessions to booms. Assuming that time runs from $t = 1$ to infinity, the parameter $\mu$ describes how the system begins. Assume further that $G_0^w > 0$, so that trade volume is always positive.

The parameters $\rho$ and $\lambda$ play important roles in two key measures associated with the business cycle. First, $\rho$ and $\lambda$ may be interpreted in terms of the expected duration of boom and recession phases, respectively. Suppose that $g_{t-1} = r$ and $g_t = b$, so that a switch to a boom period occurs at period $t$, and define $t^* \equiv \min \{ \tau > t \mid g_{\tau} = r \}$. We then define a **boom phase** as a sequence of boom periods, $\{t, \ldots, t^* - 1\}$, and the **expected duration of a boom phase** is given by

$$
\sum_{z=1}^{\infty} z \rho (1 - \rho)^{z-1} = 1/\rho
$$

In the same manner, we may define a **recession phase** and derive that the **expected duration of a recession phase** is $1/\lambda$.

A second important measure for the business cycle concerns the correlation in growth rates through time. Observe that

$$
E(g_{t+1} \mid g_t = b) - E(g_{t+1} \mid g_t = r) = [1 - \lambda - \rho][b - r],
$$
and so the expected growth rate is higher in period $t + 1$ when period $t$ is a boom period if and only if $1 - \lambda - \rho > 0$. Accordingly, we say that business cycle growth rates are positively correlated through time when $1 - \lambda - \rho > 0$, and that they are negatively correlated through time when $1 - \lambda - \rho < 0$. Finally, business-cycle growth rates are said to exhibit zero correlation when $1 - \lambda - \rho = 0$.

With the business-cycle model now developed, we return to our original focus and examine the possibilities for cooperation between countries in the setting of tariffs. In particular, suppose that the static tariff game is repeated infinitely often, where in any period $t$ governments are fully informed of (i). all past tariff choices, (ii). the current value of $g_t$ and $\varepsilon_t$ as well as all past values, and (iii). the stochastic process that governs the future evolution of $G^w_t$.¹¹ We define this game as the dynamic tariff game with international business cycles.

We select among the set of subgame perfect Nash equilibria with two additional requirements. First, we assume that equilibrium tariff strategies are symmetric across countries and sectors, so that at any date $t$ a single tariff is selected by both countries for both imports and exports. Second, we characterize the most-cooperative tariffs, which we define as the lowest tariffs that can be supported in a symmetric subgame perfect equilibrium. Following the general arguments of Abreu (1986), we find such tariffs by supposing that a deviation induces a maximal punishment. In the context of our tariff model, this is accomplished with the requirement that, if a deviation from equilibrium tariff policy occurs, then in the next period and forever thereafter the countries revert to the autarky Nash equilibrium of the static tariff game.¹²

3.3. The Cost of a Trade War

It is now apparent that countries encounter a tradeoff when making their respective tariff selections, as each must balance the one-time benefit of cheating with a deviant high tariff against the future value of maintaining a cooperative trading relationship. In other words, a tariff policy can then be supported in equilibrium only if the incentive to cheat is no higher than the expected discounted cost of a

¹¹Tariff cooperation is also possible in a finite-horizon game, since the static game admits two Nash equilibria. In this case, defection would trigger a reversion to the “bad” (i.e., autarky) equilibrium in the future.

¹²The autarky punishment is convenient because it delivers the most-cooperative equilibrium outcome. Less-severe punishments might also be considered. Our main conclusions also can be supported in equilibria with milder punishments, although the overall level of tariffs then would be somewhat higher.
trade war. Having already characterized the incentive to cheat, we turn now to a formal representation of the cost of a trade war. Combining this with the business-cycle model developed above, we then characterize the incentive constraints that equilibrium tariffs must satisfy.

For a given tariff \( \tau \) and number of traded goods \( G \), the per-period cost of a trade war, \( G^w_\omega(\tau) \), may be defined as

\[
G^w_\omega(\tau) \equiv W(\tau; G, G^*_t) - W(\hat{\tau}^a; G, G^*_t) \equiv G^w_t (1/2)[\hat{\tau}^n - \tau^2], \tag{3.7}
\]

where \( \hat{\tau}^a = 1 \) is the symmetric autarky tariff and \( \omega(\tau) \) measures the cost of a trade war per period and per export or import good.\(^{13}\) Provided that the tariff \( \tau \) is not so high as to prohibit trade, therefore, the per-period cost of a trade war is positive, larger when more goods are traded, and concave and decreasing in \( \tau \). Intuitively, the cost of a trade war is greater when more goods otherwise would be traded at cooperative tariff levels. Figure 2 illustrates.

As the cost of a trade war is experienced in future periods, it is important to specify the manner in which countries discount the future and the relationships between growth rates and the discount factor. We assume only that countries employ the same discount factor, \( \delta \), and that \( 0 < \delta r < \delta b < 1 \). These assumptions allow \( b > 1 > r \) as one possibility, in which case booms are periods of positive growth and recessions entail negative growth; another possibility is that growth is positive in either state and faster in booms. The assumption that \( \delta b < 1 \) ensures that expected discounted values are finite.

We now use methods similar to those in Bagwell and Staiger (1997) and derive the incentive constraints that equilibrium tariffs must satisfy. The Markov structure of the growth process is particularly helpful in this regard. When growth rates follow a Markov process, the expected cost of a trade war is the same in any one boom period as any other, holding fixed the level of the transitory shock \( \varepsilon \), and likewise recession periods are equivalent with one another in this sense. Equilibrium tariff functions thus may be represented as \( \tau_b(\varepsilon) \) and \( \tau_r(\varepsilon) \), where these functions indicate the equilibrium tariffs to be charged in boom and recession periods, respectively, when the current period within-phase demand shock is given by \( \varepsilon \).\(^{14}\)

\(^{13}\) In making this calculation, we have used the fact that autarky payoffs are 9/8 per export good and 5/8 per import good.

\(^{14}\) As will become clear below, equilibrium tariffs do not depend upon \( G^w_t \), since it enters as a proportional constant in both the incentive to cheat and the expected discounted cost of a trade war.
An additional benefit of the Markov structure is its recursive nature, which permits an explicit calculation of the expected discounted cost of a trade war, once the appropriate definitions are put forth. To this end, we define $\omega_b(\tau_b(\varepsilon), \tau_r(\varepsilon))$ as the per-good expected discounted cost of a trade war in period $t+1$ and thereafter, if $g_{t+1} = b$, $\tau_b(\varepsilon)$ and $\tau_r(\varepsilon)$ are the tariff functions, and the value for $\varepsilon$ in period $t+1$ has not yet been determined. Analogously, we may define $\omega_r(\tau_b(\varepsilon), \tau_r(\varepsilon))$ when $g_{t+1} = r$. Both functions are evaluated in period $t+1$ dollars.

To fix ideas, consider now the incentive constraint facing a country in period $t$, when period $t$ is a boom period and the period-$t$ within-phase shock is given by $\varepsilon_t = \varepsilon$. Simplifying notation slightly, we may represent this incentive constraint as

$$G^w_t \Omega(\tau_b(\varepsilon)) \leq \delta \{ \rho (r G^w_t / \varepsilon) \omega_r + (1 - \rho) (b G^w_t / \varepsilon) \omega_b \},$$

or more simply

$$\varepsilon \Omega(\tau_b(\varepsilon)) \leq \delta \{ \rho r \omega_r + (1 - \rho) b \omega_b \},$$

Thus, the current-period “base” level of trading volume, $G^w_t$, cancels, since all future trading volume growth will be in any event proportional to this base, but the current-period within-phase shock, $\varepsilon$, is not represented in future growth, and its value remains in the incentive constraint, with higher values for $\varepsilon$ having the effect of raising the incentive to cheat.

