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Vagueness, Blurs, and Measurement*

Attempts to resolve the sorites paradox typically require restrictions, revisions, or rejection of either classical logic or common sense. The only exception to this generalization is the epistemological approach developed by James Cargile and Richmond Campbell in [3] and [4]. They claim that vagueness is due to ignorance; although there is a sharp division point between, say, short and nonshort men, we do not know where the division point is. The main objection to this view is that Cargile and Campbell have not given a positive explanation of the source of this ignorance and they have not explained how vague predicates could have the unlimited sensitivity necessary to possess sharp division points. This paper is intended to lend support to the epistemological approach by providing criticisms of rival strategies and by showing how the explanatory gap can be filled by an appeal to "blurry" predicates.

The paradoxes of vagueness can be traced back to Eubulides' paradox of the heap. Eubulides' paradox has the form of a mathematical induction. The base step of the induction is the claim that a collection of sand containing, say, one million grains of sand, is a heap. The induction step is the thesis that any heap remains a heap if only one grain of sand is removed from it. Classical logic allows us to validly infer from these two propositions that a collection of sand containing one grain of sand is a heap.

(I) Basic Approaches

One has resolved the paradox of the heap iff one has shown how Eubulides' argument (and its variations) is unsound. Thus one can classify resolutions of the paradox in accordance with the manner in which they constitute objections to the soundness of Eublides' argument. There are two basic kinds of objections to the soundness of an argument; a challenge to the truth of its premises and a challenge to its validity.

(A) Rejecting the arguments validity

One can reject the validity of an argument in two ways. First, one can claim that the argument is not valid in the same way that, say, affirming the consequent is not valid. Here, one concedes that the concept of validity applies. For example, Stephen Weiss argues in [21] that the sorites should be resolved by adding restrictions to the inference rule of mathematical induction. These restrictions are designed to disqualify sorites arguments but to permit the nonparadoxical mathematical inductions. Of course, restrictions must also be placed on chain arguments because the sorites can also be presented as a long chain of modus ponens arguments. On the other hand, one can deny that the concept of validity applies. For example, in [13] Bertrand Russell suggested classical logic does not apply to the ordinary world, that it only applies to the Platonic heavens. Max Black in [2] suggests that classical logic be viewed in the same way as geometry--only applicable in a tentative, rough and ready way to our terrestial surroundings.

(B) Rejecting a premise

Since Eublides mathematical induction has only two premises, there are only two ways to solve the paradox by rejecting a premise. Either rejection is plagued by difficulties.

(1) Rejecting the Base Step:

The most radical response is to reject the base step of the induction. Unger in [18] and [19], along with Wheeler in [22] and [23] suggest that the argument shows that there really are no heaps. They maintain that 'heap', along with all other vague concepts, is incoherent. Unger and Wheeler correctly observe that most, if not all, observational terms are vague.

They therefore conclude that there are no ordinary things such as chairs, trees, and people. There are two main objections to this position. First, the metaphysical scepticism about ordinary things is wildly counterintuitive. Second, the vagueness of 'vague' makes the argument incoherent. Let us make this second point by first showing that 'vague' is vague.

The vagueness of 'vague' can be shown with the help of a certain sequence of disjunctive predicates. Consider the sequence of numerical predicates '1-small', '2-small', '3-small', etc. The n-th predicate on the list is defined as applying to all and only those integers that are either small or less than n. These predicates can be used to construct a sorites paradox for the predicate 'vague'.

A. '1-small' is vague.

B. If 'n-small' is vague, then 'n+1-small' is vague.

C. 'One billion-small' is vague.

The predicate '1-small' is just as vague as 'small' because both predicates clearly apply to 0 and apply exactly in the same way to all the other integers. The same holds for '2-small' and '3-small', but eventually we reach predicates where the 'less than n' disjunct eliminates some borderline cases. Once we reach a predicate in which all borderline cases are eliminated, we have reached a nonvague predicate. But it is unclear as to where the predicates with borderline cases end and the ones without borderline cases begin. In short, 'vague' is vague.

Now consider the Unger/Wheeler argument:

1. There are ordinary things only if the predicates used to describe them have extensions.

2. These ordinary predicates are vague.

3. All vague predicates lack extensions (for they are incoherent as shown by the sorites).

4. Therefore, there are no ordinary things.

Since 'vague' is vague, Unger and Wheeler must concede that 'vague predicate' lacks an extension by (3), which is inconsistent with (2).

The vagueness of 'vague' also creates difficulties for those who seek to resolve the sorites by claiming that classical logic does not apply to vague predicates. Frege and Russell wished to restrict classical logic to clear propositions. Under this view, the sorites is not valid. However, if 'vague' is vague, then 'nonvague' is vague since the complement of any vague predicate is itself vague. Thus the Frege-Russell counsel to apply classical logic only to nonvague propositions is itself vague. Their arguments concerning the proper analysis of the sorites will then be beyond the scope of classical logic and would thus count as not valid. Their proposal cannot be salvaged by reformulating the restriction to say that logic only applies to clearly nonvague predicates. For in addition to there being predicates which are clear cases of `clearly nonvague predicate', there are borderline cases of `clearly nonvague predicate'. Higher order vagueness ensures that `clearly nonvague' is vague.

(2) Rejecting the Induction Step:

The more popular approach has been to reject the induction step of the argument. The initial appeal of this move is dampened by the recognition that rejecting the induction step is tantamount to asserting its negation. And the negation of the induction step is equivalent to the proposition that there is a precise minimum number of grains of sand necessary for being a heap. In other words, there must be a sharp division point between heaps and nonheaps. Philosophers usually try to avoid this counterintuitive commitment by altering the interpretation of the negation of the induction step through departures from classical logic.

Perhaps the most direct evasion of this consequence is Putnam's in [12]. Putnam points out that the offensive negation does not follow in intuitionist logic. So by abandoning classical logic in favor of intuitionism, we can comfortably reject the induction step. However, in [4] Cargile has pointed out, the paradox can be formulated in another way by appealing to the intuitionist's least number theorem. Further intuitionist formulations of the sorites have been provided by Stephen Read and Crispin Wright in [25].

