

BLANKS:

SIGNS OF OMISSION

The notes I handle no better than many pianists. But the pauses between the notes -- ah, that is where the art resides." -- Artur Schnabel

'Antidisestablishmentarianism' is the longest word. But what is longest possible word? And what is the shortest possible word?

Reflection on these questions have prompted me write the longest essay that has ever been written. This is it. So, sit back. Nothing will ever be longer because this one contains infinitely many sentences.

Word length = ? This may seem impossible. Only finitely many symbols can be inscribed on a page. Even if I wrote smaller and smaller, I would eventually run out of inscribable surfaces. I cannot autograph an atom. Even if I had unlimited time, I would run out of space. I appear condemned to produce only finitely many sentences.

1. A line containing infinitely many sentence tokens

However, not all symbols need to be written. Under common circumstances, blanks are symbols. A blank is created when the symbolizer omits an inscription or sound. The omission must take place against a backdrop of potential action. So although the margins of this page are blank, the margins are

not blanks. Margins frame a text but are not themselves text or even the omission of text. Blanks require a foundation of character positions.

It is too much to require that each of these positions be inscribable. Consider a page marred by holes. I can use the holes as blanks: HI \circ THERE! Indeed, pairs of holes $\circ\circ$ are sometimes used as letters: L $\circ\circ$ K! Resemblance to letters or inscribable surfaces is enough to trigger recognition of the relevant communicative intention.

The size and location of a blank is just the size and position of the character position. Normally, the features of a blank are inferred from the surrounding inscriptions. However, I need to write a long string of blanks and so must use other means of specifying where each character position lies.

Features of my new accelerated font will be partly based on the font used in writing sentences in this paragraph. The height of each character is the same as these characters. The width of each character shall be specified in terms of the length of a full line. Let that length be L millimeters. The first character position occupies region $L/2$, the second occupies region $L/4$, the third occupies region $L/8$, and so on:

In general, character position n has length $L/2^n$. I can inscribe letters in many of these early positions but the inscriptions become more difficult as the width of each character position

narrows. According to The Guinness Book of Records, Horace Dall

. . . built a pantograph -- which reduces movement -- fitted with a diamond stylus, with which he engraved writing small enough to fit 140 Bibles to one square inch. In the 1990s, the movement of small atoms under an electron microscope now allows engraving of letters five atoms tall. At this size, several Bibles could printed on a single bacterium. (1997: 406)

There are physical limits to the project of miniaturizing inscriptions. The inscriber can only make finitely many movements and there are only finitely many particles that can be inscribed. I cannot fill the accelerated line by writing a line on the paper. The ink will fill in the early character positions but virtually none of the latter positions. Infinitely many of the latter positions are smaller than electrons. However, I can still use those uninscribable positions in space as blanks. And indeed, I stipulate that each of the accelerated positions is a blank.

Formally, the accelerated sequence of blanks is a set of ordered pairs, coupling each positive integer with a blank: $\{ \langle 1, \ \rangle, \langle 2, \ \rangle, \dots \}$. The accelerated sequence of blanks should be distinguished from the empty string which contains no symbols (Partee et. al. 1990, 434). Like zero, the empty string is unintuitive but theoretically central. The empty

string has length zero, is a substring of every string, and is an identity element with respect to concatenation: juxtaposing any string with the empty string yields the string itself. Strings of blanks have none of these properties.

Formal linguists emphasize the difference between a single letter and a string containing just that letter; the letter *g* differs from the string *g* (which is really $\{ \langle 1, g \rangle \}$). Similarly, the single blank should be distinguish from the string containing a single blank. Finally, strings must have finite lengths. Therefore, the accelerated sequence of blanks is a sequence of symbols that contains infinitely many strings as subsequences but is not itself a string. Given that all sentences are strings, it follows that the accelerated sequence is not itself a sentence. All sentences are finitely long (although there are non-standard systems that permit infinitely long sentences).