Building on these insights, we now represent the complete incentive system as

$$\varepsilon \Omega(\tau_b(\varepsilon)) \leq \delta \{ \rho r \omega_r + (1 - \rho) b \omega_b \} \quad (3.8)$$

$$\varepsilon \Omega(\tau_r(\varepsilon)) \leq \delta \{ \lambda b \omega_b + (1 - \lambda) r \omega_r \} \quad (3.9)$$

where

$$\omega_b = E\{ \omega(\tau_b(\varepsilon)) \varepsilon \} + \delta \{ \rho r \omega_r + (1 - \rho) b \omega_b \} \quad (3.10)$$

$$\omega_r = E\{ \omega(\tau_r(\varepsilon)) \varepsilon \} + \delta \{ \lambda b \omega_b + (1 - \lambda) r \omega_r \}. \quad (3.11)$$

We may now solve (3.10) and (3.11) for $\omega_b$ and $\omega_r$ and substitute these values back into (3.8) and (3.9). This yields the following representation of the incentive constraints:

$$\varepsilon \Omega(\tau_b(\varepsilon)) \leq E\{ \omega(\tau_r(\varepsilon)) \varepsilon \} \rho r \Delta + E\{ \omega(\tau_b(\varepsilon)) \varepsilon \} \beta \Delta \quad (3.12)$$

$$\varepsilon \Omega(\tau_r(\varepsilon)) \leq E\{ \omega(\tau_b(\varepsilon)) \varepsilon \} \lambda b \Delta + E\{ \omega(\tau_r(\varepsilon)) \varepsilon \} \sigma \Delta, \quad (3.13)$$

where

$$\Delta = \frac{\delta}{[1 - (1 - \lambda) \rho \delta r][1 - (1 - \rho) \delta b] - \delta^2 \lambda b \rho r} \quad (3.14)$$
$$\beta = b[1 - \rho - \delta r(1 - \lambda - \rho)]$$  \hspace{1cm} (3.15) \\
$$\sigma = r[1 - \lambda - \delta b(1 - \lambda - \rho)]$$ \hspace{1cm} (3.16)

Clearly, $\beta > 0$ and $\sigma > 0$. We also have that $\Delta > 0$ and that $\Delta$ increases in $\delta$ for $\delta \in (0, 1/b)$.$^{15}$

### 3.4. Solution Method

With the incentive constraints now fully captured by (3.12) and (3.13), the next task is to solve for the most-cooperative tariff functions, $\tau_b^*(\epsilon)$ and $\tau_r^*(\epsilon)$. These functions maximize welfare over the set of all tariff functions that satisfy (3.12) and (3.13). One difficulty in approaching this problem is that tariff functions affect both the incentive to cheat as well as the expected discounted cost of a trade war. Here, we expand on Rotemberg and Saloner (1986) and Bagwell and Staiger (1997) and exploit a two-step solution process, in which the expected discounted cost of a trade war is initially regarded as a constant.

Specifically, in the first step of the solution process, we view the right hand sides of (3.12) and (3.13) as fixed values, defined as

$$\tilde{\omega}_b \equiv E\{\omega(\tau_r(\epsilon)\epsilon)\} \rho \Delta + E\{\omega(\tau_b(\epsilon))\epsilon\} \beta \Delta$$  \hspace{1cm} (3.17) \\
$$\tilde{\omega}_r \equiv E\{\omega(\tau_b(\epsilon))\epsilon\} \lambda b \Delta + E\{\omega(\tau_r(\epsilon))\epsilon\} \sigma \Delta$$ \hspace{1cm} (3.18)

Using (3.12)-(3.13) and (3.17)-(3.18), the incentive constraints now appear as

$$\epsilon \Omega(\tau_b(\epsilon)) \leq \tilde{\omega}_b$$  \hspace{1cm} (3.19) \\
$$\epsilon \Omega(\tau_r(\epsilon)) \leq \tilde{\omega}_r.$$  \hspace{1cm} (3.20)

We may now define $\tau_b^*(\tilde{\omega}_b/\epsilon)$ and $\tau_r^*(\tilde{\omega}_r/\epsilon)$ as the most-cooperative tariffs when $\tilde{\omega}_b$ and $\tilde{\omega}_r$ are taken as fixed values; i.e, $\tau_b^*(\tilde{\omega}_b/\epsilon)$ is the lowest tariff satisfying (3.19) and $\tau_r^*(\tilde{\omega}_r/\epsilon)$ is defined analogously for (3.20). Using (3.4), these tariffs can be represented as follows:

$$\tau_b^*(\tilde{\omega}_b/\epsilon) = \max\{\tilde{\tau}^n - \frac{(6\tilde{\omega}_b/\epsilon)^{1/2}}{2}, 0\}$$  \hspace{1cm} (3.21)

$^{15}$Let $\Delta \equiv \delta/D(\delta)$, where $D$ is the denominator of the expression in (3.14). Simple calculations reveal that $D(0) = 1 > (b-r)/b = D(1/b)$ and $D'(\delta) < 0$ for $\delta \in [0, 1/b]$. It follows that $D(\delta) > 0$ for $\delta \in [0, 1/b]$. These properties ensure that $\Delta > 0$ and $\Delta$ increases strictly in $\delta$ for $\delta \in (0, 1/b)$.
\[ \tau^*_r(\bar{\omega}_r/\varepsilon) = \max\{\bar{\tau}^n - \frac{(6\bar{\omega}_r/\varepsilon)^{1/2}}{2}, 0\}. \] (3.22)

In short, each tariff is set as close to free trade as possible, while still being consistent with the corresponding incentive constraint.

We now proceed to the next step in this process, and present a fixed point technique through which the most-cooperative values for \(\bar{\omega}_b\) and \(\bar{\omega}_r\) may be endogenously determined. Specifically, consistency requires that the most-cooperative values for \(\bar{\omega}_b\) and \(\bar{\omega}_r\) lead through (3.21) and (3.22) to tariffs which in turn generate through (3.17) and (3.18) the originally specified values for \(\bar{\omega}_b\) and \(\bar{\omega}_r\). This requirement is captured by the following fixed-point equations:

\[
\bar{\omega}_b \equiv E\{\omega(\tau^*_b(\bar{\omega}_b/\varepsilon))\varepsilon\} pr\Delta + E\{\omega(\tau^*_b(\bar{\omega}_b/\varepsilon))\varepsilon\} \beta \Delta \] (3.23)

\[
\bar{\omega}_r \equiv E\{\omega(\tau^*_r(\bar{\omega}_r/\varepsilon))\varepsilon\} \lambda b \Delta + E\{\omega(\tau^*_r(\bar{\omega}_r/\varepsilon))\varepsilon\} \sigma \Delta. \] (3.24)

We show in the Appendix that these fixed-point equations admit a unique solution, \((\bar{\omega}_b, \bar{\omega}_r)\). Once these values are determined, the most-cooperative tariffs are then defined by

\[
\tau^c_b(\varepsilon) \equiv \tau^*_b(\bar{\omega}_b/\varepsilon) \] (3.25)

\[
\tau^c_r(\varepsilon) \equiv \tau^*_r(\bar{\omega}_r/\varepsilon). \] (3.26)

In this way, the problem of solving for the most-cooperative tariff functions is reduced to the alternative task of solving for two fixed point values.\(^{16}\)

### 3.5. The Most-Cooperative Tariffs

The most-cooperative tariffs are set to balance the current incentive to cheat against the long-term cost of a trade war. Viewed from this perspective, it may be anticipated that cooperation will be easier in periods in which the expected rate of future trade growth is large, since the cost of a trade war is then also large. This suggests that lower tariffs can be enforced in such periods, even though the incentive to cheat is thereby raised. Transitory within-phase shocks represent an additional influence on the most-cooperative tariff functions. Drawing on the structure developed above, it is natural to anticipate that attempts to liberalize trade will be frustrated by high transitory shocks, as a period of unusually high

\(^{16}\)The approach pursued here presumes that the most-cooperative tariffs are found by lowering tariffs as much as possible in each state, as is evident from (3.21) and (3.22). This presumption is appropriate in the present model, because incentive constraints are complementary, with more cooperation in any one state fostering greater cooperation in the other as well.
trade volume exacerbates the short-term incentive to cheat without raising commensurably the cost of a trade war. We develop and elaborate upon these ideas in this subsection.