A more influential response is to attempt to reflect the intuition that vague concepts have degrees of applicability by introducing intermediate truth values (Goguen, Sanford, Machina) or accuracy values (King). It can then be maintained that a one grain difference cannot be the crucial difference between whether a collection of sand is a heap or not. For one can claim that the grain only makes a difference to the degree of truth or accuracy there is in the claim that the body of sand is a heap. One can then explain the plausibility of the induction step in terms of the high degree of truth possessed by each conditional of the form 'If an n grain collection of sand is not a heap, then an n + 1 grain collection of sand is not a heap'. Although each conditional of this form has a high degree of truth, the degree of some them is slightly less than full truth. These small differences can accumulate into a big difference. Thus the induction step of Eubulides' argument should be rejected because it implicitly states that "If it is true to degree x that 'An n grain collection of sand is not heap', then it is also true to degree x that 'An n + 1 grain collection of sand is not a heap'".

One cost of this solution is the revision to logic. Intuitively, 'This collection of sand is either a heap or not a heap' is a tautology and so should have a degree of truth equal to 1. But given that the collection of sand is a borderline heap, 'This is a heap' and 'This is not a heap' will have degrees of truth equal to less than 1. The standard many-valued rule for determining the truth value of a disjunction is to assign the disjunction the higher of the truth values assigned to its disjuncts. So according to this rule, the truth value of 'This is a heap or not a heap' will be less than 1. Thus, one must either follow Sanford in [15] and deny the truth functionality of the logical connectives (to preserve classical theorems) or follow Machina in [10] and [11] and deny the classical theorems (to preserve truth functionality).

Supervaluationists such as Kit Fine in [6], soften this dilemma by appealing to truth value gaps rather than intermediate truth values. According to Fine, simple propositions about borderline cases lack truth values. However, a complex proposition whose component propositions lack truth values may have a truth value. For it may be the case that the complex proposition comes out true regardless of the way we fill in the truth value gaps of its component propositions. Logical truths come out true under every (admissible) valuation of their components. When dealing with vague propositions, one need only consider the admissible precisifications to see that the vagueness of propositions does not prevent us from showing that combinations of those propositions can be logical truths. Thus 'Either this is a heap or not a heap' is a logical truth on the supervaluational approach because any precisification of 'heap' will ensure that one of the disjuncts is true while the other is false. In fact, all of the classical theorems will be preserved by this approach. So Fine claims that the departure from standard logic occurs only in the meta-theory. Since the truth values of complex propositions do not depend on the (actual) truth values of their components, supervaluational logic is not truth functional. However, the important relation between the proposition and its component is preserved in the form of determination of the whole by hypothetical assignments of truth values to its parts.

Critics of the supervaluationists complain that hypothetical precisifications are irrelevant. Sanford points out in [19] that the supervaluational approach distorts the existential quantifier and disjunction. For the supervaluationists are claiming that 'Either this is a heap or this is not a heap' can be true even if 'This is a heap' is not true and 'This is not a heap' is not true. To deny the induction step, they must say that there is a minimum number of grains necessary for a heap. Yet they say that no number is actually the minimum number. In [11] Machina further objects that the supervaluational approach requires us to abandon classical rules of inference even though the classical tautologies which mirror those rules remain. In general, critics of the supervaluationists have maintained that the conservativeness of the theory is illusory.

According to the epistemological approach, the induction step is false and there is no need to alter standard logic. There are precise division points. However, it does not follow that we are in a position to know where those division points are. Thus the existence of sharp division points is reconciled with our inability to specify them. According to Cargile, this ignorance is due to our imperfect understanding of vague words. Campbell describes our ignorance as "semantic uncertainty". Both urge the acceptance of the epistemological thesis on the grounds that it is far more plausible than any departure from classical logic.

Viewed from the epistemological perspective, there is a resemblance between the sorites and the lottery paradox. This resemblance can be highlighted by expressing a sorites as a chain of alternations. We begin with the observation that a 2000 millimeter man is not a short man but a 1000 millimeter man is a short man. We then consider the following alternations:

1. A 2000 millimeter man and a 1999 millimeter man are either both short or both not short.

2. A 1999 millimeter man and a 1998 millimeter man are either both short or both not short.

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1000. A 1001 millimeter man and a 1000 millimeter man are either both short or both not short.

According the epistemic proposal, exactly one of the above propositions is false but we do not know which. Thus each of these propositions has a high epistemic probability; high enough to merit the belief of man who had the acceptance rule: believe a proposition iff its probability exceeds. .99. However, the probability of the conjunction of these propositions is lower than .0l. Therefore, the follower of the acceptance rule would believe each member of the conjunction of (1)-(1000) and yet believe the negation of the conjunction. So he would have inconsistent beliefs. Given a lottery of 1000 tickets with only one prize, the same sort of inconsistent beliefs would be formed concerning propositions of the form 'Ticket number i is a losing ticket'. The lottery situation is paradoxical since it appears to show that one can have rational inconsistent beliefs. The same holds for the above 1000 propositions. This resemblance is useful to the proponent of the epistemic view because he can use it to explain our willingness to assent to each step of the chain argument and our reluctance to accept the argument's conclusion. He can admit that vagueness engenders inconsistent beliefs without concluding that we are irrational or that vague language is incoherent. The "incoherency" is restricted to the doxastic level.

Nevertheless, two strong objections to the epistemic view can be raised. First, we have not been given a positive explanation of why vague predicates should induce such ignorance. The negative explanation is that unless we assume such ignorance, the paradoxes of vagueness will force us into unacceptable conclusions about either classical logic or the ordinary world. But this is not enough. There is an explanatory gap.

The second objection is that epistemological approach makes an unrealistic assumption about the sensitivity of vague concepts. As King points out in [8], proponents of the epistemic approach must say that a millimeter difference can make the difference between a runner starting from New York being far from San Francisco and his not being far from

San Francisco. According to King, there can only be division points if there are determinants. The determinant for 'far' cannot be conventional, for ordinary usage is indecisive. The determinant cannot be natural, because there are no natural boundaries between far and nonfar points from San Francisco. Since the determinant must be either conventional or natural, there is no determinant and thus no sharp division point.

The sensitivity objection has also been used by Unger in [18] against the many valued theorists and the supervaluationists. For the many valued theorists are committed to saying that a one atom difference can affect the degree to which a predicate like 'stone' applies to an object. Yet it seems absurd to suppose that the degree of truth or accuracy of 'This is a stone' can change from say .7399995 to .7399994 by the removal of an atom. Likewise, the supervaluationists are committed to saying that a one atom difference can make the difference between a proposition having a truth value or having no truth value.

