Coulomb's Law

I. Equipment

conducting ball on monofilament
2 conducting balls on plastic rods
housing with mirror and scale
vinyl strips (white)
wool pads (black)
acetate strips (clear)
cotton pads (white)

II. Introduction

At its most essential level, Coulomb's law tells us that the force between two charges depends (1) linearly on the strength of each charge, and (2) inversely on the square of the distance between them. Mathematically we would write this as

$$F \propto \frac{q_1 q_2}{r^2}$$

where you should read the "$\propto$" as "goes like" or "is proportional to." In order to change the "$\propto$" to an "=" we need to worry about what units the various quantities are measured in.* In this lab you will *explicitly* not be worried about the units: you will be trying to verify if $F \propto 1/r^2$ and if $F \propto q_1 q_2$.

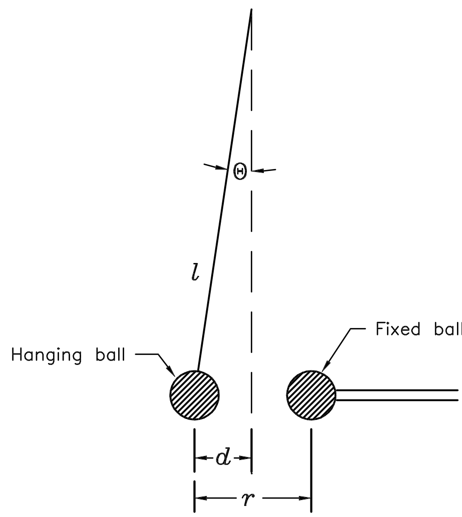
III. Pre-Lab

The apparatus you will use is quite simple but surprisingly effective. You have two conducting balls, one of which is suspended from a thread, and a second one that is mounted on a plastic rod. There is a scale that allows you to measure the horizontal position of each ball, and some plastic strips and wool pads for generating charge.

1. In order to get a good measurement, you'll need a pretty decent charge on the balls; you need to get the product $q_1 q_2$ as large as possible. Let's give some thought as to how to do this. Here's a good way. First, charge one of the balls by induction. To do this, bring a charged strip in close to the ball, and then touch the opposite side; bringing the charged strip in close polarizes the ball, and touching the opposite

* In the cgs unit system, which was "designed" for electricity and magnetism, the unit of charge is *defined* by Coulomb's law (with an = instead of a $\propto$), providing that the force is measured in dynes (=gm cm/sec^2) and r is measured in cm. In SI units, the Coulomb is defined by a measurement involving currents, so that a Coul^2/meter is not a Newton. We have to stick in some constant (with units) to fix the proportionality. When a single international set of units was chosen, it was agreed that this constant would be written $1/4\pi\epsilon_0$. This discussion is down here in a footnote because it is important to separate the *physics* (meaning $F \propto q_1 q_2/r^2$) from the details of what units we use, or exactly what constant is required to make them work.

side allows some of the charge to go to ground (you may want to review pp. 616 – 618 in your book). The first ball is now charged. Touching it to the other ball allows the charge to flow to cover both balls. The nice thing about this procedure is that it can be repeated several times to build up a good hefty charge on both balls. *Draw a diagram in your notebook showing this procedure; draw little + and − signs to indicate what's happening to the charge.* It doesn't matter which sign you choose for the charged strip.



2. The figure shows how the balls will look in the apparatus. Since we're going to use this to measure forces, we have to analyze the forces on the hanging ball. On page 430 in your book, there's an analysis of a swinging pendulum. The situation is similar here, but the Coulomb force on the hanging ball will balance the tendency for the ball to swing; it will hang in equilibrium. The component of gravity which drives a swinging pendulum acts along the arc of the swing, but if the deflection from the vertical is small (as it is here) it acts almost horizontally. *Use all this to argue that the gravitational restoring force pushing the ball to the right is very nearly* $mg(d/l)$. Once the ball is in equilibrium, this must be equal to the electric force F_e, which is what we're trying to measure. Question: *Which of* d, l, m, *and* g *will we actually have to measure to show that* $F_e \propto 1/r^2$?