In characterizing the most-cooperative tariffs, it is interesting to determine those environments in which countries achieve free trade in all possible states, \( \tau_b^c(\varepsilon) \equiv \tau_r^c(\varepsilon) \equiv 0 \). Of course, complete liberalization is sure to fail if \( \varepsilon \) is sufficiently big, as the temptation to cheat is then irresistible when the within-phase shock is near its upper bound. To create the possibility of complete liberalization, we thus restrict the size of \( \varepsilon \) with the following assumption:

\[
\delta b > \frac{\varepsilon}{[3 + \overline{\varepsilon}]}.
\] (3.27)

This assumption admits a simple interpretation. It implies that even a maximal transitory shock is insufficient to disrupt free trade, when the business cycle is described by maximal growth (i.e., \( g_t = b \) with probability one at all dates).

It is also interesting to characterize those environments in which some protection is required in the most-cooperative equilibrium. This motivates the following assumption:

\[
\frac{\varepsilon}{[3 + \overline{\varepsilon}]} > \delta r.
\] (3.28)

This inequality implies that a maximal transitory shock would be incompatible with free trade, were the business cycle one of minimal growth (i.e., \( g_t = r \) with probability one at all dates). Together, as will become clear, inequalities (3.27) and (3.28) describe an international business cycle in which complete liberalization is possible if and only if the expected duration of a boom (recession) phase is sufficiently long (short).

With these assumptions in place, we are now prepared to describe the conditions under which free trade can be achieved in all states. To this end, we define the following functions:

\[
\tilde{\lambda}(\rho, \overline{\varepsilon}) = \frac{1 - (1 - \rho)\delta b}{1/\lambda^*(\overline{\varepsilon}) - \delta b}
\] (3.29)

\[
\tilde{\rho}(\lambda, \overline{\varepsilon}) = \frac{1 - (1 - \lambda)\delta r}{1/\rho^*(\overline{\varepsilon}) - \delta r},
\] (3.30)

where \( \lambda^*(\overline{\varepsilon}) = 1 - \rho^*(\overline{\varepsilon}) \) and

\[
\lambda^*(\overline{\varepsilon}) = \frac{\overline{\varepsilon}/[3 + \overline{\varepsilon}] - \delta r}{\delta(b - r)}
\] (3.31)
\[
\rho^*(\varepsilon) = \frac{\delta b - \varepsilon/[3 + \varepsilon]}{\delta(b - r)}.
\] (3.32)

Under assumptions (3.27) and (3.28), we find that \(\lambda^*(\varepsilon) \in (0, 1)\) and \(\rho^*(\varepsilon) \in (0, 1)\). We also have that \(\lambda(0, \varepsilon) \in (0, 1)\) and \(\rho(0, \varepsilon) \in (0, 1)\). These properties are illustrated in Figure 3.

As we show formally in the Appendix, when \(\lambda \geq \hat{\lambda}(\rho, \varepsilon)\) and \(\rho \leq \hat{\rho}(\lambda, \varepsilon)\), then the most-cooperative tariffs support free trade in all states, i.e., \(\tau^c_b(\varepsilon) \equiv 0\). This free-trade region of the parameter space is marked as Region I in the parameter box represented in Figure 3. The essential point is intuitive. When \(\lambda\) is large and \(\rho\) is small, the expected duration of a recession is brief and the expected duration of a boom is long. Thus, the expected growth rate in the future is close to the boom level, \(b\), regardless of whether the current period is a boom or a recession period. In this situation, under assumption (3.27), free trade can be supported even when a maximal transitory shock is encountered. Notice that the free trade region expands as the difference between \(\varepsilon/[3 + \varepsilon]\) and \(\delta r\) shrinks, since then free trade becomes possible in all states even for a business cycle that has long exposures to recessions.

Free trade is no longer possible in all states when \(\lambda < \hat{\lambda}(\rho, \varepsilon)\) or \(\rho > \hat{\rho}(\lambda, \varepsilon)\). Some protection is then required and a central issue is whether protection is greater in boom or recession periods. As we show formally in the Appendix, the cyclical properties of protection are determined entirely by the correlation in growth rates. Growth rates are positively correlated in Region II of Figure 3, and in this case expected future growth is higher when the current period is a boom period. This means in turn that the expected discounted cost of a trade war is higher when the current period is a boom, since cheating today would result in the sacrifice of a high level of expected gains from trade in the future. Consequently, a higher incentive to cheat can be tolerated in boom periods, and so the most-cooperative tariffs are (weakly) lower in boom than recession periods, given the level of transitory shock. In other words, when growth rates are positively correlated, the most-cooperative tariffs are countercyclical \((\tau^c_b(\varepsilon) \leq \tau^c_r(\varepsilon))\).

While free trade cannot be supported in all states in Region II, it may be possible in some states. Figure 4a illustrates one possibility. Here, free trade can be achieved in both boom and recession periods provided that the level of transitory shock is small. When higher shocks arrive, however, free trade is possible only in boom periods. Finally, if the transitory shock is higher yet, then the most-cooperative tariff must be positive for both boom and recession periods.
although the recession-period tariff remains higher. In sum, if international business cycles exhibit persistence, as captured in our model by the specification of positively correlated growth rates, then protection is countercyclical with respect to business cycle phases, and high transitory shocks to trade volume may require that protection be temporarily increased.

The next region to consider is the region marked as Region III in Figure 3. Here, growth rates are negatively correlated, indicating that the prospects for cooperation are most favorable now in recession periods. Accordingly, we find that protection is procyclical \( (\tau_c^b(\varepsilon) \geq \tau_c^r(\varepsilon)) \) when growth rates are negatively correlated through time. As before, high transitory shocks raise the short term incentive to cheat, forcing a temporary retreat from liberalization. Figure 4b illustrates the negative-correlation case.\(^{17}\)

The main points may now be summarized as follows:

**Theorem 2:** In the dynamic tariff game with international business cycles,

(i) The most-cooperative tariffs involve free trade in all states if and only if the expected duration of a boom phase is sufficiently long and the expected duration of a recession phase is sufficiently short.

(ii) The most-cooperative tariffs are countercyclical (procyclical) when growth rates are positively (negatively) correlated through time.

(iii) Regardless of the nature of correlation in growth rates, a higher transitory shock to trade volume results in a (weakly) higher most-cooperative tariff.

To the extent that international business cycles are well described by positively correlated growth rates, therefore, the theory developed here suggests that tariffs will be higher in recessions and in periods in which the trade volume experiences a transitory surge. These findings are consistent with the empirical analyses of protection noted in the Introduction. In particular, the model predicts countercyclical movements in protection in the presence of procyclical movements in trade volume, consistent with the large empirical literature relating to cyclical properties of protection and imports. But for a given phase of the business cycle the model also predicts that protection levels rise in response to increases in trade volume, and this finding is consistent with Trefler’s (1993) observation that protection rises with increases in import penetration, even after controlling for

\(^{17}\)A final possibility is that growth rates exhibit zero correlation, in which case \( 1 = \lambda + \rho \). In this event, expected trade volume growth in the future is independent of whether the current period is a boom or a recession, and so the most-cooperative tariffs are acyclic \( (\tau_c^b(\varepsilon) = \tau_c^r(\varepsilon)) \).
The results developed here also generalize an earlier finding of ours (Bagwell and Staiger, 1990), in which we model transitory surges only and offer an equilibrium interpretation of the GATT escape clause (Article XIX) and various “managed trade” practices (e.g., VERs). According to this interpretation, high transitory shocks to the volume of trade necessitate an increase in protection above the relevant “baseline” level, if the cooperative agreement is to be credibly enforced. Temporary retreats from liberalization that are brought about by unusual surges in trade volumes thus may serve to maintain the credibility of the cooperative trade agreement. From this perspective, the results developed here offer an equilibrium interpretation of GATT safeguard procedures and managed trade practices that arise in response to transitory trade volume surges that occur within broader business cycle phases.19

18Trefler’s (1993) analysis is based on cross-section data for 1983 (and changes in import penetration between 1980 and 1983). Controlling for business cycle conditions as measured by industry growth and unemployment, he finds that U.S. manufacturing industries tended to receive higher protection in 1983 if they experienced rising import penetration between 1980 and 1983. Our model depicts growth in trade volume as growth in the overall number of traded goods \((G + G^*)\), and therefore does not directly yield predictions about import surges and protection at the industry level. However, the model can be reinterpreted to yield industry-level predictions if it is viewed as a model of trade in a single industry and \(G (G^*)\) is taken to be the number of economically active (but geographically distinct) import regions in the home (foreign) economy. See also Bagwell and Staiger (1990), where the industry-level relationship between import surges and protection is direct.