The sensitivity objection is significant in judging the relative advantages of the rival solutions to the paradox. For if our predicates have the sort of unlimited sensitivity needed to meet the objection, the many-valued and supervaluational positions are left unmotivated. Both of these positions require us to obtain our rejection of the induction step at the price of revisions to logic. But if vague predicates have unlimited sensitivity, the rejection of the induction step immediately follows; eliminating any motive to change logic.

The thesis of unlimited sensitivity is counterintuitive. Arguing for its acceptance will require an explanation of how the unlimited sensitivity of predicates could be compatible with the limited abilities of language users to discern whether the predicates apply. This argument will fill the explanatory gap left by Cargile and Campbell.

II. Problems with Limited Sensitivity

Part of my strategy for gaining acceptance of the unlimited sensitivity thesis is to emphasize the difficulties of its negation. Let me begin by specifying what sensitivity is. A predicate is <u>sensitive</u> to unit u iff there is a possible positive instance and a possible negative instance which only differ by one u. Thus, 'short man' is sensitive to meters but apparently not millimeters. For a one meter man is short and a two meter man is not short, while two men differing only by one millimeter in height apparently must be either both short or both nonshort. Now consider the predicate 'is sensitive to unit u'. Is this a vague predicate? Here's an argument that should persuade the proponent of limited sensitivity. Consider the following sequence of propositions:

1. A pair of men differing by 1 millimeter can differ in that only one is short.

2. A pair of men differing by 2 millimeters can differ in that only one is short.

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1000. A pair of men differing by 1000 millimeters can differ in that only one is short.

Our proponent of limited sensitivity maintains that 'short man' is not sensitive to millimeters but is sensitive to meters. Thus he denies (1) and affirms (1000). We now ask him which of the members of the sequence is the first false proposition. Perhaps he will be confident that (2) is false and also deny (3), (4), and (5). But eventually his confidence diminishes. He does not know which member is the first false member of the sequence and there appears to be no way he could find out. So he concludes that there is no first false member of the sequence even though he believes that there are some false members. This is hard saying because it violates the least number theorem which asserts that if there is a number that has a property, then there is a least number possessing that property. But the proponent of limited sensitivity cannot very well maintain that there is an unknowable first false proposition since he would then be left without an objection to the view that there is an unknowable first short man in a sequence of 1001 men descending in height from two meters to one meter by one millimeter decrements. The proponent of the limited sensitivity thesis will argue for a restriction, revision, or rejection of the least number theorem. He will maintain that vague predicates show that such modification is necessary. The proponent of the limited sensitivity thesis draws the moral that 'sensitivity' is vague; that 'limited sensitivity' has limited sentivity.

Another way of looking at this issue is from the point of view of someone trying to construct a sorites argument. Given that his predicate is 'short man', he cannot use increments of one meter in his induction step to "prove" that all men are short. Nor
an he use .9999 meter increments. What is the largest increment he can use and still have a true induction step? The question is made more vivid by considering a "meta-sorites".

1. A sorites argument concerning 'short man' has a false induction step if the step's increment equals or exceeds ten thousand millimeters.

2. If a sorites argument concerning 'short man' has a false induction step if the step's increment is n millimeters, it it also has a false induction step if the step's increment

is n -1.

*3. Therefore, all sorites arguments concerning 'short man' having induction steps with increments co*nvertible to millimeters have false induction steps.

I believe this argument is sound. I accept the induction step because the unlimited sensitivity of 'short' guarantees that all sorites concerning 'short' have false induction steps. The proponent of limited sensitivity is forced to agree with me but on different grounds. His commitment to (2) springs from commitment to the vaguely limited sensitivity of 'short'. Since he cannot reject (1), he can only escape the conclusion by denying the validity of the argument. And he must try to escape the conclusion because his talk of limited sensitivity was designed to support the truth of the sorites' induction steps. Thus, these considerations constitute a third objection to Unger's position. For Unger claims that the sorites is valid and his commitment to limited sensitivity saddles him with acceptance of (1) and (2). So Unger's attempt to shore up the truth of the sorites' induction steps through appeal to limited sensitivity ultimately backfires.

III. Unlimited Sensitivity and Blurs

There are a number of strong objections to the thesis that vague predicates have unlimited sensitivity. One wonders how our predicates could distinguish between cases that the language users could not distinguish between. After all, 'red', 'heap', and 'loud' were created by human beings who cannot make fine judgements. These words are taught and used by people in a rough and ready way. Isn't the sensitivity of a predicate limited by the perceptual, pedagogical, and memory limits of the users of the predicate?

Not necessarily. We can envision a language whose discriminatory power far exceeded that of its users. Indeed, this language could be equivalent to English. This possibility
can be illustrated with the help of "blurry" predicates.

Brouwer once noted that although one can know that there are three consecutive 7's in the decimal expansion of pi, one cannot know there are not. For the three might be found further down the expansion. This asymmetric unknowability is due to the fact that pi is an irrational number. My derivation of blurry predicates requires examples of symmetrically unknowable propositions. For example, 'The precise meter length of this page is rational' can neither be known to be true nor known to be false. Unfortunately, the vagueness of `page` and its alternatives makes such propositions unsuitable. For I want the definiens of my blurry predicates to be unquestionably precise. I think the proper sort of symmetrically unknowable propositions can be derived from the purely mathematical sort of asymmetrical propositions Brouwer considered.

Instead of pi, we will make use of an infinite sequence of irrational numbers. The sequence is composed of the square roots of the prime numbers 2, 3, 5, ... Call the first member
of this sequence index-1, the second member index-2, and so on. Since the n-th root of any prime is irrational, it should be noted that in addition to our "horizontal" reference numbers obtained from successive primes along the number line, we can if need be, derive "vertical" reference numbers from the infinitely many roots of each prime. Consider the decimal expansions of our horizontal reference numbers.

1.1 The decimal expansion of index-1 contains 1 consecutive 7.

1.2 The decimal expansion of index-1 contains 2 consecutive 7's.

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1.n The decimal expansion or index-1 contains n consecutive 7's.

At first blush one might feel certain that there is a false proposition in the index-1 sequence. If there is a false proposition, it is impossible to know that the proposition is false. However, the strings of 7's may be finite but of arbitrarily large size making all of the propositions true. For example, ever expanding strings of 7's might be sandwiched between strings of 8's as in 878...8778..87778.... Those impressed by the notion that all possibilities are realized in the infinite might believe that such ever expanding strings of7's surely exist in the decimal expansion of the square root of two, $2^{1/2}$. Proponents of this view should also expect the expanding strings of 7's in the cube root of two, $2^{1/3}$.