The accelerated blanks can be fashioned into an infinite sequence of sentence tokens. I stipulate that each blank in the accelerated sequence means BUFFALO. Instead of marking the end of sentences with periods. I use the following rule: End the first sentence after one word, the second sentence after the next two words, the third sentence after the next three words, etc. The sequence of sentences is:

1. BUFFALO
 2. BUFFALO BUFFALO
 3. BUFFALO BUFFALO BUFFALO
 4. BUFFALO BUFFALO BUFFALO BUFFALO
 5. BUFFALO BUFFALO BUFFALO BUFFALO BUFFALO
- etc.

Tokens of 'buffalo' can be singular nouns, plural transitive verbs, and plural noun (as can 'bear', 'perch', and 'steer'). This ensures that iterations of these words always yield grammatical sentences even though they are unacceptably difficult to parse in daily discourse (Valian 1990: 121). The buffalo sentences have various readings. A finite labeled digraph can represent the finite state automaton that will generate the intended readings (Tymoczko and Henle, 1995: 105). They can also be conveyed by specifying their logical translations. Read 'Bx' as 'x is a bison' and 'Ixy' as 'x intimidates y':

- 1'. $(x)Bx$
 - 2'. $(x)(Bx \ \& \ (y)Ixy)$
 - 3'. $(x)(Bx \ \& \ (y)(Ixy \ \& \ By))$
 - 4'. $(x)(Bx \ \& \ (y)(Ixy \ \& \ By \ \& \ (z)Iyz))$
 - 5'. $(x)(Bx \ \& \ (y)(Ixy \ \& \ By \ \& \ (z)Iyz \ \& \ Bz))$
- etc.

Each sentence in the sequence is contingent and has a distinct meaning from the other sentences. For instance, 4' means

`Bison [that] bison intimate [also] intimidate [something]' while sentence 5' means `Bison [that] bison intimidate [also] intimidate bison'.

In the terminology of formal language theorists, `buffalo' is a star (*) word. S^* denotes the set of sentences of the form $SSS \dots S$. A word S is a star (*) word if any sequence $SSS \dots S$ is a grammatical sentence. The two rules for a regular sublanguage that generates these sentences: $S \rightarrow SS$ and $S \rightarrow \text{buffalo}$. Since these two rules can be implemented by a finite state automaton, a computer can be programmed to recognize all the grammatical sentences of the buffalo sublanguage. This is a useful property to those who wish to build a computer that communicates -- a property not shared by some languages possessing re-write rules.

The feat of producing infinitely many sentence tokens must be distinguished from the ability of any speaker to produce any sentence token from an infinite range of distinct sentences. Any natural language has infinitely many sentence types because every natural language has predicates that express recursive functions. In English, `father' recurses in the infinite sequence `The father of Darwin was human', `The father of the father of Darwin was human', `The father of the father of the father of Darwin was human', etc. This is an infinite sequence of sentence types rather than sentence tokens. Linguists have stressed that the finitude of each speaker ensures that he can only utter a finite subset of sentences. Since there are only finitely many speakers and the

union of finitely many finite sets is a finite set, it follows that there are only finitely many sentence tokens. Since each sentence token is composed of finitely many symbols, it follows that only finitely many symbol tokens have been produced.

Until just recently. The accelerated sequence of blanks breaks the finitude barrier. The line has infinitely many sentence tokens and therefore contains more (and longer) sentences than hitherto produced by mankind. A learnable language can only have aleph-0 sentences. Consequently, the number of sentences of a natural language cannot be increased beyond the aleph-0 buffalo sentences just produced. The buffalo sentences could become a proper subset of another infinite set of sentence tokens of size aleph-0. But the two sets have the same size because the set of buffalo sentences can be put into a one to one correspondence with all the sentence tokens that will ever be uttered -- even by future exploiters of blanks. The record can be tied, but will never be broken. The only sentence tokens that will ever be actually produced are those from learnable languages. Since each learnable language has only countably many sentence types, all actual speakers will produce only countably many sentence tokens.

Some formal language theorists recognize the theoretical possibility of infinite languages. These feature an infinite stock of primitive terms or infinitely many rules. They can be learned by hypothetical beings that can perform super-tasks.