19The model also can be generalized to allow for within-phase shocks that are of intermediate duration. This can be formalized with the assumption that the within-phase shock is transitory with probability \(\theta \in (0, 1)\) and permanent with probability \(1 - \theta\). Specifically, let

\[
G^w_t = g_t[\theta G^w_{t-1}/\varepsilon_{t-1} + (1 - \theta)G^w_{t-1}]\varepsilon_t
\]

where \(g_t\) obeys (3.6) and \(\varepsilon_t\) is iid. Assuming that governments don’t know when setting tariff policy in period \(t - 1\) whether the period \(t - 1\) shock is in fact transitory or permanent, the incentive constraints can be derived as before, except that \(\varepsilon_{t-1}/[\theta + (1 - \theta)\varepsilon_{t-1}]\) now replaces \(\varepsilon_{t-1}\). In the pure case of permanent shocks (\(\theta = 0\)), we find that within-phase shocks have no effect on the most-cooperative tariffs whatsoever, since the shock affects the incentive to cheat and the cost of a trade war in the same proportion. More generally, the most-cooperative tariffs are more responsive upward to within-phase shocks when the shocks are expected to be more transitory in nature (i.e., when \(\theta\) is higher).
4. Protection and National Business Cycles

We now relax the assumption of an international business cycle and suppose instead that each country’s business cycle evolves independently of the other’s. After defining the national business cycle model, we derive the corresponding incentive constraints and show that the main qualitative conclusions developed above continue to hold. However, we now find that a country’s tariff policy depends not only on the state of its own business cycle, but also on the state of its trading partner’s business cycle.

4.1. The Dynamic Tariff Game with National Business Cycles

We begin by developing the national business cycle model. Our approach is to specify directly a multi-state Markov-growth process for the total volume of trade, $G^w_t$, and then to interpret the associated trade volume growth states in terms of the respective national business cycles. Under this approach, national business cycle fluctuations are summarized entirely by various growth rates for $G^w_t$, and so the modeling framework developed above can be extended in order to characterize the associated most-cooperative tariffs.20

In particular, our first assumption is that the total volume of trade alternates stochastically between three possible growth rates: $g_{bb}$, $g_{br}$ and $g_{rr}$, where $g_{bb} > g_{br} > g_{rr}$, $\delta g_{bb} < 1$ and $g_{rr} > 0$. The interpretation is that total trade volume grows at the fast rate $g_{bb}$ when both national economies are experiencing a boom, while the total trade volume grows at the slower rate $g_{rr}$ when the national economies are each in a recession. An intermediate growth rate, $g_{br}$, arises when one economy is in a boom and the other is in a recession.

Our second assumption specifies the Markov transition probabilities associated with the three states for total trade volume. The specification is motivated by the interpretation that domestic and foreign national business cycles evolve independently but are described by the same underlying set of transition probabilities. To this end, let $S_t$ be a two-dimensional vector with elements $(s_t, s^*_t)$, where $s_t \in \{B, R\}$ and $s^*_t \in \{B, R\}$ represent the general state of the business cycle in period $t$ in the domestic and foreign countries, respectively. Then the

20 An alternative approach is to directly specify independent Markov-growth processes for the domestic and foreign business cycles, and then to examine the implied cyclical behavior for $G^w_t$ between the two countries. While this approach is conceptually attractive, it does introduce significant technical complexities. The most-cooperative tariffs may then also depend on the current levels $G_t$ and $G^*_t$, representing an increase in the dimensionality of the state space.
transition probabilities for the total trade volume are fully specified under the assumption that

\[ \text{Prob}(s_t = R \mid s_{t-1} = B) \equiv \rho \equiv \text{Prob}(s_t^* = R \mid s_{t-1}^* = B) \]  
\[ \text{Prob}(s_t = B \mid s_{t-1} = R) \equiv \lambda \equiv \text{Prob}(s_t^* = B \mid s_{t-1}^* = R), \]  
(4.1)

where \( g_t = g_{bb} \) if \( S_t = (B, B) \), \( g_t = g_{br} \) if \( S_t \in \{(B, R), (R, B)\} \) and \( g_t = g_{rr} \) if \( S_t = (R, R) \). With this structure in place, the transition probabilities associated with the three states for total trade volume are easily calculated. For example, the probability of moving from the “boom, boom” state with growth rate \( g_{bb} \) to the “recession, recession” state with growth rate \( g_{rr} \) is \( \rho^2 \).

We may now define the nonstationary process that \( G_t \) is assumed to follow as

\[ G_t^w = g_t(G_{t-1}^w/\varepsilon_{t-1})\varepsilon_t, \]  
(4.3)

which is the same as (3.5), except that the period-\( t \) growth rate \( g_t \) now assumes one of three possible rates, \( g_t \in \{g_{bb}, g_{rr}, g_{br}\} \), with the associated transition probabilities now defined by (4.1) and (4.2). As before, we assume that \( \varepsilon_t \) is iid through time with full support over \( [\varepsilon, \overline{\varepsilon}] \) where \( E\{\varepsilon_t\} = 1 \in (\varepsilon, \overline{\varepsilon}) \) and \( \varepsilon > 0 \).

With the national business cycle model now fully specified, we may define the dynamic tariff game with national business cycles in terms the infinite repetition of the static tariff game, in which in any period \( t \) all governments are fully informed of (i). all past tariff choices, (ii). the current value of \( g_t \) and \( \varepsilon_t \) as well as all past values, and (iii). the stochastic process given in (4.1)-(4.2) that governs the future evolution of \( G_t^w \).

4.2. The Most-Cooperative Tariffs

We turn now to a representation of the incentive constraints associated with the dynamic tariff game with national business cycles. As in the international business cycle model, the equilibrium tariff in period \( t \) may be expressed as a function of the period-\( t \) growth rate for total trade volume, which is now either \( g_{bb}, g_{br} \) or \( g_{rr} \), and the period-\( t \) transitory shock, \( \varepsilon \). We thus write the equilibrium tariff functions in the form \( \tau_{bb}(\varepsilon), \tau_{br}(\varepsilon), \) and \( \tau_{rr}(\varepsilon) \). The most-cooperative tariffs are the lowest such tariffs, and they are denoted as \( \tau_{cc}^{bb}(\varepsilon), \tau_{cc}^{br}(\varepsilon), \) and \( \tau_{cc}^{rr}(\varepsilon) \).