However, they cannot reasonably have this expectation for all decimal expansions of number of the form $2^{1/n}$ collectively.

Since there are infinitely many expansions of this form, the belief that all possibilities are realized in the infinite should lead one to the conclusion that some of these "vertical" expansions have a largest string of consecutive 7's. Of course, this argument does not guarantee that there are expansions having maximal strings. The argument is only intended to lower the probability to insignificance.

Nevertheless, to cover the possibility that there are no maximal strings, we shall define "mystery-1" as equal to 1 iff there are no largest strings of 7's in expansions of the form $2^{1/n}$. Otherwise "mystery-1" equals x + 1 where x equals the position of the first false proposition in the index-1 sequence (or the nearest sequence with reference number $2^{1/n}$ that has a false member). In a like manner, we can define "mystery-2" in terms of the index-2 sequence which consists of propositions of the form "The decimal expansion of index-2 contains m consecutive 7's". So if all of the expansions of of $3^{1/n}$ lack maximal strings of 7's, "mystery-2" equals 1. Otherwise "mystery-2" equals x + 1 where x equals the position of the first false proposition in the index-2 sequence (or of the nearest sequence with reference number $3^{1/n}$. An infinite stock of other unknown numbers can be obtained with the index-3 sequence, the index-4 sequence, and so on.

Now consider the set containing the following propositions.

1. Mystery-1 is an even number.

2. Mystery-2 is an even number.

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100. Mystery-100 is an even number.

The epistemic probability of each of the members of the set is .5 since we have no more reason to believe that a particular mystery number is even than we do to believe that it is odd. Further, the probabilities are independent. Call the number of true propositions in this set "murk". Call a number "miny" iff it is less than or equal to murk. 0 is a miny number because the minimum number of true propositions in the set is 0. 101 is not miny because the maximum number of true propositions in the set is 100. Is 1 miny? Well, the probability of 1 not being miny $.5^{100}$ since all of the propositions in the set would have to be false. Thus the probability of 1 being miny is very close to unity. (By selecting a larger set, the probability can be made arbitrarily close to unity.) The probability of 2 being miny is also extremely high. Likewise for 3, 4, and 5. But gradually the probability diminishes. When we reach 100, we are virtually certain that it is not miny.

A predicate F is blurry iff there is a sequence of F's and non-F's such that all of the F's precede all of the non-F's but the position of the last F is unknowable. In other words, blurriness is a matter of not knowing where to draw the line between F's and non-F's. Here "not knowing" must be given the strong reading "not having any way of finding out". Blurriness is compatible with the fact that you may be more ignorant than you need to be. For one can make better or worse probability judgements as to where the dividing line is. However, even the best judgement will leave you in ignorance. Blurriness imposes a graduated limit on knowledge.

Is 'miny' a vague predicate? It has the characteristic binomial probability distribution of many vague predicates. It has an unknowable division point. Should it be introduced into ordinary English, it would behave like a vague predicate. My neighbor permits me to pick miny apples from his tree. If I pick none, I am within the bounds of his permission. Likewise, I can be sure that picking four apples is permitted. Should my neighbor discover that I picked 80 apples we might quarrel over whether I took more than a miny number of apples. Neither of us can be certain, but it is possible to have better grounds.

Controversy can be the result of honest mistakes due to the various difficulties and fallacies associated with calculating probability distributions and varying rules of acceptance. Of course, the debate can be sham where one party abuses the inherent uncertainty of the situation. To prevent abuse, quarrels, and to promote clarity, precising definitions could be introduced to reduce the blurriness. My neighbor could stipulate that 'miny' is to be understood as 20. Perhaps the stipulation would catch on and the original definition abandoned. But it is more likely that 'miny' would survive many ad hoc redefinitions. Notice that such precising definitions would not be purely stipulative. There are acceptable and unacceptable precising definitions. There would be arbitrariness in drawing the line but it would be false to say that anything goes. Despite the disadvantage of blurriness, there are important advantages. First, assertion conditions are easier to satisfy.

Casual observation is usually sufficient. From a glance I know that there are miny ice cubes in my freezer but more than miny flakes of Wheaties in my cabinet. A rudimentary grasp of 'miny' can be quickly acquired by children through a few examples and tips. Trial and error will refine their usage. Even if they never learn the formal definition of 'miny', they can teach it to others. 'Miny' can survive even if I take its definition to the grave. However, as the probability distribution associated with 'miny' is slowly but surely distorted, the authority of my definition is eroded, and the meaning changes. The second advantage is that we can communicate whole probability distributions. Casual observation of a dalmation dog does not enable one to know how many spots the dog has. One could estimate by means of a range specification; you might say that there are between 10 and 100 spots on the dog. However, this description fails to convey characteristics of your probability distribution.

There is a loss of information. If I tell you that I will arrive between 8:20 AM and 11:40 AM, you have no more reason to expect that I will show up within the 9:45 to 10:15 interval than to expect that I will show up within the 9:10 to 9:40 interval. But if I tell you that I will show up within miny minutes of 10 AM, you would consider it more likely that I will arrive within the 9:45 to 10:15 interval than within the 9:10 to 9:40 interval. Blurriness increases predictive power by allowing us to efficiently express and evoke confidence distributions.

Although 'miny' bears an interesting resemblance to vague predicates, one might still have doubts as to whether it is vague. For if 'miny' is vague, it is a counterexample to the compositionality of precision. The defiendum would be vague even though its defienens are precise. And despite the unknowability of the division point, we can nevertheless be sure that it exists. Consequently, the predicate has unlimited sensitivity.

In light of these peculiarities, one might deny that 'miny' is vague. If so, the possibility of eliminating vagueness from ordinary language arises. For example, one might define 'noonish' as 'any time that is within a miny number of minutes from noon'. Here one would be attempting to mirror the probability distribution people have for propositions of the form 'Time x is noonish'. Psychological research might disclose that our probability distribution for these propositions could be better reflected with a predicate other than 'miny'. If so, a superior redefinition could be offered by constructing another predicate from the mystery numbers. No matter what our ordinary probability distribution is, an identical one can be constructed with a predicate based on the mystery numbers.

