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Logically equivalent – but closer to the truth

Some statements vary in how close they are to the truth without varying in what they entail.

1 Equivalent measurements with greater verisimilitude

Consider two cubes that differ slightly in length. The following three statements are increasingly distant from the truth:

A. The two cubes are equal in length.

B. The two cubes are equal in area.

C. The two cubes are equal in volume.

Statement A is closer to the truth than B. After all, area increases with the *square* of the length. Statement B is in turn closer to the truth than C. After all, volume increases with the *cube* of the length. Since the amount of error increases from A to B to C, so does the distance from the truth.

Statements A, B, and C strictly imply each other: any two cubes that are equal in length must also be equal in area and equal in volume. Since these statements are geometrically interderivable, their strict equivalence is *a priori*.

Verisimilitude theorists aim to systematically rank scientific theories by how close they come to the truth. Almost all of these theorists accept the principle that logically equivalent statements are equidistant from the truth.

The logical equivalence principle is a consequence of the criteria they use to measure resemblance to the truth. Karl Popper [1963, p. 397] measured verisimilitude as matter of having true consequences and avoiding false consequences. Logically equivalent statements have exactly the same consequences.

Counting consequences is trickier than Popper thought (Miller [1974], Tichy [1974]). By exercising the ingenuity Nelson Goodman [1954] made famous in his new riddle of induction, one can use disjunctive predicates akin to `grue' and `bleen' to gerrymander calculations of verisimilitude. A theory that is closest to the truth relative to one language can be furthest from the truth relative to another language.

Many confirmation theorists try to solve Goodman's problem by ranking some languages as better than others. Rudolf Carnap [1962] frowned upon predicates that specify positions in space or time. W. V. Quine [1969] sanctions vocabulary to the degree that it mirrors divisions in nature. Nelson Goodman preferred entrenched predicates that have worked well in past inductions. There are echoes of these proposals in the literature on verisimilitude. Some use mereological criteria to prevent double-counting (Britton [2004]). The patient propose that science itself will develop the right language for calculating verisimilitude. Others worry that this strategy is unfairly circular; we cannot let the winner retrospectively legislate the rules by which the winner prevailed. We need to judge verisimilitude *now* to arrive at an authoritative future science. Graham Oddie [1986] calculates verisimilitude on the basis of primitive properties and relations. He puts the specimen in disjunctive normal form and measures the average distance to the truth of its canonical consequences. Although Oddie relies on scientists to discover what the primitive properties and relations are, he does prevent much gerrymandering.

The cube argument survives these expurgations and regimentations because it uses basic geometrical vocabulary. It also survives the shift from computing content to measuring modal similarity:

> A theory is close to the truth to the extent that our world resembles some world where that theory is exactly true. A true theory is closest to the truth, because our world *is* a world where the theory is true. As for false theories, the ones that can come true in ways that involve little dissimilarity to the world as it really is are thereby closer to the truth than those that cannot. (Lewis [1986], p. 24)

Statements A, B, and C are true in the same possible worlds. Therefore, a possible worlds analysis must rank them as equidistant to the truth.

2 Verisimilitude in Mathematics

In his 1644 <u>Cogitata Physica-Mathematica,</u> Marin Mersenne stated that the number $M_n = 2^n - 1$ is prime for n = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, and 257 and composite for all other *n* less than 257. No one knows his reasoning.

> At any rate, he was astonishingly close to the truth. Only in 1947, when desktop calculators became available, was it finally possible to check his claim. Mersenne

had made just five mistakes: M_{67} and M_{257} are not prime, and M_{61}, M_{89}, and M_{107} are prime. (Devlin [1999], p. 13)

The verisimilitude of Mersenne's conjecture is measured by an inventory of 256 statements. The conjecture is closer to the truth because it has a large ratio of true substitutions to false substitutions. It would be uninformative to measure the conjecture's verisimilitude by an inventory of all its logical consequences. Since Mersenne's conjecture is a necessary falsehood, it strictly implies every statement.

In other cases, verisimilitude is measured by syntactic resemblance to a true statement. The statements in the following sequence change by an increment of one.

1. Every positive integer can be expressed as the sum of the squares of one or fewer integers.
2. Every positive integer can be expressed as the sum of the squares of two or fewer integers.
3. Every positive integer can be expressed as the sum of the squares of three or fewer integers.
4. Every positive integer can be expressed as the sum of the squares of four or fewer integers.

Generalization (3) works for the first six substitutions. 7 is a counterexample because $7 = 2^2 + 1^2 + 1^2 + 1^2$. But "believing that [(3)] is close to the truth, we might ask whether each positive integer is the sum of at most four squares." (Gemignani 1968, 25) This modified generalization is true. So instead of calculating the ratio of truths to falsehoods we note the proximity of (3) to (4) in a sequence. (3) is a near-miss. Although there is no difference between what (2) and (3) strictly imply, (3) is closer to the truth.