In analogy with (3.8)-(3.11), the incentive constraints may now be represented as:

\[ \varepsilon \Omega(\tau_{cc}^{bb}(\varepsilon)) \leq \delta \{ \rho^2 g_{rr} \overline{\tau}_{rr} + 2(1 - \rho) \rho g_{br} \overline{\tau}_{br} + (1 - \rho)^2 g_{bb} \overline{\tau}_{bb} \} \]  
(4.4)
\[\varepsilon \Omega(\tau_{br}(\varepsilon)) \leq \delta \{(1-\lambda)\rho g_{rr}\bar{\omega}_{rr} + [(1-\lambda)(1-\rho) + \rho \lambda]g_{br}\bar{\omega}_{br} + \lambda(1-\rho)g_{bb}\bar{\omega}_{bb}\} \quad (4.5)\]

\[\varepsilon \Omega(\tau_{rr}(\varepsilon)) \leq \delta \{(\lambda^2 g_{bb}\bar{\omega}_{bb} + 2(1-\lambda)\lambda g_{br}\bar{\omega}_{br} + (1-\lambda)^2 g_{rr}\bar{\omega}_{rr}\} \quad (4.6)\]

where

\[\bar{\omega}_{bb} = E\{\omega(\tau_{bb}(\varepsilon))\varepsilon\} + \delta \{(1-\lambda)\rho g_{rr}\bar{\omega}_{rr} + 2(1-\lambda)\rho g_{br}\bar{\omega}_{br} + (1-\rho)g_{bb}\bar{\omega}_{bb}\} \quad (4.7)\]

\[\bar{\omega}_{br} = E\{\omega(\tau_{br}(\varepsilon))\varepsilon\} + \delta \{(1-\lambda)\rho g_{rr}\bar{\omega}_{rr} + [(1-\lambda)(1-\rho) + \rho \lambda]g_{br}\bar{\omega}_{br} + \lambda(1-\rho)g_{bb}\bar{\omega}_{bb}\} \quad (4.8)\]

\[\bar{\omega}_{rr} = E\{\omega(\tau_{rr}(\varepsilon))\varepsilon\} + \delta \{\lambda^2 g_{bb}\bar{\omega}_{bb} + 2(1-\lambda)\lambda g_{br}\bar{\omega}_{br} + (1-\lambda)^2 g_{rr}\bar{\omega}_{rr}\} \quad (4.9)\]

As before, for any given total trade volume growth rate and transitory shock, the short-term incentive to cheat cannot exceed the long-term cost of a trade war. In representing the cost of a trade war for each of the three possible period-\(t\) growth rates, we define \(\bar{\omega}_{bb}\) as the per-good expected discounted cost of a trade war in period \(t+1\) and thereafter, if \(g_{t+1} = g_{bb}, \tau_{bb}(\varepsilon), \tau_{br}(\varepsilon)\) and \(\tau_{rr}(\varepsilon)\) are the tariff functions, and the value for \(\varepsilon\) in period \(t+1\) has not yet been determined. Analogous interpretations apply for \(\bar{\omega}_{br}\) and \(\bar{\omega}_{rr}\).

With the national business cycle model defined and the incentive constraints represented, the analysis now proceeds similarly to that presented above for the international business cycle model. We thus relegate additional derivations to the Appendix and describe here the main findings. In analogy with analysis above, we say that a national business cycle growth rate is positively correlated (negatively correlated) through time if \(1 - \lambda - \rho > 0\) \((1 - \lambda - \rho < 0)\), while zero correlation occurs when \(1 - \lambda - \rho = 0\). Similarly, we say that the most-cooperative tariffs are countercyclical when \(\tau_{bb}(\varepsilon) \leq \tau_{br}(\varepsilon) \leq \tau_{rr}(\varepsilon)\), while they are said to be procyclical when \(\tau_{bb}(\varepsilon) \geq \tau_{br}(\varepsilon) \geq \tau_{rr}(\varepsilon)\). With these definitions made, our main findings can be reported:

**Theorem 3:** In the dynamic tariff game with national business cycles,

(i) The most-cooperative tariffs are countercyclical (procyclical) when growth rates are positively (negatively) correlated through time.

(ii) Regardless of the nature of correlation in growth rates, a higher transitory shock to trade volume results in a (weakly) higher most-cooperative tariff.

This theorem is proved in the Appendix.

Two main lessons emerge from this theorem. A first point is that the central results reported above in Theorem 2 for the case of international business cycles carry over to the situation in which countries experience independent national
business cycles. The most-cooperative tariffs are again lower when the growth rate for total trade volume is higher, provided that business cycles are described by positive correlation, and higher transitory shocks to total trade volume continue to require higher most-cooperative tariffs. A second lesson concerns the determinants of a country’s tariff policy. A country’s most-cooperative tariff is fundamentally determined by the growth rate of total trade volume, but this rate is in turn determined by the combination of business cycle states experienced in the domestic and foreign national economies. In other words, the most-cooperative tariff selected by a country at a point in time is a function of the current states of the business cycle both at home and abroad.

5. Conclusion

Adopting the view that trade agreements must be self-enforcing, we explore the ability of countries to overcome their beggar-thy-neighbor incentives and enforce liberal trade policies. Cooperative trade policies can be enforced when countries recognize the ongoing nature of their relationship, since each country’s short-term incentive to pursue beggar-thy-neighbor policies is then balanced against the long-term costs of a consequent trade war. Business cycle fluctuations result in an initial imbalance between these short- and long-term considerations, requiring an adjustment in the equilibrium tariff level in order to maintain some measure of cooperation. In this general fashion, we forge a link between the state of the business cycle and the level of protection.

We demonstrate the usefulness of this general approach with two main predictions. First, we find that the most-cooperative tariffs are countercyclical, as countries are able to sustain low tariffs in a persistent boom phase characterized by fast growth in the volume of trade. A second finding concerns the implications of transitory or acyclic increases in the level of trade volume. We show that transitory shocks to the trade volume level result in more protection. As we discuss in the Introduction, these predictions are consistent with empirical regularities observed in the relationship between protection and the business cycle. The findings also offer an interpretation of GATT safeguard procedures and managed trade practices as responses to transitory trade volume surges that occur.

\[ \text{21} \text{It is also possible to derive the region over which free trade occurs in all states. As in the case of international business cycles, this region is described by low values for} \ \rho \text{ (i.e., a large expected duration for a boom phase) and high values for} \ \lambda \text{ (i.e., a small expected duration for a recession phase).} \]
within broader business cycle phases. We also demonstrate that our predictions are robust, as they arise whether business cycles are international or national in nature.

Our theory suggests a number of empirical tests that might be pursued in future work. A first possibility would be to organize import volume data into persistent and transitory components. In this way, the two main predictions of our approach could be thoroughly investigated as part of a unified empirical analysis. Second, it would be interesting to empirically investigate the relationship between country size and tariff cycles. In our theory, cyclical movements in trade policy derive from the ability of countries to manipulate the terms of trade. An implication is that the countercyclical behavior of tariffs should be more pronounced in large countries. Finally, our analysis suggests that anticipated changes in future trade volumes can affect current trade policies. This principle - behavioral changes can precede (anticipated) structural changes - may lead to interesting empirical tests.²²

We began this paper with a discussion of the two logical possibilities that might explain the observed countercyclical behavior of trade barriers. While it is commonly argued that governments increase protection in recessions for domestic political economy reasons, a convincing model of this process has not yet been offered. We have endeavored here to present a formal model of the alternative possibility, whereby beggar-thy-neighbor tendencies across countries lead to countercyclical tariffs. The theory that we have developed is consistent with available evidence, and it also suggests a number of new testable implications that might be investigated in future empirical work.