Most of our ordinary language words could not be defined simply in terms of mystery integers; mystery rationals would have to be employed. For example, 'succotash' is a mixture containing an appropriate ratio of corn kernals and lima beans. If I mix 100 corn kernals with 100 lima beans, I have created succotash. But if I mix 100 corn kernals with 1,000,000,000 lima beans, I do not have succotash. A 1:1 ratio is permitted but a 1:10,000,000 ratio is not. But what is the minimum ratio? We do not know and I think we cannot know. However, this does not prevent the construction of blurry counterparts to succotash. For example, one proposal is that the actual ratio be between (murk +1):(murk +10) and (murk +10):(murk +1). This proposal yields a confidence distribution similar to that given by "'Succotash' is roughly a 1:1 mixture of corn kernals and lima beans". However, to completely eliminate the vagueness one would have to define 'mixture'. In the ordinary sense of the word, mixtures must have their parts close to each other. They need not touch for I can spill my succotash on the floor and still have succotash to clean up. However, there are limits to how far I can scatter my succotash. If I enclose each kernal and bean in separate envelopes, and send the envelopes to friends and admirers all over the world, that is the end of my succotash. Further, the kernals and beans have to be integrated. If I pour half a barrel of lima beans into half a barrel of corn, I don't yet have a barrel of succotash. I still need to mix them up. As I begin to mix them, I wonder whether I have succotash yet. After 10 minutes of mixing, I am certain that I have succotash. But I do not know exactly when I first had succotash. This ignorance is a symptom of the vagueness of the integration requirement. I am not sure what the measure of integration is but am willing to guess for the sake of exposition. The integration of a collection of objects is a function of homogenous distance and heterogenous distance. The homogenous distance between the kernals is the average distance from one kernal to another kernal; likewise the homogenous distance between the beans is the average bean to bean distance. The heterogenous distance between kernals and beans is the average distance from a kernal to a bean. A collection is integrated to the extent that the result of dividing the homogenous distance by the heterogenous distance is small. The blurry version of the integration requirement might read "The degree of integration must be a miny number". The question of how scattered the parts of the succotash might then be answered by determining the average distance between the kernals and beans. It could be proposed that this average distance be miny millimeters.

Regardless of the success of our definitions for 'appropriate ratio' and 'mixture', there still remains the problem of defining the vague words 'corn kernal' and 'lima bean'. These definitions would in turn appeal to parts of the kernals and beans. New definition will be required for these parts and their subparts. Perhaps, eventually, we reach the atomic level. If there is no vagueness at this level, the replacement reduction is complete. But must there be an ultimate nonvague level? Could the vagueness continue infinitely? There is a sense in which an infinity of levels would threaten the completeness of the reduction and a sense in which it would not. The reduction would be incomplete in the sense that there would always be an unreduced element. However, it would be complete in the sense that no particular element is unreducible. I am inclined to think that a reduction that is only incomplete in the first sense is nevertheless satisfactory. In other words, I think a replacement reduction is in trouble if there are particular things it cannot reduce; but it is not in trouble if it is merely the case that some of the things must be left unreduced.

As previously mentioned, there are many different ways to obtain the desired probability distributions. Each is equally successful in explaining our linguistic behavior. However, most will be undetectably inadequate from the point of view of truth. For example, by considering the second hundred mystery numbers, we can come up with a number resembling murk; call it "nurk". Nurk is the number of true propositions in the set of propositions of the form 'Mystery-(100 + n) is an even number' (for n between 1 and 100). Just as we defined 'miny' as less than

or equal to murk, 'niny' can be defined as less than or equal to nurk. The predicates 'miny' and 'niny' give rise to same probability distribution over propositions of the form 'n is miny' and 'n is niny'. However, it is unlikely that the predicates have the same extensions because it is unlikely that murk = nurk. If there really is a number n such that 'noonish' means 'within n minutes of noon' spawning the same probability distribution as murk and nurk, it is likely that the number is neither murk nor nurk. However, the number will be identical to one of the cousins of murk and nurk. The good news is that blurry counterparts to vague expressions can have the same probability distributions and extensions. Indeed, since identical numbers are necessarily identical, the blurry counterparts will have the same intensions as well. The definitions will hold across all possible worlds. The bad news is that we cannot know which of the definitions are correct. We can narrow down the field of candidates. But we cannot determine the winners. This indeterminacy of translation is reminiscent of the indeterminacy associated
with the reduction of numbers to sets. Whether one follows von Neumann in stipulating that 2 = {0} U {{0}} or one follows Zermelo in stipulating that 2 = {{0}} is irrelevant to the reduction of number theory.

As I entered the discussion of blurry predicates, I mentioned two positions that can be taken toward blurry predicates. First, one could hold that blurry predicates are vague predicates. Second, one could maintain that they are not vague and then wonder whether vagueness is eliminable from ordinary language. I favor the first view. My conjecture is that the vagueness of natural languages, such as English, is blurriness. Blurry predicates have unlimited sensitivity. They have metaphysically sharp division points. However, their division points are unknowable. Nevertheless, the location of the division point can be estimated.

Someone who accepts the thesis that the behavior of blurry predicates is indistinguishable from the behavior of vague predicates might wonder 'What grounds do we have for supposing that the predicates in question really are blurry?'. Even if they are indistinguishable, might not the predicates be vague in the sense elaborated by the supervaluationists or many-valued theorists? The answer is that once it is conceded that the behavior of blurry predicates is indistinguishable from that of vague predicates, the full weight of logic favors the identity thesis that vague predicates are blurry predicates. Our logical beliefs are central beliefs. We are only warranted in revising central beliefs in the face of extraordinary difficulties. But no such difficulties remain if vague predicates and blurry predicates are indistinguishable. Only the identity thesis allows us to retain both the bulk of common sense and standard logic. So reflective equilibrium would force the issue. However, this all presupposes that the indistinguishability thesis holds. We must now turn to a serious objection to this thesis based on the differential learnability of the two types of predicates.

IV. The Language Learning Objection

A key objection to the view that vagueness is blurriness (call this "blurism") appeals to language learning. Perhaps some blurry words, such as 'miny', can be introduced into English through stipulative definition. Perhaps vague words can be defined in strictly blurry terms. But how could such blurriness arise naturally? None of the rules of English make direct reference to unknowable numbers like murk and nurk. And none of us are aware of performing probability calculations in order apply a vague predicate. So it does not seem realistic to claim that the vagueness of English is blurriness. And if the vagueness of language is not the same as blurriness, the sorites remains unresolved.