A super-task is an action composed of infinitely many sub-actions. Some commentators claim that super-tasks are logically impossible (Chihara 1965) and many more say they are physically impossible (Pitowsky 1990). But if supertasks are possible, we can go on to speculate about accelerated Turing machines that can inscribe all the buffalo sentences by writing ever faster. These non-standard machines would not need to symbolize by omission.

Albert Einstein's twin paradox suggests a way of mimicking the performance of a super-task by division of labor. The first agent, a Turing machine, has the chore of checking a generalization that has an infinite domain of discourse, say, Goldbach's conjecture that every even number greater than 2 is the sum of two primes. The second agent, corresponding to the twin who travels near the speed of light, is an observer who has access to the computer's history but experiences only a finite lapse of time. The computer will signal if it finds a counterexample to Goldbach's conjecture. If the computer sends no signal, the observer will know after finite time that the Turing machine never halted, so Goldbach's conjecture is true. John Earman and John Norton (1993, 1996) argue that this scheme is logically possible and might even be physically possible.

The accelerated font permits a single finite being to mimic the performance of a limited kind of super-task. It lacks the versatility of an accelerated Turing machine or the observer-machine duo. The accelerated font cannot yield an

answer to Goldbach's conjecture. But the accelerated font has the advantage of being implementable by ordinary people.

There was never an opportunity to create the longest possible sentence. Although sentences must be finitely long (because they must be understandable by finite speakers), there is no maximum length of a sentence. The reason is that for any sentence, there is a grammatical sentence that is one word longer. The same connection between understanding and finitude ensure that there can be no longest possible word.

There can still be a longest actual word. Prior to the production of the accelerated line, there was also room for the longest actual sentence. But now there can no longer be a longest actual sentence -- or even a sentence that has a novel length. To see that sentences of novel lengths are no longer possible, note that the accelerated line provides a specimen sentence token of each possible word length. This further ensures that there will always be a buffalo sentence longer than any future sentence token.

The accelerated line contains sentences longer than any sentence that has been previously produced. Indeed, it contains infinitely many. To see this, observe that there is a buffalo sentence whose length corresponds to each positive integer. There are infinitely many finite positive integers but only finitely many sentences were uttered prior to the production of the accelerated line. Hence, there was a longest sentence whose length equals some positive integer n . Since

there are infinitely many positive integers greater than n , the accelerated line contains infinitely many sentence tokens longer than any previously uttered sentence.

The permanence of the record is underscored by the indestructibility of the sentences in the accelerated line. Members of the accelerated line cannot be erased. They can be over-written by someone attempting to deface my monument to loquaciousness. This micro-graffiti artist would obscure my buffalo sentences. But he would not have obliterated them. Blanks can have irrelevant inscriptions running through them. For instance, if you write OUT TO LUNCH in bold red across this page and post it in on your door, readers of the sign will ignore the original writing that survive between the spaces of your words.

If I defined my character positions as positions on a page, then burning the page containing the accelerated line would destroy its blanks. But I defined the positions as those in the space occupied by the page. After all, in 1911 Ernest Rutherford's experimentally demonstrated that the page is almost entirely empty space anyway. Given the indestructibility of space, the Buffalo sentence tokens will last forever. Egyptian hieroglyphs were also intended to last forever. But the Egyptians erred in choosing granite as their medium.

2. Oulipo

A crossing guard who repeatedly uses her STOP paddle issues distinct orders with the same sentence token. If the paddle were immortal, then it could be re-cycled without limit.

Parts of sentence tokens can also be combined to make utterances of distinct sentence types. This possibility was exploited by the French literary group Oulipo. The name comes from *Ouvroir de Litterature Potentielle*, which translates as 'Workshop of Potential Literature'. Members of Oulipo explore mathematical features of literature. Oulipo's first manifesto was "A Hundred Thousand Milliard Poems". This was a book composed of 10 basic sonnets written on sliced pages. By flipping the sliced pages, one can mechanically obtain many distinct, well formed sonnets. Combinatorial analysis verifies the manifesto's title: there are indeed 100,000 billion sonnets. John Allen Paulos worries that it is impossible to verify that all the sonnets make sense because "there are vastly more texts in the 10^{14} different sonnets than in all the rest of the world's literature" (1991, 166). However, the recursive nature of transformational grammar obviates the need to check by actually reading all the sentences. Thus The Guinness Book of Records (1997, 254) errs in listing the ancient Chinese encyclopedia The Yongle Dadian as the largest publication.