²²Further extensions of the representation of the business cycle that we have adopted in this paper may also be warranted, especially in relation to future empirical investigation. In this regard, Hamilton’s (1989) description of the U.S. business cycle can be criticized because: (i) it implies a lack of duration dependence for both booms and recessions, whereas Diebold, Rudebusch and Sichel (1993) find no evidence of duration dependence for booms but strong evidence of positive duration dependence for recessions in postwar U.S. data; and (ii) it implies that the sign of the estimated autocorrelations between growth rates will be the same across lags, whereas Evans and Reichlin (1994) find using post-war U.S. data that the estimated autocorrelations are positive for low lags and negative for longer lags. In other work (available on request), we have investigated the theoretical properties of a three-state Markov process that can address (i) and (ii) above, and find that the key predictions of the present paper can be extended naturally to this more complicated business cycle environment.
6. Appendix

Lemma 1: In the dynamic tariff game with international business cycles, the fixed point values \((\bar{\omega}_b, \bar{\omega}_r)\) satisfying (3.23) and (3.24) exist uniquely and satisfy

(i). \((\bar{\omega}_b, \bar{\omega}_r) > 0\)

(ii). \(\text{sign}\{\omega_b - \bar{\omega}_r\} = \text{sign}\{1 - \lambda - \rho\}\)

Proof: We begin by characterizing the function

\[
E(\bar{\omega}) \equiv E\{\omega(\tau^*(\bar{\omega}/\varepsilon))\varepsilon\},
\]

where

\[
\tau^*(\bar{\omega}/\varepsilon) = \max\{\bar{\tau}^n - \frac{(6\bar{\omega}/\varepsilon)^{1/2}}{2}, 0\}
\]

Using (3.7), calculations reveal that

\[
E(\bar{\omega}) = 1/8, \text{ if } \bar{\omega}/\varepsilon \geq 1/24
\]

\[
E(\bar{\omega}) = 3/32 + (1/8)(6\bar{\omega})^{1/2}E(\varepsilon^{1/2}) - 3\bar{\omega}/4, \text{ if } \bar{\omega}/\varepsilon \leq 1/24
\]

\[
E(\bar{\omega}) = 1/8 - (1/32) \int \frac{\varepsilon dF(\varepsilon)}{24\bar{\omega}} + [(6\bar{\omega})^{1/2}/8] \int \frac{\varepsilon^{1/2} dF(\varepsilon)}{24\bar{\omega}} - (3\bar{\omega}/4) \int dF(\varepsilon), \text{ otherwise}
\]

where \(F\) is the distribution function for \(\varepsilon\) and \(E(\varepsilon^{1/2})\) is the expected value of \(\varepsilon^{1/2}\). It may now be confirmed that \(E(\bar{\omega})\) is continuous and positive for \(\bar{\omega} \geq 0\), and has infinite slope when \(\bar{\omega} = 0\). In addition, \(E(\bar{\omega})\) is increasing and concave for \(\bar{\omega} \in [0, \varepsilon/24]\) and constant for \(\bar{\omega} \geq \varepsilon/24\).

With this notation in place, the fixed point equations (3.23) and (3.24) may be rewritten as

\[
\bar{\omega}_b = E\{\bar{\omega}_r\}pr\Delta + E\{\bar{\omega}_b\}\beta\Delta
\]

\[
\bar{\omega}_r = E\{\bar{\omega}_b\}\lambda b\Delta + E\{\bar{\omega}_r\}\sigma\Delta.
\]

Notice that neither constraint is satisfied at the origin. In correspondence with (6.6) and (6.7) when \(\bar{\omega}_b = \bar{\omega}_r\), we may define

\[
f_b(\bar{\omega}) \equiv E(\bar{\omega})[pr + \beta]\Delta - \bar{\omega}
\]

\[
f_r(\bar{\omega}) \equiv E(\bar{\omega})[\lambda b + \sigma]\Delta - \bar{\omega}.
\]
Thus, e.g., when \( f_b(\omega) = 0 \), it follows that the boom-period incentive constraint (6.6) is satisfied on the 45 degree line at \( \omega_b = \omega_r = \omega \). Observe that \( f_b(\omega) \) and \( f_r(\omega) \) are positive with an infinite derivative when \( \omega = 0 \). These functions are also concave for \( \omega \leq \pi/24 \), decrease linearly at higher values for \( \omega \), and become negative when \( \omega \) is sufficiently large.

Let \( \omega_b \) be the unique root satisfying \( f_b(\omega_b) = 0 \), and let \( \omega_r \) be the unique root satisfying \( f_r(\omega_r) = 0 \). Clearly, \((\omega_b, \omega_r) > 0, f'_b(\omega_b) < 0, \) and \( f'_r(\omega_r) < 0 \). Observe next that

\[
\{(pr + \beta) - (\lambda b + \sigma)\} = (b - r)(1 - \lambda - \rho)\Delta.
\]

We thus have that

\[
f_b(\omega_r) = f_b(\omega_r) - f_r(\omega_r) = E(\omega_r)(b - r)(1 - \lambda - \rho)\Delta,
\]

which with \( E(\omega_r) > 0 \) implies that

\[
\text{sign}\{(\omega_b - \omega_r)\} = \text{sign}\{1 - \lambda - \rho\}.
\]

The function \( f_b \) thus has a larger root than does the function \( f_r \) under positive correlation.

We next differentiate the boom-period fixed point equation (6.6) to get

\[
\frac{\partial \omega_b}{\partial \omega_r} \mid_b = \frac{E'(\omega_r)pr\Delta}{1 - E'(\omega_b)\beta\Delta} = \frac{E'(\omega_r)pr\Delta}{E'(\omega_b)pr\Delta - f'_b(\omega_b)}.
\]

Similarly, the recession-period fixed point equation (6.7) satisfies

\[
\frac{\partial \omega_b}{\partial \omega_r} \mid_r = \frac{1 - E'(\omega_r)\sigma\Delta}{E'(\omega_b)\lambda b\Delta} = \frac{E'(\omega_r)\lambda b\Delta - f'_r(\omega_r)}{E'(\omega_b)\lambda b\Delta}.
\]

Differentiating once more, we have that

\[
\frac{\partial^2 \omega_b}{\partial \omega_r^2} \mid_b = \frac{E''(\omega_r)pr\Delta + E''(\omega_b)\beta\Delta[\frac{\partial \omega_b}{\partial \omega_r} \mid_b]^2}{1 - E'(\omega_b)\beta\Delta}
\]

\[
\frac{\partial^2 \omega_b}{\partial \omega_r^2} \mid_r = -\frac{E''(\omega_r)\sigma\Delta + E''(\omega_b)\lambda b\Delta[\frac{\partial \omega_b}{\partial \omega_r} \mid_r]^2}{E'(\omega_b)\lambda b\Delta}.
\]

Observe from (6.13) and (6.15) that the boom-period incentive constraint is concave if it is positively sloped. Using (6.16), we see that the recession-period incentive constraint is convex.
Using (6.13), (6.14), $f'_b(\bar{\omega}_b) < 0$ and $f'_r(\bar{\omega}_r) < 0$, it is now a simple matter to see that
\[
\frac{\partial \bar{\omega}_b}{\partial \bar{\omega}_r} |_{\bar{\omega}_b = \bar{\omega}_r = \bar{\omega}_b} = 0,
\]
(6.17)
\[
\frac{\partial \bar{\omega}_b}{\partial \bar{\omega}_r} |_{\bar{\omega}_r = \bar{\omega}_b = \bar{\omega}_r} = 0.
\]
(6.18)

It follows that the respective fixed point curves eventually slope upward through their respective 45 degree line crossings. In particular, there must exist values $\omega_b$ and $\omega_r$ with $0 < \omega_b < \bar{\omega}_b$ and $0 < \omega_r < \bar{\omega}_r$ such that (6.6) is satisfied at $(\bar{\omega}_b, \bar{\omega}_r) = (\omega_b, 0)$ and slopes upward from $(\omega_b, 0)$ through $(\bar{\omega}_b, \bar{\omega}_r)$ and on, while (6.7) holds at $(\bar{\omega}_b, \bar{\omega}_r) = (0, \omega_r)$ and slopes upward from $(0, \omega_r)$ through $(\bar{\omega}_r, \bar{\omega}_r)$ and on. With $\bar{\omega}_b$ on the y axis, the boom-period fixed point equation thus crosses the 45 degree line at $(\bar{\omega}_b, \bar{\omega}_b)$ from above, while the recession-period fixed point equation crosses at $(\bar{\omega}_r, \bar{\omega}_r)$ from below. Neither crosses the 45 degree line at any other point.