This is a difficult objection to address. Obviously there is no direct reference made to unknowable numbers in normal dictionaries or ordinary language learning situations. It is also plain that ordinary speakers would have a difficult time recognizing the blurist's definitions as definitions of the speaker's ordinary language expressions. And people rarely require calculation to speak. I think this is all true but I am not sure how this constitutes a special problem for blurism. If we picture language as a set of rules for speaking, it is a general puzzle as to why speakers are unable to specify the very rules we are supposing them to follow. Nor do speakers seem to recognize or remember the complicated rules constructed by ingenious linguists and philosophers. Are we to conclude that speakers of English know the rules unconsciously or innately? Is it less obscurantist to appeal to tacit or implicit knowledge? Must all the speakers know all the rules of the language? Need they know the same rules? Or do the ongoing changes in language preclude the existence of a shared corpus of rules? Do natural kind terms show that our psychological grasp of linguistic rules would still fail to determine meaning?

Given that one regards learnability as an adequacy condition for a theory of meaning and one retains the rule-following picture of language, the difficulties associated with that picture will be inherited by one's theory of meaning. Adopting blurism will then increase one's explanatory burden and limit one's hopes for discovery. First, one will have the difficulty of explaining the mechanism by which blurs arise in language learning. Second, one will have to resign oneself to a description of what type of rule we must be using. The actual rule will be unknown since it is underdetermined by linguistic behavior. Thus one's hope of discovering each and every rule of the language will be frustrated. Despite all of its mysteries and conundrums, I find the rule following picture of language attractive and accept learnability as an adequacy condition. So I do think that the language learning objection points to a problem peculiar to my account. The problem common to all theories about the meaning of expressions is 'How does meaning connect with the understanding of ordinary speakers?'. The problem peculiar to my account is that it makes this difficult question more difficult. I think that there are some possible solutions to the problem of how blurry predicates make a natural entrance into language. For example, we could speculate about whether our brains have internal random number generators. Blurry predicates can remain interesting as a mathematical model of vagueness independent of the truth value of these speculations. Nevertheless, blurry predicates would be of greater interest if the conjectures showed some promise of being developed into live historical hypotheses about how vague predicates might actually be blurry predicates. So I will now try to improve the blurist's prospects for claiming historical accuracy by developing one line of speculation in some detail.

V. A Conjecture about Measurement and Blurs

Given high standards of precision, measurement error is inevitable. A reliable measurement of an object must be a limited measurement for our judgements have only limited sensitivity. With a meter rod I may be able to determine that my table is 1.5 meters. With a tape measure, I may be able to determine that the table is 1.503 meters. More sophisticated equipment might extend the last known value to the sixth decimal place. But regardless of the sophistication of my equipment, there will always be a last known value, and thus a first unknown value. The actual length of my table is unknowable (given high standards). However, we can improve our probability judgements as to its actual length without limit. Our knowledge of an object's length in terms of meters is finite but unbounded. Measurement predicates are blurry because there is always a sequence which poses the problem of where to draw the line. For example, 'is less than one meter' can be shown to be blurry by considering a sequence of one trillion rods whose lengths grow in increments from a low of .9 meters to a high of 1.1 meters. Units of measurement are established by fiat. Creators of the original meter rod could know without measurement that the bar was precisely one meter in length because 'meter' refers to the length of the original meter rod. (Or perhaps more precisely, the distance between the two scratches marked on the bar.) However, they only had limited knowledge of the meter rod's length in terms of inches.

Now consider the following situation. Peter pours some water into a can and tells his assistant "A glip of water is the same amount of water as the amount in this can. Now put ten glips of water in my barrel." His assistant obeys by filling a can with the same amount of water as the unit setting can.

Notice that Peter has managed to establish a unit of measurement even though he only has a rough idea of how much water was in the can (in terms of other units such as pints and liters). Also note that the assistant was able to follow orders even though he shares Peter's ignorance of precisely how much water is in the original can. More importantly, Peter's assistant can obey the order even though he does not know precisely how much water is contained in vessels that must match the original. Suppose Peter's assistant had made the following objection. "I cannot possibly succeed. No matter how carefully I try to make a match with the original, it is overwhelmingly probable that the match will not be perfect. There is sure to be a difference in the amounts of water." Is Peter's assistant right? After all, he appears to be making the same point that students of measurement make about inevitable measurement error. Should we conclude that Peter's assistant can only hope to approximately carry out the order?

No. Peter's order can be exactly executed because Peter's unit of measurement is actually a range of quantities. The 'same' in 'same amount of water' is an absolute term like 'flat', 'clean', and 'certain'. In [20] Unger characterizes absolute terms as ones which permit comparisons of the form 'x is more F than y' but allow us to claim 'x is F' only if there is no z such that 'z is more F than x'. Thus if a surface is really flat it must be the case that nothing is flatter than that surface. Absolute terms have hidden quantifiers. A surface is flat iff it has no bumps, curves, or irregularities. It is clean iff it has no dirt. I am certain iff there is no possibility of being mistaken. A sceptic might argue that nothing is flat. For example, the sceptic will argue that your floor is not flat because it has little bumps on it. No matter how much you sand and polish your floor, the sceptic will still deny its flatness on the grounds that (albeit littler) bumps are sure to be found. Likewise the sceptic will complain that your house really isn't clean since there is sure to be some uneliminated dirt. And of course we are familiar with the sceptic's reasons for denying that we can be certain of anything. Now suppose you open your refrigerator door and say 'There is no beer'. Force of habit may prompt the sceptic to object 'False! There is plenty of beer in Milwaukee.' The sceptic's habit is the habit of keeping his quantifiers wide open. When you claimed that there was no beer, your domain of discourse was restricted to the contents of your refrigerator. The sceptic's quantifiers are unrestricted. When you describe your floor as flat, your domain of discourse is restricted to fair sized portions of your floor. None of those portions are bumps.

Two things are the same iff there is no difference between them. The sceptic will conclude that the only thing something can be the same as is itself. Even if you explicitly limit your domain by saying "Tom is the same <u>height</u> as Bill', the sceptic will insist that there is sure to be some difference whether it be an inch or a billionth of an inch. As David Lewis in [9] has noted, the sceptic can often get away with this since it is easy to raise the standards in conversation but it is difficult to lower them.