3. When you are done with the first part of lab, you will have a table consisting of list of different separations r between the balls, and for each separation, a something that is directly proportional to F_e. *Figure out how to plot this data so that if Coulomb was right, you will get a straight line. Describe in your notebook.*

4. Pay attention to the weather on lab day. If the day is warm or rainy or snowy, leave your coat and as much of your stuff as possible *outside* the lab room. Check your shoes for snow, ice, or water. Don't bring that kind of moisture into the lab. Maybe you want to carry a pair of sneakers to lab and change into those. If the lab room becomes too humid, then the charge will quickly leak off the balls, and it will be a challenge to do the experiment. If we have typical dry winter weather (and everyone tries not to breathe too much) then the balls should hold their charge for many minutes.

IV. Procedure

1. Carefully check out the features of your apparatus. Note how the hanging ball is suspended from two threads so it can only move in one direction. Pretty clever. Make sure the hanging ball is aligned properly: slide in a fixed ball and position it next to the hanging one. Look in the apparatus from the top: make sure the balls line up along the path of the fixed ball. Look in from the side: make sure they are at the same height. The fishing line that suspends the hanging ball is held in little slots. By gently tugging the line through the slots, you can change the height and lateral position of the ball (both at once). By adjusting the two threads separately, you can arrange for the hanging ball to line up with the fixed one.

2. Using the procedure you worked out before coming to lab, measure the force between two charged balls for a large number of positions of the 'fixed' ball (you want at least a dozen measurements over as wide a range as you can get). Make a quick determination whether it is proportional to $1/r^2$.

   Some hints:

   a. Use the vinyl strips (white) and wool pads (black) for charging the balls. Be careful not to let the strip touch the ball while you are charging it or you will have to start over.

   b. The box contains a mirror and a scale. Figure out how to use the mirror to eliminate (OK, reduce) parallax errors. These are errors caused by the fact that there's a gap between the scale and the balls – your reading can be thrown off if you don't look at the assembly perfectly square-on.

   c. Even with the mirror, it will be difficult to take extremely accurate measurements. The standard way to compensate for this is to take lots of data points.

3. Try to determine if F_e is directly proportional to the amount of charge. This is more difficult than the first part.

   a. If your apparatus has a piece of glass on the front, you need to remove it for this part.

   b. The idea is to let the 'fixed' ball stay actually fixed, and discharge it several times in a controlled way; after each time, you can measure the deflection of the 'swinging' ball. First you have to decide where to put the fixed ball. In order to get a measurement for small values of q_{fixed}, you want the balls to be close together when they're uncharged. Start with them uncharged, and position the fixed ball so the gap between it and the swinging ball is 2 to 5 mm. Then charge up both balls as heavily as you can.

   c. Measure the force on the hanging ball.

   d. You have a second ball mounted the same way as your "fixed" ball. Make sure this one is discharged. Then reach in the front of the apparatus and gently touch this ball it to the actual fixed ball. Now, carefully remove the second ball. However much charge the fixed ball had before, it now has half as much. (Why?) Note the new force.

e. Repeat the discharging procedure to cut the fixed ball's charge in half again. Do it as many times as you can and still measure a deflection. (It will be small at the end, which is why you arranged the fixed ball so they'd be close together at this point).

f. Make a quick plot of your data to see if F_e is linearly proportional to q_{fixed}.

4. Experiment with the acetate sheets and cotton pads. Can you get charge opposite in sign to what you get with the vinyl and wool?