Given these properties, the two incentive constraints must cross at exactly one point, with the recession-period constraint being steeper at that point. Furthermore, the intersection point, $(\hat{\omega}_b, \hat{\omega}_r)$, must satisfy $(\hat{\omega}_b, \hat{\omega}_r) > 0$ and sign$(\hat{\omega}_b - \hat{\omega}_r) = $ sign$(\bar{\omega}_b - \bar{\omega}_r) = $ sign$(1 - \lambda - \rho)$.

Lemma 2: In the dynamic tariff game with international business cycles, the most-cooperative tariffs support free trade in all states $(\tau^*_b(\varepsilon) \equiv \tau^*_r(\varepsilon) \equiv 0)$ if and only if $\min\{pr + \beta, \lambda b + \sigma\} \Delta \geq \varepsilon/3$.

Proof: Observe that $\min\{pr + \beta, \lambda b + \sigma\} \Delta \geq \varepsilon/3$ is equivalent to $(pr + \beta)\Delta/8 \geq \varepsilon/24$ and $(\lambda b + \sigma)\Delta/8 \geq \varepsilon/24$, and consider the solution candidate $(\hat{\omega}_b, \hat{\omega}_r) = ((pr + \beta)\Delta/8, (\lambda b + \sigma)\Delta/8)$. Given that (3.21) implies $\tau^*_b(\bar{\omega}_b/\varepsilon) = 0$ for $\bar{\omega}_b \geq \varepsilon/24$, with (3.22) yielding the analogous conclusion for $\tau^*_r(\bar{\omega}_r/\varepsilon)$, it follows that $\tau^*_b(\bar{\omega}_b/\varepsilon) \equiv 0$ and $\tau^*_r(\bar{\omega}_r/\varepsilon) \equiv 0$. Substitution of these free-trade values into the fixed-point equations (3.23) and (3.24) yields $(pr + \beta)\Delta/8$ and $(\lambda b + \sigma)\Delta/8$ on the respective RHS’s, confirming that the proposed solution is indeed a fixed-point solution. Next, suppose a fixed-point solution exists and $\tau^*_b(\varepsilon) \equiv \tau^*_r(\varepsilon) \equiv 0$. Using (3.21)-(3.24), it is then necessary that $\tau^*_b(\bar{\omega}_b/\varepsilon) = 0 = \tau^*_r(\bar{\omega}_r/\varepsilon)$, $\bar{\omega}_b \geq \varepsilon/24$ and $\bar{\omega}_r \geq \varepsilon/24$, and $\bar{\omega}_b = (pr + \beta)\Delta/8$ and $\bar{\omega}_r = (\lambda b + \sigma)\Delta/8$. It thus must be that $(pr + \beta)\Delta/8 \geq \varepsilon/24$ and $(\lambda b + \sigma)\Delta/8 \geq \varepsilon/24$, or equivalently $\min\{pr + \beta, \lambda b + \sigma\} \Delta \geq \varepsilon/3$. 
Lemma 3: In the dynamic tariff game with international business cycles, the most-cooperative tariffs support free trade in all states \((\tau_b^c(\varepsilon) \equiv \tau_r^c(\varepsilon) \equiv 0)\) if and only if \(\lambda \geq \hat{\lambda}(\rho, \varepsilon)\) and \(\rho \leq \tilde{\rho}(\lambda, \varepsilon)\).

Proof: Using (6.10), \(\min\{\rho r + \beta, \lambda b + \sigma\} \Delta = (\lambda b + \sigma) \Delta\) if and only if \(1 - \lambda - \rho \geq 0\). Lemma 2 thus implies that the most-cooperative tariffs involve free trade in all states under positive correlation if and only if \((\lambda b + \sigma) \Delta \geq \varepsilon/3\). But calculations reveal that this occurs if and only if \(\lambda \geq \hat{\lambda}(\rho, \varepsilon)\), where \(\hat{\lambda}(\rho, \varepsilon)\) is defined in (3.29). Similarly, free trade occurs in all states under negative correlation if and only if \((\rho r + \beta) \Delta \geq \varepsilon/3\), which is true if and only if \(\rho \leq \tilde{\rho}(\lambda, \varepsilon)\), where \(\tilde{\rho}(\lambda, \varepsilon)\) is defined in (3.30). Finally, as illustrated in Figure 3, under positive correlation, \(\lambda \geq \hat{\lambda}(\rho, \varepsilon)\) implies \(\rho \leq \tilde{\rho}(\lambda, \varepsilon)\) while under negative correlation the reverse implication holds.

Lemma 4: In the dynamic tariff game with international business cycles,
(i) under positive correlation when \(\lambda < \hat{\lambda}(\rho, \varepsilon)\), the most-cooperative tariffs are countercyclical \((\tau_b^c(\varepsilon) \leq \tau_r^c(\varepsilon))\) and nonincreasing in the level of transitory shock, \(\varepsilon\).
(ii) under negative correlation when \(\rho > \tilde{\rho}(\lambda, \varepsilon)\), the most-cooperative tariffs are procyclical \((\tau_b^c(\varepsilon) \geq \tau_r^c(\varepsilon))\) and nonincreasing in the level of transitory shock, \(\varepsilon\).
(iii) under zero correlation when \(\lambda < \hat{\lambda}(\rho, \varepsilon)\), the most-cooperative tariffs are acyclic \((\tau_b^c(\varepsilon) = \tau_r^c(\varepsilon))\) and nonincreasing in the level of transitory shock, \(\varepsilon\).

Proof: We prove here part (i); the other cases are similar. Under Lemma 1, we have that \(\hat{\omega}_b > \hat{\omega}_r\). Furthermore, given that \(\lambda < \hat{\lambda}(\rho, \varepsilon)\), it follows from Lemma 3 that the most-cooperative tariffs are sometimes positive, and so it must be that \(\hat{\omega}_r < \varepsilon/24\). With \(\tau^*\) defined by (6.2) and (weakly) decreasing in \(\hat{\omega}/\varepsilon\), we thus have that \(\tau_b^*(\varepsilon) = \tau^*(\hat{\omega}_b/\varepsilon) \leq \tau^*(\hat{\omega}_r/\varepsilon) = \tau_r^c(\varepsilon)\), with the inequality being strict at \(\varepsilon\). It also follows that higher values for \(\varepsilon\) cannot lower the most-cooperative tariff; in fact, in a recession phase, and if \(\varepsilon\) is near its upper bound, a higher value for \(\varepsilon\) is sure to raise the most-cooperative tariff. Together, Lemmas 1-4 prove Theorem 2.