Nevertheless, it is clear that speakers who attribute equal heights to different people have restricted domains of discourse. They are concerned with <u>relevant</u> differences. Thus they are often correct.

Our common sense use of 'same' has to be defended in the same way as we defend common sense use of other absolute terms. Rather than concede that hardly anything is flat, clean, empty, rigid, octangular, pure, safe, or known, we must make appeal to contextual factors that determine the appropriate domains of discourse for absolute terms. In short, the common sense position on absolute terms must be defended by an appeal to relevance. Of course, the sceptic has a right to an account of relevance. And there have been some contributions to this account, especially by Lewis in [9] and Dretske in [5]. However, I do not have a general account of relevance. Nevertheless, I think I can say that, in measurement contexts, discernibility plays a major role in determining relevant differences between things. Of course, discernibility varies with epistemic resources. What is indiscernible on the basis of casual inspection may be discernible with a microscope. Discernibility may also be a function of the type and purpose of the task at hand, as well as of the importance of the measurement.

When Peter declared that a glip of water is the same amount of water as the water contained in his can, he had a restricted domain of discouse. He meant that a glip of water is not discernibly different from the original. Given their modest resources, a glip of water might vary a couple of ounces from the amount of water in the standard setting can. Exactly how much could it vary? Within the maximum indiscernible difference from the quantity in the original can. How much is the maximum indiscernible difference? This amount is difficult to measure, but psychologists have made progress. With advances in psychology, fairly accurate measurement of maximum discernible differences will be possible. Of course, those of us impressed by inevitable measurement error in physics will also view the psychologists as only narrowing their ignorance. What is the difference between glips and meters? Structurally, there is no difference. But there is an important content difference. The epistemic resources of physicists far surpass that of Peter and his assistant. Thus indiscernibility differs. When officials declared that a meter is the length of the Paris meter rod, anything indiscernibly different in length from the rod did equal a meter. Like a glip, the meter of 1889 was a range of lengths. Enhanced epistemic resources have increased discernibility and thus forced revisions of the original length.

Indeed, the meter was redefined in 1960 by the Eleventh General Conference of Weights and Measures as follows: "The meter is the length equal to 1,650,763.73 wavelengths in vacuum of the radiation corresponding to the transition between the levels 2 p^{10} and 5 d^{5} of the krypton-86 atom." Since the 1889 meter rod is discernibly different from this wavelength, the 1889 meter rod does not equal a 1960 meter. Dissatisfaction with measurement units arises with improvements in discernibility.

According to the above analysis, units of measurement are defined by fiat. These units are intervals whose upper and lower bounds are unknowable due to the twofold uncertainty generated by our ignorance of the precise actual value of the standard setting object and our ignorance of maximum indiscernible differences. This analysis can be extended beyond unit words to many of our ordinary expressions. For example, 'succotash' is learned by fiat. In the past, someone referred to a mixture of corn kernals and lima beans and declared that any mixture which is the same as that mixture is succotash. This standardization view of word origin must be distinguished from resemblance theories. According to Crispin Wright in [24], 'succotash' refers to anything resembling an example of succotash. He correctly points out that a consequence of this view is that a sequence of mixtures beginning with a clear example of succotash and ending with a bowl of pure corn would be a sequence of succotash samples as long as each was indiscernibly different from its predecessor. Under the standardization view, there would be a last bowl of succotash even though the bowl is indiscernibly different from the next bowl. For succotash must be indiscernibly different from the standard setting bowl; not just any example of succotash. Thus indiscernible differences between succotash and nonsuccotash are allowed as long as the succotash is not the standard setting succotash.

VI. Monistic versus Pluralistic Standardization

It might be objected at this point that I have assumed that all standardization is monistic rather than pluralistic. A monistic standardization of a term proceeds by referring to exactly one object as one's standard. For instance, the modern British system of length measurement was initiated by King Edward I in an Act of 1305: "And it is to be remembered that the Iron Ulna [ulna = yard] of our Lord the King contains iii feet and no more, and the foot must contain xii inches measured by the correct measure <u>of this kind of Ulna</u>; that is to say the thirty-sixth part of the said Ulna makes i inch neither more nor less; and five and a half Ulnae make i rod, sixteen feet and a half, by the aforesaid Iron Ulna of our Lord the King." Of course, secondary standards can be constructed. But they do not play a role in defining the unit. Their function is to facilitate the dissemination of the unit and to check the stability of the primary standard. Iron rods do shrink. Parliamentary copies of succeeding primary yard standards have progressely shortened at an average rate of one part in a million in thirty years. Even with the precaution of constructing the 1889 kilogram out of a durable mixture of platinum and iridium and storing it under carefully controlled conditions (including shielding under triple bell jars), scientists are concerned by the microscopic accumulation of film on the standard kilogram. A pluralistic standarization involves references to at least two objects as one's standard. Thus the introducer of 'succotash' may have said 'Any mixture that is the same as these mixtures is succotash'. There appear to be many examples of pluralistic standardization in the history of measurement. For example, 'inch' used to be defined as the breadth of a thumb. Of course, people realized that the breadth of thumbs vary. Likewise, we realize that the informal distance unit 'one hour drive' varies from driver to driver. When this variation is not neglible, we have two options. Either we make the unit an aggregation of the various values or we relativize the units to particular values. For example, an inch might be determined by the average breadth of a thumb. Alternatively, inches would be relativized to particular thumbs or even temporal stages of a thumb. Thus we could speak of a Sorensen inch as opposed to a Reagan inch. Indeed, we do speak of British yards and American yards, troy ounces and avoirdupois ounces, and so on. By following the relativization route, we make pluralistic standardization collapse into monistic standardization. Likewise, when we follow the aggregation route, pluralistic standardization becomes just a complicated form of monistic standardization. Neither relativization nor aggregation is essential to preserve consistency. For the multiple standards may actually have precisely the same length. The pluralistic standardization of the 1960 meter was encouraged by subatomic theory which states that every krypton-86 atom is identical to every other ensuring identical wavelengths. Some quantum physicists hope to discover "chronons"; uniform indivisible timelets which would provide the basis for a natural time unit. So, in general, I think pluralistic standardization does not pose a special problem since it can be ultimately reduced to monistic standardization if there is intolerable variation amongst one's many standards.