Biblical inerrantists are frequently challenged to explain why *1 Kings 7:23* and *2 Chronicles 4:2* imply that pi equals 3. The fallibilists draw an invidious comparison with the value given in the Rhind Papyrus: $(16/9)^2 = 3.160493827. . .$. The fallibilists complain that the Bible is further from the truth than a value established a thousand years earlier by the ancient Egyptians.

Regress also mars the career of William Shanks (1812-1882). Over fifteen years, he churned out the first 707 digits of pi. Only 527 of the digits were correct. He revised his calculation three times. New errors crept in. Sadly, Shanks' first calculation turned out to be closest to the truth.

Verisimilitude theorists combat skepticism about the expression `close to the truth' by demonstrating its currency in scientific discourse. They also appeal to intuition. Given that there are five kingdoms of complex organisms, `There are six kingdoms of complex organisms' is closer to the truth than `There are sixty kingdoms of complex organisms'. The same appeal to intuition can be made for mathematics. Given that there are five Platonic solids, `There are six Platonic solids' is closer to the truth than `There are sixty Platonic solids'. We can also use automated word searches to demonstrate that phrases such as `close to the truth' have a frequency in mathematics that is comparable to their frequency in science. In short, the same type of data that legitimates the concept of verisimilitude for science exists for mathematics.

Indeed, the data overlaps because much mathematics is integrated into the empirical sciences.

> The numbers we get from this random sample are most likely wrong, but we can use the theorems of mathematical statistics to determine how to sample and measure in an optimum way, making sure that, in the long run, our numbers will be closer to the truth than any others. (Salsburg [2001], p. 172)

Scientists often get closer to the empirical truth by getting closer to the mathematical truth.

Verisimilitude theorists may neglect mathematical verisimilitude because they do not feel there is a problem about explaining mathematical progress. Pessimistic induction from the falsehood of past empirical theories suggests that our current scientific theories are false and our future ones will be false as well. Scientific progress must therefore be a matter of getting closer to the truth. Pessimistic induction for mathematics is implausible. Mathematicians make mistakes but still manage to establish stable results. Therefore, we are free to stick with the assumption that mathematical progress is to be measured in terms of the full truth.

However, the full truth does not exhaust all mathematical progress. Verisimilitude still counts. Mathematicians do reason statistically and analogically about their subject matter even though they give privileged status to deductive proof.

Even if the concept of verisimilitude were not needed to explain mathematical progress, the mathematicians' frequent judgments that a statement is close to the truth would require explanation by a general theory of verisimilitude. African elephants are useless because they will not follow human directions. But the zoologist ought not to restrict his study to Indian elephants. `Elephant' is a general term, so an understanding of elephants requires understanding of both the useful and useless species. The same goes for `verisimilitude'.

The issue of scientific progress dominates commentary on verisimilitude. But the concept has wider explanatory potential.

Consider ethics. We have less disapproval of lies that are close to the truth. Were it not for the Biblical mistake about pi, I might back this assertion by citing the authority of *Exodus 23:7*, "Keep thee far from a false matter".

Consider the psychology of regret. We aim for the truth. Even the pessimists who think that they will never reach the truth try to get close to the truth. News that your theory is false is disappointing. News that your theory is far from the truth is more disappointing.

Albert Einstein regretted adding the cosmological constant to his equation for the expansion of the universe. In a conversation with George Gamow, he characterized it as "the greatest blunder of my life" (Gamow 1970, p. 44). Gamow, not to be outdone, published a paper that contained an equation that was off by a factor of 10^{24}. He prepared an erratum in advance that assured the reader the error "does not affect the result." (Gell-Mann 1985) The concept of verisimilitude is needed to make sense of Gamow's ambition to make the biggest mistake.

3 Inconsistent Scientific Theories

Physics students still begin their study of the atom with Neils Bohr's solar system model. A positively charged nucleus corresponds to the sun and negatively charged electrons correspond to the planets. Since the electrons are accelerating (due to their circular orbits), they should lose energy and spiral into nucleus after a few orbits. To stop this, Bohr imposed a quantization condition; instead of losing energy continuously, the electron changes energy in discrete jumps (quanta). However, an accelerating charged particle must lose angular momentum continuously. Despite this inconsistency, Bohr's model delivered predictions that were uniquely accurate:

> Bohr's theory of the Balmer series is based upon several novel hypotheses in greater or less contradiction with ordinary mechanics and electrodynamics, . . . yet the representation afforded by it of the line spectrum is so extraordinarily exact that a considerable substratum of truth can hardly be denied to it. Therefore, it is matter of great theoretical importance to examine how far really it is inconsistent with ordinary electrodynamics, and in what way it can be modified so as to remove the contradictions. (Schott [1918], p. 243)

If logically equivalent statements are equidistant from the truth, then inconsistent theories should not vary in how close they are to the truth. But physicists got closer to the truth by improving on Bohr's model even when those improvements fell short of making the theory consistent.