Solution method for the national business cycle model:

We begin by deriving the incentive constraints for the national business cycle model. In analogy to (3.12)-(3.13), the incentive constraints given in (4.4)-(4.9) may be written as:

\[
\varepsilon\Omega(\tau_{bb}(\varepsilon)) \leq C_{bb}^{bb}E\{\omega(\tau_{bb}(\varepsilon))\varepsilon\} + C_{bb}^{br}E\{\omega(\tau_{br}(\varepsilon))\varepsilon\} + C_{bb}^{rr}E\{\omega(\tau_{rr}(\varepsilon))\varepsilon\}\]

(6.19)
where $C_{ij}^{kl} > 0$ are constants determined as functions of parameters of the model. To solve for the most-cooperative tariffs, we first treat the RHS’s of (6.19)-(6.21) as constants and rewrite the incentive constraints as

\[
\varepsilon \Omega(\tau_{bb}(\varepsilon)) \leq \bar{\omega}_{bb}
\]

(6.22)
\[
\varepsilon \Omega(\tau_{br}(\varepsilon)) \leq \bar{\omega}_{br}
\]

(6.23)
\[
\varepsilon \Omega(\tau_{rr}(\varepsilon)) \leq \bar{\omega}_{rr}.
\]

(6.24)

Solving for the lowest tariffs consistent with (6.22)-(6.24) and using the definitions given in (6.1) and (6.2), we now describe the fixed-point equations as

\[
\bar{\omega}_{bb} = C_{bb}^{bb} E\{\bar{\omega}_{bb}\} + C_{bb}^{br} E\{\bar{\omega}_{br}\} + C_{bb}^{rr} E\{\bar{\omega}_{rr}\}
\]

(6.25)
\[
\bar{\omega}_{br} = C_{br}^{bb} E\{\bar{\omega}_{bb}\} + C_{br}^{br} E\{\bar{\omega}_{br}\} + C_{br}^{rr} E\{\bar{\omega}_{rr}\}
\]

(6.26)
\[
\bar{\omega}_{rr} = C_{rr}^{bb} E\{\bar{\omega}_{bb}\} + C_{rr}^{br} E\{\bar{\omega}_{br}\} + C_{rr}^{rr} E\{\bar{\omega}_{rr}\}
\]

(6.27)

We argue below that the fixed-point equations admit a unique solution, $(\bar{\omega}_{bb}, \bar{\omega}_{br}, \bar{\omega}_{rr})$. Once these values are determined, the most-cooperative tariffs are then defined by

\[
\tau_{bb}^c(\varepsilon) \equiv \tau^* (\bar{\omega}_{bb}/\varepsilon)
\]

(6.28)
\[
\tau_{br}^c(\varepsilon) \equiv \tau^* (\bar{\omega}_{br}/\varepsilon)
\]

(6.29)
\[
\tau_{rr}^c(\varepsilon) \equiv \tau^* (\bar{\omega}_{rr}/\varepsilon).
\]

(6.30)

which completes the description of the solution technique for the national business cycle model.

**Lemma 5:** In the dynamic tariff game with national business cycles, the fixed point values $(\bar{\omega}_{bb}, \bar{\omega}_{br}, \bar{\omega}_{rr})$ satisfying (6.25)-(6.27) exist uniquely and satisfy

(i). $(\bar{\omega}_{bb}, \bar{\omega}_{br}, \bar{\omega}_{rr}) > 0$

(ii). \text{sign}\{\bar{\omega}_{bb} - \bar{\omega}_{br}\} = \text{sign}\{\bar{\omega}_{br} - \bar{\omega}_{rr}\} = \text{sign}\{1 - \lambda - \rho\}.

**Proof:** In examining the fixed-point equations (6.25)-(6.27), we first relate the magnitudes of the associated constants to the sign of correlation. Calculations reveal that

\[
\text{sign}\{C_{bb}^{ij} - C_{br}^{ij}\} = \text{sign}\{C_{br}^{ij} - C_{rr}^{ij}\} = \text{sign}\{1 - \lambda - \rho\}
\]

(6.31)
for any \((i, j) \in \{(b, b), (b, r), (r, r)\}\). Next, we fix \(\bar{\omega}_{br}\) at a constant level \(\bar{\omega}_{br} \geq 0\) and focus on the fixed-point equations for the boom-boom and recession-recession states. Define

\[
f_{kl}(\bar{\omega}) \equiv E(\bar{\omega})[C_{kl}^{bb} + C_{kl}^{rr}] + E(\bar{\omega}_{br})C_{kl}^{br} - \bar{\omega}
\]

(6.32)

for \((k, l) \in \{(b, b), (r, r)\}\). Each function is positive with infinite derivative at \(\bar{\omega} = 0\) and has a unique root. Let \(\bar{\omega}_{kl} = \bar{\omega}_{kl}(\bar{\omega}_{br})\) be the unique root satisfying \(f_{kl}(\bar{\omega}_{kl}) = 0\). We have \(\bar{\omega}_{kl} > 0 > f'_{kl}(\bar{\omega}_{kl})\). Using (6.31), we find that

\[
\text{sign}\{\bar{\omega}_{bb} - \bar{\omega}_{rr}\} = \text{sign}\{1 - \lambda - \rho\},
\]

(6.33)

which is analogous to (6.12). Thus, for any given value of \(\bar{\omega}_{br}\) and under positive correlation, the function \(f_{bb}\) has a larger root than does the function \(f_{rr}\).

Continuing to hold \(\bar{\omega}_{br}\) fixed at \(\bar{\omega}_{br}\), we may proceed as in (6.13) and (6.15) and differentiate the boom-period fixed-point equation (6.25), finding that \(\bar{\omega}_{bb}\) is concave in \(\bar{\omega}_{rr}\) when it is increasing. Similarly, as in (6.14) and (6.16), we may differentiate the recession-period fixed-point equation (6.26), discovering that \(\bar{\omega}_{bb}\) is convex in \(\bar{\omega}_{rr}\). Following (6.17) and (6.18), we may then exploit that \(f'_{kl}(\bar{\omega}_{kl}) < 0\) in order to conclude that the boom-period (recession-period) fixed-point equation crosses the 45 degree line with a nonnegative slope that is less than one (greater than one). Given \(\bar{\omega}_{br} = \bar{\omega}_{br}\) and (6.33), the two fixed-point equations are uniquely satisfied at positive values \(\bar{\omega}_{bb}(\bar{\omega}_{br})\) and \(\bar{\omega}_{rr}(\bar{\omega}_{br})\) where

\[
\text{sign}\{\bar{\omega}_{bb}(\bar{\omega}_{br}) - \bar{\omega}_{rr}(\bar{\omega}_{br})\} = \text{sign}\{1 - \lambda - \rho\}.
\]

(6.34)

Straightforward differentiation reveals that \(\bar{\omega}_{bb}(\bar{\omega}_{br})\) and \(\bar{\omega}_{rr}(\bar{\omega}_{br})\) are nondecreasing functions.

We return now to the boom-recession fixed-point equation (6.26), now written as

\[
\bar{\omega}_{br} = C_{br}^{bb}E\{\bar{\omega}_{bb}(\bar{\omega}_{br})\} + C_{br}^{br}E\{\bar{\omega}_{br}\} + C_{br}^{rr}E\{\bar{\omega}_{rr}(\bar{\omega}_{br})\}.
\]

(6.35)

It is direct to verify that the RHS is nondecreasing, positive at \(\bar{\omega}_{br} = 0\), and constant for sufficiently large \(\bar{\omega}_{br}\). Thus, there exists a positive value \(\bar{\omega}_{br}\) satisfying (6.35), and so the unique fixed-point solutions are given by the positive values \(\bar{\omega}_{bb} = \bar{\omega}_{bb}(\bar{\omega}_{br})\), \(\bar{\omega}_{rr} = \bar{\omega}_{rr}(\bar{\omega}_{br})\) and \(\bar{\omega}_{br}\). We then have that (6.34) yields

\[
\text{sign}\{\bar{\omega}_{bb} - \bar{\omega}_{rr}\} = \text{sign}\{1 - \lambda - \rho\}.
\]

(6.36)

Finally, we may fix \(\bar{\omega}_{rr} = \bar{\omega}_{rr}\) and then \(\bar{\omega}_{bb} = \bar{\omega}_{bb}\) and establish by related arguments that

\[
\text{sign}\{\bar{\omega}_{bb} - \bar{\omega}_{br}\} = \text{sign}\{1 - \lambda - \rho\},
\]

(6.37)
\[
\text{sign}\{\tilde{\omega}_{br} - \tilde{\omega}_{rr}\} = \text{sign}\{1 - \lambda - \rho\}.
\]  
(6.38)

The proof of Theorem 3 now follows directly from (6.2), (6.28)-(6.30), and (6.36)-(6.38).

7. References


Figure 1
Figure 2

$G_t^w \omega(\tau)$

$G_t^w \left( \frac{1}{8} \right)$
Figure 4a

Positive Correlation
Figure 4b

Negative Correlation