One of best known members of Oulipo is Georges Perec. He constructed a palindrome (about palindromes!) containing more than 5,000 letters. (A palindrome is an expression that

reads the same backwards as forwards, like 'radar' and 'Step on no pets'.) Perec is better known for his novel La Disparition. It does not contain a single e. This makes La Disparition the longest lipogram (a string of sentences that omits one or more letters of the alphabet).

Or was. Each sentence in the accelerated sequence omits all the standard letters of the alphabet. Hence, it contains infinitely many lipograms that are longer and more parsimonious than any lipogram that will ever be written. No lipogram can be longer or restrict itself to fewer symbol types.

The accelerated sequence also contains the longest palindromes. The sequence is complete in that it contains palindromic sentences of every finite length. Offhand, it may seem impossible to reverse infinitely many sentences. However, the buffalo sentences in the accelerated sequence have a further characteristic that helps us finesse this problem: each is an ambigram. An ambigram is an expression that looks the same in a mirror vertically (OTTO) or horizontally (CHOICE) or when rotated (NOON). The sentences in the accelerated sequence are ambigrams with respect to all of these operations. Consequently, the mirror reflection of the accelerated sequence verifies that each of its sentences is a palindrome.

3. Inscribed "blanks" and Alan Turing

The idea of redefining the blank to mean buffalo is inspired by Benoit Mandelbrot's (1954) interchange of the roles of 'e'

and blank. His symbol swap arose in the course of his discussion of George Zipf's (1935) law that the frequency of a word varies inversely with its length. Mandelbrot conceded that the law holds but maintained that it is misleading to describe Zipf's law as a principle about human language. He showed that the law held equally well for "texts" produced through a wide variety of arbitrary processes. Take a newspaper and interchange the role of blank and `e'. The resulting `words' obey Zipf's law just as well as the real words in the newspaper.

Tests of Zipf's law are natural to implement by computer. The ASCII code (American Standard Code for Information Interchange) for blank (or "space") is the first item on the character list: 0100000. The ASCII code for `e' is 1100101. Thus an algorithm for calculating the dispersion of 010000s in ASCII can be re-keyed to measure the dispersion of 01100101s.

Word processing programs produce sentences with functions that take only inscriptions as input values. Hence, programmers are forced to use inscriptions to play the role of blanks. Hence, their "inscribed blanks" look like # or _____. If `blank' were merely a functional term like `word separator', then whatever performed the role of a blank would be a blank -- just as whatever plays the role of a paperweight is a paperweight. However, blanks are metaphysically different from inscriptions. Indeed, they are absences of inscription -- which makes `inscribed blank' an oxymoron. False teeth

perform the role of teeth, can be called "teeth" for short, and are sometimes superior to organic teeth. But they are not teeth. Parallel points hold for inscribed blanks.

Inscribed blanks suffer the same constraints as any other inscription. Inscribed blanks take energy to write, erase, and transmit. Moreover, blanks can be mis-inscribed and garbled. Since there can only be finitely many inscriptions, a word processing program cannot match the feat of producing infinitely many sentence tokens.

Computers cope with omissions by treating them as commissions. When I tap the space bar, the computer records an inscription in the same category as any other key. When I omit to assign a value to a variable in BASIC, the computer assigns the default value of 0. The rationale for treating omissions as commissions is that computers are symbol manipulating machines. Each symbol token causes a substantive effect in the machine. That is, the machine acquires a new power to change other things or to be changed by other things. Blanks do not have intrinsic properties. They only exist in the way that a dieter's abstentions exist. A blank owes its width and height to other things such as inscriptions, sounds, fiats, and conventions. Like a hole, a blank only has extrinsic properties. It is a parasitic entity.