Newtonian cosmology assigns inconsistent forces to a test mass (Norton [2002], p. 186). But it comes closer to the truth than Ptolemy's cosmology – even if Ptolemy's theory is consistent.

One explanation of this verisimilitude ranking is that Newtonian cosmology has a consistent subtheory that strongly outranks Ptolemy's. For instance, David Malament [1995] plausibly argues that Newtonian cosmology can be rendered consistent by retracting the assumption that there are preferred inertial states of motion in space.

The old quantum theory of black body radiation is inconsistent about whether energy levels can vary continuously (Norton [1987]). The search for a consistent variation is more complicated. But John Norton is sufficiently satisfied to offer the general proposal that scientists react to inconsistency with meta-reasoning:

> If we have an empirically successful theory that turns out to be logically inconsistent, then it is not an unreasonable assumption that the theory is a close approximation of a logically consistent theory that would enjoy similar empirical success. The best way to deal with the inconsistency would be to recover this corrected, consistent theory and dispense with the inconsistent theory. However, in cases in which the corrected theory cannot be identified, there is another option. If we cannot recover the entire corrected theory, then we can at least recover some of its conclusions or good approximations to them, by means of meta-level arguments applied to the inconsistent theory. (Norton [2002], p. 193)

Norton's characterization of an inconsistent theory as being a "close approximation" of a logically consistent theory sets the stage for a verisimilitude ranking. For if the consistent theory is true, the inconsistent theory will be close to the truth by virtue of being a close approximation of that truth. Joel Smith, Philip Kitcher, and other commentators agree that there will always be a similar, consistent theory that enjoys all the successes of the inconsistent theory – even if that consistent theory is hard to find. The truth is out there.

A few theorists deny that this consistent variation must exist. Dudley Shapere cautions, "there can be no guarantee that we must always find a consistent reinterpretation of our inconsistent but workable techniques and ideas" (1984a, p. 235). Shapere has in mind classical (relativistic) electrodynamics. Lorentz's classical theory of the electron precludes the existence of point charges while the theory of relativity requires that charged particles be pointlike (Shapere 1984b, p. 360).

Mathias Frisch [2005] devotes half a book to vindicating Dudley Shapere's warning. Frisch denies that anyone could tidy up classical electrodynamics in the way David Malament tidied up Newton's cosmology. The inconsistency is here to stay. According to Frisch, scientists only demand that theories generate reliable models:

> If acceptance involves only a commitment to the reliability of a theory, then accepting an inconsistent theory can be compatible with our standards of rationality, as long as inconsistent consequences of the theory agree approximately and to the appropriate degree of accuracy. Thus, instead of Norton's and Smith's condition that an inconsistent theory must have consistent subsets which capture all the theory's acceptable consequences, I want to propose

> that our commitment can extend to mutually inconsistent subsets of a theory as long as predictions based on mutually inconsistent subsets agree approximately. (Frisch [2005], p. 42)

Frisch is requiring accuracy of the predictions (not mere agreement). The predictions should be close to the truth even if one of the predictions is closer than the other.

Although one cannot wittingly believe two mutually inconsistent statements, one can believe that each is close to the truth. For instance, if you think the time is about noon, you can believe `The time is 11:58 AM' is close to the truth and `The time is 12:02 PM' is close to the truth.

The goal of minimizing one's average distance from the truth is sometimes best served by methods that ensure some departure from the truth. I may know that the answer to a question is an irrational number but use an heuristic that can only yield a rational number. I sacrifice the remote possibility of getting a true answer for sake of securing a false answer that is close to the truth.

A fallibilist who believes that some of his beliefs are false makes it impossible for all of his beliefs to be true. If all of his scientific beliefs are true, then his meta-belief about the existence of some false beliefs is mistaken. If his meta-belief is true, then some of his remaining beliefs must be false. Either way, he has some false beliefs. The self-reflective fallibilist is resigned to the inconsistency of his overall belief system. But he is still free to hope that his belief system is close to the truth. He can still hope that his belief system will come closer to the truth in long run – despite never becoming consistent. The inconsistency of his belief system does not foreclose the applicability of the concept of verisimilitude.