According to the standardization theory of language learning, the history and nature of language is best illuminated by the history and nature of measurement. Historians of measurement stress the fact that the meanings of measurement terms vary and are determined by a variety of sometimes conflicting forces. Fiats do not always succeed in determining the meanings of words. Political leaders have long realized the advantages of universal and univocal units of measurement. They have tried to achieve this end through official declarations and sanctions. The major reason for their only modest success is the transition cost. There is the public cost of dissemination and enforcement, the cost of replacing the old measuring devices, and the costs accruing from the new unit's inadequacy for special purposes. Some units are adequate for some measurement tasks but inadequate for others. For example, many terrestial tasks are best accomplished through large temporal units like 'day', 'month', and 'year' rather than through the 'second'. These units all have currency despite the fact that they are not really multiples of one another. If we were to outlaw all temporal units except 'second', the law would be either widely ignored or our measurements would become more cumbersome. Old units can become economically entrenched. The decimalized day, introduced in France in 1793 was immediately unpopular. People did not want to scrap their valuable clocks and portable timepieces.

In addition to failing because of the transition costs, fiats can have rivals. For instance, the Bohr magneton, the natural and smallest possible unit of magnetic moment, is defined in ways that sometimes conflict. The chad, a unit of neutron flow or flux, has two widely differing definitions: (a) a flux of 10,000 neutrons per square meter per second, and (b) 10^{16} neutrons per square meter per second. The latter is 10^{12} times greater than the former. Political scientists must choose from among a large variety of measures of class diversity.

Other measurement schemes suffer from the fact that the variable in question is a complex or combination of diverse concepts. There is no single or internationally accepted measure of hardness despite its great importance for industry and technology. Hardness is a function of the relative ability of different substances to scratch each other, the relative ability of substances to take and keep a cutting edge, and the relative ability of substances to resist abrasion and wear. These are just the principal subvariables. A substance can rate high with respect to one subvariable without rating high on another. The subvariables for intelligence may be even more diverse.

Even once established, a unit can be redefined by succeeding fiats, through the drift of common usage, and by the desire to have our terms correspond to newly discovered natural divisions.

The libra unit of weight illustrates the drift of usage. In the course of its migration from Rome to the New World via Spain, it increased in weight by about 40%. The influence of natural divisions can be observed with 'summer'. The cycle of seasons was discovered long ago. 'Summer' was regarded as the warmest season of the year. Advances in astronomy are responsible for the gradual supplantation of this vague definition by the more precise definition of 'summer' as the period between the vernal equinox and the autumnal equinox. The vague definition of 'summer' lingers on and is responsible for observations like 'This may be the first day of summer but it has been summer in Arizona since March!'.

This desire to reflect divisions in nature is satisfied at the cost of at least temporary increases in ambiguity and an increased need for linguistic division of labor. Knowing the beginning of summer now requires communication links between ordinary people and astronomers. But advances in science and technology have created an increasingly powerful infrastructure which facilitates the supplantation of vague terms by more precise ones.

The difficulty of obtaining universality and univocality for measurement units also arises for our ordinary vocabulary. The meanings of words can differ slightly from speaker to speaker if their mastery of those words are based on different standardizations. My standards for 'blue', 'coat', and 'sunny' differ slightly from the standards of other people. Thus these words have slightly different meanings for me. Such words are analagous to the old anatomical units 'nail', 'inch', 'palm', predating their regimentation. The meaning variance of these old units did not seriously undercut their practical value. Nor did the variance entail inconsistency. For consistency is preserved through aggregation and relativization. Likewise for our contemporary 'blue', 'coat', and 'sunny'. On the other hand, my standard for 'meter', 'summer', and 'gold' is a widely shared standard. The ruling standards here are determined by scientific and governmental organizations. The chain of authority converges upwards through many branches reaching the same standards. The resulting community wide standards ensure exact uniformity of meaning amongst ordinary speakers. Not any chain of authority will do. Linguistic division of labor only establishes uniformity when there is harmony amongst the experts. Consider the disharmony concerning 'adult' and 'insane' amongst sociologists, psychologists, and those in the legal and medical profession. Standardization can account for both variation and uniformity of meaning. In addition to linguistic variation occurring from speaker to speaker, there will be variation over time. These variations may appear to be intolerable to those who hope to define language mastery in terms of knowledge of a specific set of rules. However, I think this vision of the role of rules in language should be set aside. The corpus of rules assigned to each speaker of the language should be allowed to vary within certain limits. These limits on variation are a more accurate guide to the individuation of uniformity of meaning. In addition to linguistic variation occurring from speaker to speaker, there will be variation over time. As linguists like to say, language is a living, growing thing. Its changes are a function of the changes undergone by its speakers. But these speakers need not be changing in the same way.

VII. Summary

The essence of my proposal is that vagueness is blurriness. Blurriness is introduced into language by the fact that most of our words enter language through standardization. Standardization almost always results in a definition involving intervals with unknowable upper and lower limits. There is a fact of the matter as to what these limits are. For the limits are determined by combination of the properties of the standard setting object and psychological properties of the word definer and his audience. Vague predicates have unlimited sensitivity because these limits have actual values regardless of which unit of measurement is used to express those values. Although the limits of the interval are unknown, probabilistic judgements can be made as to whether something lies within the interval. The confidence distributions induced by the uncertainty generated by standardization are responsible for our linguistic behavior concerning vague predicates. Although there is no direct reference to unknowable numbers in our definitions of most vague words, there is indirect reference to such numbers through an appeal to standards. Thus "semantic uncertainty" is reconciled with the absence of explicit random number generators in everyday language.

Our temptation to claim that there is no fact of the matter concerning borderline cases is due to the same forces that make verificationism appealing. People who deny generalizations but are unable to produce counterexamples are suspicious characters. People who affirm generalizations but rule out the possibility of checking the consequences of the generalization are equally suspicious characters. Appeals to ignorance are associated with obscurantism and occultism. Respectable inquiry thus carries the presumption of epistemological optimism. Epistemological optimism is embodied in little mottoes like 'Anything that's true can be known to be true' and 'Anything you can know, I can know' which create a vision of an endless epistemic frontier. However, there are limits to knowledge. Indeed a description of the limits of knowledge is one of the traditional tasks of the epistemologist. Violation of these limits breeds paradox. In the case of vagueness, the paradoxes run deep, so deep that we fail to be persuaded by sheer logic itself. Thorough-going epistemological optimism only leaves us one choice when confronted with unknowable propositions; deny that there is a fact to be known. Reality is thus forced to conform to our capacity to map it. But reality does not bend.

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