So here is a qualitative difference between people and mechanical symbol inscribers: only people can produce an actual linguistic infinite. If people were limited to (occurently) believing sentence tokens (perhaps as utterances in their

"language of thought"), then the number of their beliefs is limited by the number of sentence tokens. But now we see that a believer with a finite brain could create an actual infinite number of objects of belief.

According to the Turing test, a computer thinks if its sentences are indistinguishable from the sentences produced by a human being. Alan Turing (1950) envisages an extended conversation featuring an interrogator who attempts to detect whether his interlocutor is a person rather than a machine by examining the content of the sentences appearing on a teletype printer. However, our meditations on blanks suggests that the interrogator could instead concentrate on the number of sentences. A human respondent can produce infinitely many sentence tokens but a finite computer cannot.

Of course, the computer can fake the production of infinitely many sentences by writing the few sentences I used to specify the infinite sequence of blanks. But this is not the kind of faking Turing intended. Turing wants the machine to genuinely produce the sentences. Faking is restricted to the origin of the sentences. Turing's idea is that if the interrogator cannot tell whether the sentences are produced by a human being or by a computer, then the computer thinks. A computer that fakes the production of the sentences themselves is akin to a computer that fakes garbled transmission of sentences.

It might be suggested that a Turing machine already has an infinite stock of symbols in the form of its unused tape.

This "infinite" tape is composed of squares which can hold a single inscription from its alphabet. Many computability theorists regard each blank as a symbol from this alphabet (for instance, Boolos and Jeffrey 1989, 21). This enables the theorists to reduce the operation of erasing a symbol to the operation of writing a blank on a previously inscribed square. To erase is to write a blank. Under this interpretation, the Turing machine comes pre-equipped with infinitely many symbols. Hence, the Turing machine contrasts with John Locke's "blank slate" picture of the newborn's mind. Instead of being the unsymbolized surface upon which the moving finger of experience writes, the tape is an unbounded repository of innate ideas.

This suggestion fails because the memory of a Turing machine is unbounded but finite. Although there is no upper limit on how many blank squares the machine can have, it cannot have infinitely many.

Another suggestion is to equip the Turing machine with an accelerated tape. This finitely long tape contains infinitely many blanks by virtue of having cells that are narrower and narrower without limit in the same pattern as the accelerated font. Since the tape is finitely long, the scanner can easily pass over infinitely many cells in a finite amount of time. However, the scanner would still need an infinite amount of time to read the tape. Each inspection of a cell is a discrete act of the scanner.

The reading problem could be solved if we relax the constraint that the Turing machine have an upper bound on how fast it operates. An "accelerated" Turing machine could solve many uncomputable problems (Copeland 1998). But in addition to deviating from standard computational theory, an accelerated machine violates the physical principle that no signal moves faster than the speed of light. The accelerated Turing machine cannot produce any symbol tokens in the actual world. Consequently, the accelerated Turing machine is not going to make it into The Guinness Book of Records.

The limits physics imposes on the manipulation of inscriptions were appreciated early in the history of computing. Alan Turing first realized that Turing machines could be built because of his war-time encounter with new electronic technology. Electronic signals travel nearly at the speed of light, so Turing also realized that electronic symbols could not be moved significantly faster. Of course, they could and were moved with greater reliability, efficiency, and in greater quantity. But the fundamental limit imposed by the speed of light is evident in the slim size of contemporary super-computers. They must be built slim because it takes time for signals to travel from one side of the computer to the other.

Uninscribed blanks are uniquely free of these physical limits. True, other symbols can go unstated. I knew a logician who would sternly lecture on the proper uses of quotation marks but then tell his students that his own quotation marks

would henceforth be so faint as to be invisible. His idea was that any needed quotation mark is present but unstated -- just as a premise is present but unstated in an enthymeme. There are common conventions for relaxing some requirements for formal languages. Well formed formulas are supposed to be flanked by brackets. But authors of meta-theory books will informally declare that they will drop the outermost brackets (for instance, Hunter 1996: 56). Blanks should not be assimilated to these abbreviatory phenomena. Blanks are not enthymematic because there is nothing to fill in. Blanks are officially invisible. No special rule is needed to allow them to go uninscribed.