4 Meaningless statements

Does two liters plus three meters equal two meters plus three liters? If 2/2 is less than 2/1, is 2/1 less than 2/0? What time is it at the North Pole?

Meaningless statements are logically equivalent in the degenerate sense that there is no difference in what they entail. Since they lack a truth-value, meaningless statements do not have any consequences. Yet some meaningless statements resemble meaningful statements. My digital clock reports the time is noon by displaying *12:00 PM.* (After 11:59 AM, the illuminated AM goes off and the PM display lights up.) Strictly speaking, the sentence `The time is 12:00 PM' is ill-formed. `PM' abbreviates the Latin phrase *post meridiem*, which means `after noon'. `AM ' abbreviates *ante meridiem* which means `before noon'. So there is a representational limit in the AM-PM system. Noon and midnight are singularities. Nevertheless, `The time is 12:00 PM' is closer to the truth than `The time is 12:43 PM'.

Indeed, the 12:00 PM report is closer to the truth than reporting the time as 12:03 PM or 12:02 PM or 12:01 PM. Given that the time is precisely noon, reporting the time as 12:00 PM is also closer to the truth than reporting it as 12:00:10 PM, or 12:00:01 PM, or 12:00:00:10 PM. And so on. Therefore, a meaningless statement can be arbitrarily close to the truth.

Two referees for this article suggested that `12:00 PM' might now be well formed. We need not be slaves to the Latin *post meridiem*. The fact that `myriad' originally meant 10,000 is compatible with `myriad' currently meaning `a large number'.

Logical positivists and ordinary language philosophers made far too many attributions of meaninglessness. In a reaction to this excess, philosophers now set a high standard for these attributions. Although I have trouble meeting this standard for any particular case, I hope to persuade my colleagues that there are some genuine cases by reminding them of the wide array of candidates.

Ludwig Wittgenstein regarded `The standard meter rod is 1 meter long' as meaningless. But even if Wittgenstein is right, this meaningless statement is closer to the truth than the meaningful statement `The meter rod is 1.1 meters long'.

Consider incomplete thoughts. Scientists ask an Eskimo to go out and measure the temperature. The Eskimo reports `The temperature is –40 degrees'. The English scientist asks `Is the scale Fahrenheit or Centigrade?'. The Eskimo confesses that he did not realize that there was a difference and so did not pay attention to which type of thermometer he was reading. The French scientist points out that - 40 degrees Fahrenheit equals –40 degrees Centigrade so there is no need for the Eskimo to make a second reading. Although the Eskimo's statement is meaningless (rather than ambiguous), it comes out true under all ways of supplying the missing relatum. It was nearly true.

Let us pass from absent units to defective units. Kenneth Arrow [1951] described cardinal utility under certainty as "a meaningless concept" but believed that there were meaningful substitutes. For instance, some principles that were rendered meaningless by their use of utiles had analogues that only used ordinal preference rankings. The meaningless statements were close enough to the truth to be worth paraphrasing.

The pioneers of set theory assumed that any description can be used to specify a set. After the discovery of Russell's paradox, theorists imposed grammatical restrictions that characterize many formulas of naïve set theory as ill-formed Nevertheless many of these meaningless elements of naïve set theory were regarded as close to the truth. The reformers were tinkering with what to count as ungrammatical to avoid paradoxes.

Theories about the limits of what can be meaningful tend to imply their own meaninglessness. The penultimate sentence of Ludwig Wittgenstein's Tractatus boldly declares that his book is senseless:

> 6.54 My sentences are illuminating in the following way: to understand me you must recognize my sentences—once you have climbed out through them, on them, over them – as senseless. (You must, so to speak, throw away the ladder after you have climbed up on it.)
>
> You must climb out through my sentences; then you will see the world correctly.

One way to reconcile informativeness of the Tractatus with its meaninglessness is to say that it is close to the truth. This also brings us back into accord with the principle of charity. The reader can comply with the author and interpret the Tractatus as meaningless and still satisfy Donald Davidson's [1984], p. 169) constraint that the reader maximize his agreement with the author.

On analogy with Mathias Frisch's thesis that some inconsistency is ineliminable, one might emulate Wittgenstein's boldness and say that there is no meaningful variant of the Tractatus that is just as successful. Meaninglessness does play an essential role in Natalie Dorsch's poem, "Just Because,"

I walked up the door,
shut the stairs,
said my shoes,
took off my prayers,
turned off my bed,
got into the light,
all because
you kissed me goodnight.

Disentangling this poem into intelligible sentences would be a step backwards. One would only be untying the knots Dorsch needs to capture the delightful confusion of the romantic interlude. Likewise, the attempt to make the Tractatus well-formed would fail to convey the human drive to transcend the limits of language.

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