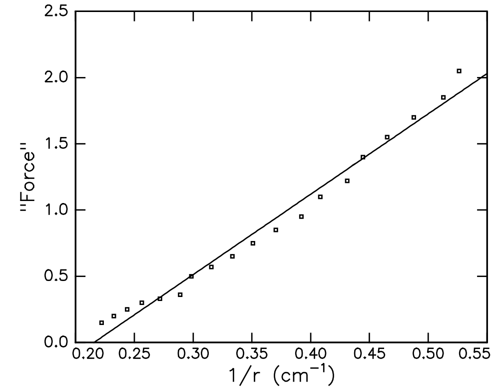
V. Analysis

This will be somewhat tedious if you don't use a spread-sheet or data analysis program. If you do use one, you should be able to do all of this quickly. The strategy is to enter your data (such as readings of r and F) in columns in the spreadsheet, then use the formula function of the spreadsheet to compute things such as $1/r^2$ based on the values of the other columns. This makes it very easy to repeat calculations without having to type all your data over again. There are also tools available for making plots based on data in the spreadsheet. Failing all else, you can always plot up the computed values by hand in your lab notebook, though this is rather laborious.

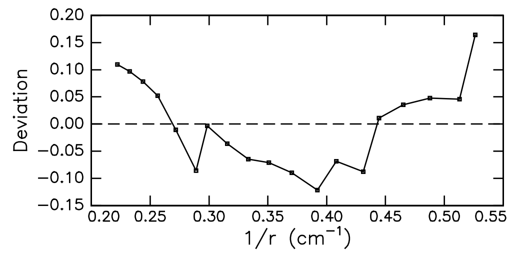
Let's assume that Coulomb's law goes like $F \propto r^n$. The question is, what is the best choice for n based on your data? Start by guessing that $n = -2$. Plot your data so that it will be a straight line if that is the case. Do you get a good straight line? Try guessing that $n = -1.5$ or $n = -2.5$. Do these work better or worse? Can you see a difference at all? Try other values until you can make a statement about the *largest* and *smallest* values of n which are consistent with your data.

It's possible to push the analysis a lot farther if you want. For the various values of n that you test, you can fit a straight line to the data using the linear regression feature of the spreadsheet (or your calculator), and plot that line on the graph. (To do this, you may have to read off the slope and intercept values of the fit, then enter those as a formula for one of the spreadsheet columns, and then plot that column versus the separation column). If the points are all above (or below) the line in the middle and below (or above) on the ends, the value of n is not well-chosen. It may be helpful to compute the values of the line at each data point (with the spreadsheet as outlined a few sentences earlier), and subtract these values from the data points to form what are called 'residuals' to the best fit (that is, how much the fit 'misses' the data points). You can then plot these with a connect-the-dots type plot. Up or down curvature on such a plot indicates a bad value of n, while random scatter is consistent with a good fit.

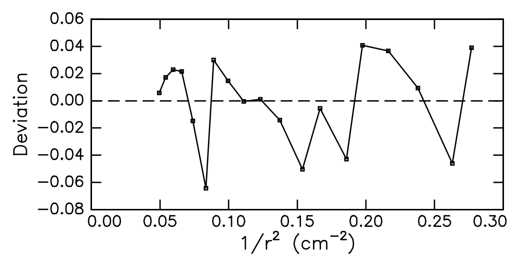
Here's an obvious example of a bad guess for n, with a straight-line fit drawn in (these are actual data from this experiment, plotted with the guess that $n = -1$.)



Here's a plot of residuals, for a bad guess at n; note that it crosses zero only a couple of times and shows obvious trends at the ends.



Here's a much better fit (the guess is that $n = 2$); there's scatter in the data, but it's consistent with random noise.



Note the overall shape of the deviation plot: definitely concave for the bad fit, while the good fit doesn't show a trend one way or the other. Also note that the vertical scale of the two plots is different. The deviations are much larger in the case of the bad fit.

Carry out a similar analysis for your force vs. charge data. You probably can't carry the things as far with any real meaning, since you will have far fewer data points to work with.

Start with the assumption that $F \propto q^m$. Plot your data under the assumption that $m = 1$. Does this look reasonable? Try $m = .5$ and 2. Are these obviously worse? Make the best statement you can about the proportionality of force to charge.