The invisibility of blanks is just a side-effect of the more fundamental property of being uninscribed. Writing in invisible ink takes just as long as writing in visible ink. When letters are written as cut-outs, then the holes are the letters and the opaque (visible) surfaces have blanks.

Holes are visible by contrast with their surroundings. Similarly, normal uninscribed blanks can be seen by contrast with inscriptions. Like other symbols, they are sometimes used because they best exhibit a pattern. For instance, computer scientists frequently replace 0s with blanks to make a pattern more salient. Psychologists have long known that adjacent letters interfere with the perception of characters (Woodworth 1938). Blanks and other non-letters facilitate word recognition (Shaw and Weigel, 1973) by clearing this interference.

4. The economic foundation of blanks

The desirability of using the absence of an inscription as a symbol grows as the cost of inscription rises. Books for the blind are expensive because they must be read by touch. After a history of diverse approaches, embossed dots became the standard solution. In particular, the Braille alphabet employs blanks to form letters from a more primitive six bit code of ones and zeros. Specifically, each Braille cell is composed of six dot positions:

1	*	*	4
2	*	*	5
3	*	*	6

$2^6 - 1 = 63$ complex characters can be formed by raising at least one dot. Concatenating cells yields words:

C	O	Y
* *	*	* *
	*	*
	*	* *

One further character, the blank, is formed by leaving the Braille cell empty. Thus the Braille blank is composed of six primitive blanks. The Braille blank illustrates the general principle from action theory that there are different levels of omission. I let my magazine subscription lapse by not responding to three pleas to renew. Acts form hierarchies via the "by" relation: I turn the light on by flipping the switch and

turn the switch on by moving my finger. Omissions mirror act hierarchies via the same "by" relation. In particular, I produce a Braille blank by not writing a Braille inscription and omit this Braille inscription by not inscribing a dot in a Braille cell.

The cost of producing a letter increases with its number of embossed dots. Hence, the more common the character, the fewer the dots that should be employed. Although 'e' is the most common letter, blank is the most common character in written language. Its high frequency is due to the punctuation requirement that words be separated with a blank. The syntactic job of word separation could be done with other characters such as the black box ■. This ■would ■work ■but ■at ■the ■cost ■of ■more ■time ■and ■ink. The cost would be even greater if smaller boxes replaced the spaces separating letters.

One could dispense with separators altogether. *Script does not use letter separators.* And we do not use word separators in oral communication. We speak continuously. Word separating blanks made their first appearance with the Carolinian reforms in script (Boorstin 1981, 497). Manuscripts prior to the Age of Charlemagne string sentences together without the benefit of spaces between words, commas, or other punctuation.

In some special purpose languages, blanks are refined to permit various levels of separation. Although Morse code may appear to be a binary code, there are actually three symbols: the dot, the dash, and the blank (Hamming 1980, 12-14). The

blanks come in different lengths: one unit of time to separate dots and dashes from each other, two units of time to separate letters, and six units of time to separate words. Theoretically, patient telegraphers could increase their stock of higher order blank-types indefinitely by using greater and greater units of time.

A speedier method is to create qualitatively different blanks is by drawing on the variety of surrounding inscriptions. For instance, the first letter of the alphabet could signified by a blank preceded by a 1, the second letter by a blank preceded by a 2, and so on. ABE would be 1 2 5 . ABBBBBBBBBBE would be 1 2 5 . A blank can only convey a single character on its own because it so non-descript. But this very passivity makes the blank a versatile chameleon. In practice, all communication systems honor this versatility by making the absence of a signal mean something. No news is good news.

Computer designers have long been aware of the economies offered by blanks (Richards 1955, chapter 6). One desideratum of machine codes is that they minimize the number of 1's. Normally, the digit 1 indicates that some electronic device is on and 0 indicates the device is off. Hence, electricity is conserved by minimizing the number of 1s. Since these 0s are symbolized by absence of electrical inscription, the computer is using blanks to represent zero.

In six bit printer synchronizer code 0000 00 means A rather than blank. So the first letter of the alphabet is

symbolized by blanks at the primitive level but not at the higher level code. This again illustrates the importance of relativizing blanks to different levels of inaction.

More generally, blanks need to be relativized to a medium of representation. Even inscribed blanks have this context dependence. Lines such as _____ are blanks on bureaucratic forms because they are devoid of inscriptions of the appropriate sort (alphanumeric characters and punctuation marks). Similarly for other inscribed blanks such as ***** and ----- . I have focused on uninscribed blanks because they avoid the stereotypical constraints on symbol manipulation.

5. The adjectival use of `blank'

My focus has been `blank' in its use as a count noun. However, `blank' also has a philosophically pregnant use as an adjective. The point of describing a wall as blank is to exclude the presence of something which is supplied by context such as pictures or graffiti. It resembles `empty', `clean', and `sober', in being an excluder term. Roland Hall conjectures that `blank' only gradually acquired this status in the last few hundred years:

One word that has developed into an excluder is `blank', originally meaning just `white', but coming via `blank paper' to mean `not written on or filled in', so that in time we get `blank cheque', `blank passport', `blank verse', `blank

expression' (of face). So there must have been a time (probably at the end of 16th century) when it would have been hard say whether `blank' was an excluder or not, . . . (Hall 1963, 72-73)

When a term excludes Es, it becomes indirectly sensitive to standards that govern Es. What counts as a bump or curve or irregularity depends on one's purposes. Hence, what counts as flat will vary with these purposes. Ditto for `certain' via the relativity of `doubt'. Some objections to Locke's tabula rasa or the Eastern cultivation of blank minds have a logical kinship to Peter Unger's (1975) objections to flat tables and knowledge. The antidote to this genre of skepticism is to focus on relationality and context shifting. In particular, to be blank is to be blank of something -- a something whose threshold of significance depends on standards.

Consider my claim that I have erased the blackboard. An erasure skeptic might point out that parts of some of the inscriptions are still visible. If I erase those, he may complain that I have just reduced the size of the parts to illegibly small fragments like a paper shredder. The inscription parts, in the form of chalk dust, still cover the board. Inscription parts persist even after a scrubbing with soap and water. Furthermore, the skeptic will raise the possibility that invisible traces of the letters remain on the slate itself. Writing makes slight indentations in the slate. Although it is currently

impossible to retrieve this information, it is still there. The blackboard never forgets.

Indeed, Sigmund Freud and other psychologists claim that human beings never forget. The same is now frequently said of computers -- erasing a file only impairs access to the information by removing the address.

Information is destroyed when it cannot be recovered. Hence, erasure is relative to the means of recovery. We tacitly relativize erasure claims to modes of recovery. As in the case of other privational terms, the skeptic equivocates by shifting these background relata.

6. Philosophical significance varies inversely with word length

The blank has the prospect of being the most important symbol. Terms attract more attention from philosophers as they grow shorter. The correlation can be traced to the fact that languages develop in accordance with the principle of least effort. Symbols shorten as they increase in currency. For instance, 'airplane' became clipped to 'plane' as airplanes became more important. Usually, words cannot be streamlined to a single letter or syllable because some other word has already taken up this favored niche. The established word normally sustains its privileged position by virtue of its outstanding service to speakers. Hence, the new word will have trouble supplanting it -- especially if the two words tend to be used in the same circumstances. The unrelated senses of

`sole' are benignly ambiguous. The intimately related senses of `is' are confusing. Ambiguities become less acceptable as the danger of equivocation rises. However, the advantages of brevity will tend to push this tolerance to the limit. Hence, we should expect the most useful words to suffer the highest degree of malignant ambiguity.

Consequently, the shortest words tend to be the ones most deserving of philosophical study. And indeed, much commentary has been devoted to `not', `the', `all', `say', `can', `now', `be', `my', `is', `if', `it', `or', `a', `I'. But symbols can be shorter than words. Quotation marks, emphasis indicators, and punctuation marks are also philosophically rich. Blanks occupy the logical limit of this hierarchy. Since blanks are omissions, they do not require inscription.

Relations between symbols can also be expressed without inscriptions. Instead of using parentheses, Polish notation uses precedence rules. These order based rules are familiar from the arithmetic convention that multiplication precedes division and addition precedes subtraction. (Recall the mnemonic rule "My Dear Aunt Sally".)

Concatenation is also free of inscription. Two expressions are concatenated when they are put together -- made immediate neighbors. For instance, conjunction in sentence logic is sometimes expressed with concatenation, that is, `p and q is expressed as `pq'.

Since blanks are symbols rather than relations between symbols, they are not as dependent on other inscriptions as precedence rules and concatenation. Blanks only depend on other symbols epistemologically. We normally need to look at inscription to locate blanks. But blanks can exist independently of inscriptions. In contrast, the relations of precedence and concatenation depend metaphysically on their relata. This explains why a line cannot be composed of just precedences or concatenations but it can be composed of just blanks.

Short terms also tend to be the most ambiguous. Blanks are used to separate words, sentences, and paragraphs and to mark the presence of unmentionable words (" -you!"). Some blanks are numerals for zero. Others serve as schemas, variables, and unknowns.

Ambiguities are especially hazardous when the term lacks a distinctive inscription. This increases the tendency to confuse the symbol with the roles it performs. For instance, blanks are so closely associated with word separation that people find it paradoxical that 'e' could exchange roles with blanks.

Since blanks seem like nothing, they attract the interest of those wishing to perform an eliminative reduction. For instance, Rudolph Carnap (1947) tries to avoid commitment to variables by analyzing expressions with x's and y's in terms of expressions with ----'s and *****'s. But his paraphrases are

grammatical only if the blanks play the role of variables (Geach 1949).

Skepticism about Carnap's reductions is compatible with acceptance of W. V. Quine's distinction between variables and schemas. Variables form open sentences, such as 'x is water'. Prefixing the open sentence with a quantifier yields a closed sentence with a truth value, such as '(x) x is water'. Schemas are sentence diagrams used in the meta-language rather than sentences in the object language. This is vivid when infinitely many formulas in the object language conform to a single axiom schema of the meta-language. Schemas and variables should in turn be distinguished from unknowns such as '2x + 3 = 11'. A formula with an unknown has a truth value and makes reference to a particular thing, here, the number 4. The generality of an unknown is epistemic (all the values x might have, for all we know) rather than semantic (variables) or syntactic (schemas).

7. Hilbert and the specious limits of symbol manipulation

David Hilbert proposed that the absolute consistency of mathematical systems be proved with the help of dis-interpretation. This syntactic regime drains sentences of meaning until only their formal exoskeletons remain. Approved inference patterns are codified as transformation rules that license formulas of one shape to be

re-written whenever formulas of another given shape appeared. Proof becomes a formal game.

Just as a chess commentator can demonstrate that a king and a rook suffice to checkmate the adversary's lone king, the meta-mathematician is in a position to demonstrate that axioms of a system suffice for the derivation of a given formula. The chess commentator can also demonstrate that a king and a knight do not suffice to checkmate a lone king. The meta-mathematician would be able to derive similar results to the effect that it is impossible to derive p and not p from the axioms of the system. This consistency proof would be absolute in the sense that it does not assume the consistency of another system.

Hilbert required finitude for the sake of verifiability. If the system had infinitely many symbols or rules, we could not learn them, read them and change them. He does not require any economizing beyond finitude. Derivations can have billions of steps. Hilbert's second theme is explicitness. The proofs cannot be enthymemes. Everything must be written out. Each symbolization should be an affirmative act rather than the residue of an omission.

Hilbert's preference for explicitness may have led him to prefer inscribed blanks over uninscribed blanks. It is also probable that he thought blanks were unnecessary. Indeed, many of their disparate roles can be executed less ambiguously by re-assigning these tasks to inscriptions and

negative fractions be left braces, {, and the ones corresponding to the positive fractions by right braces, }. The set corresponding to aleph-0 can then be conveyed with the last line of this paper:

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