PRE-LAB 1: ANALYZING DATA

December, 1994

Required materials: your lab book, a calculator, and a clear plastic ruler.

At its most basic level, the testing of a scientific theory or hypothesis consists of three steps:

1) Use the hypothesis to make a prediction about the outcome of an experiment.
2) Perform the actual experiment.
3) Compare the predictions with the data and decide if the hypothesis is correct.

Certain experiments can be more complicated than this, or sometimes simpler, but these three steps are pretty universal. The purpose of this pre-lab is to take you through step 1 above. You will do steps 2 and 3 in lab. As you will see, doing step 1 for this particular case will require brushing up on a bit of calculus, and will involve some work designed to make step 3 easy.

Also in this pre-lab exercise, we are going to try to familiarize you with the process of linearizing data. People work well with straight lines—it is very easy for us to tell when a line is straight, and when it is not. The graph at the top of the next page contains some data on the position of two objects as a function of time. Can you tell which line of points is straight? (Try holding the page at an angle and looking directly along the lines.) On the other hand, if a line is curved, it is almost impossible to guess the exact mathematical form of the curve just by looking. So, whenever possible, we'll get into the habit of analyzing our data so that if it does what we expect, it will appear as a straight line. That way, the difference between our predictions and our experimental results will be easy to spot—our line will be curved instead of straight.

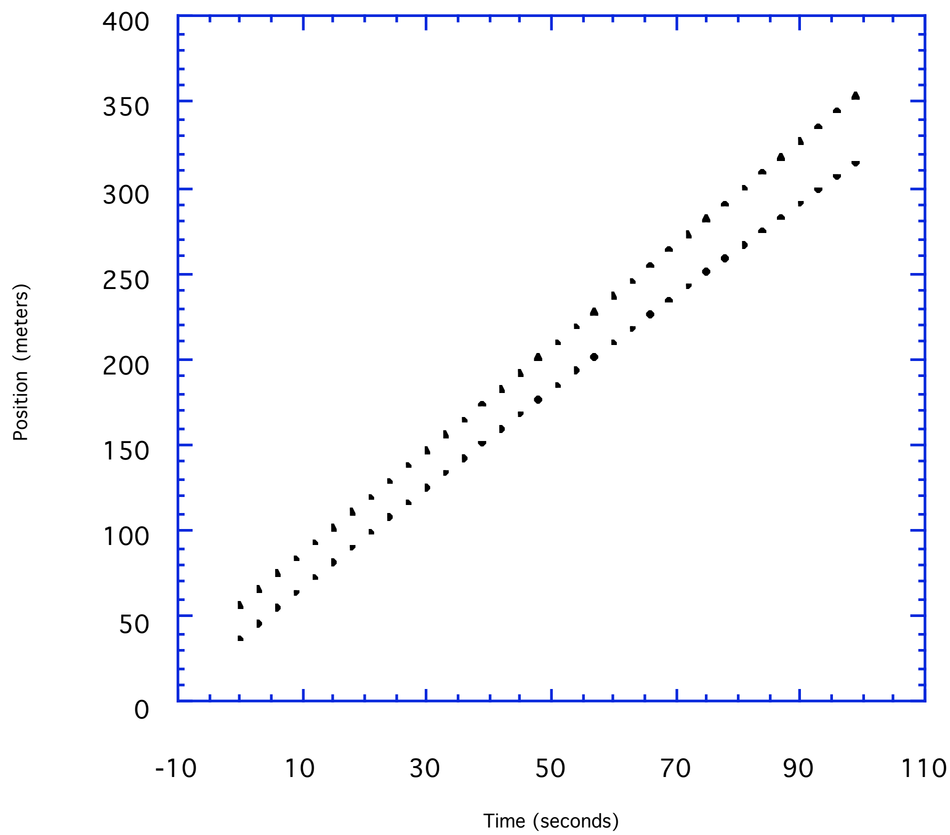


Figure 1: A plot of position vs. time for two objects.

Well, what better place to begin than the beginning? Here is a scientific hypothesis which can be tested.

HYPOTHESIS: A freely falling object accelerates at a constant rate.

Now, we want to turn that hypothesis into an actual *prediction*.. In order to do that, we need to get specific. What is acceleration? It is the second derivative with respect to time of the position of an object. So, we'd better have a variable to describe the position of an object. Let's choose *y(t)* as this variable. Now we'll choose a variable to describe the acceleration of the object: *a(t)*. The *definition* of acceleration says that

$$a(t) \equiv \frac{d^2y(t)}{dt^2}. \qquad \text{(P1.1)}$$

Nothing very profound has been said yet, we're just making sure we all agree on the definitions. Now look back at our hypothesis. It says something about a "constant rate." If the acceleration is constant, then it doesn't change. That means that the acceleration *has no time dependence*. In other words, we can drop the *(t)* business and just write acceleration as

$$a \equiv \frac{d^2y(t)}{dt^2}. \tag{P1.2}$$

How do we make a prediction out of this? We need to think about what we can do to manipulate the equation to turn our hypothesis into a prediction about something we can measure. Well, what can we measure? Well, in this particular lab, you will have equipment that will let you measure *position* and *time*. So, if we can express our hypothesis as a prediction about the position of a falling object as a function of time, we will be able to make an experimental test.

If we integrate equation (P1.1) twice, we will come up with an equation for the position as a function of time. So our prediction should be for *the position of the falling object as a function of time*. In other words, we want a prediction for *y(t)*.

Now it's your turn to do some work. Integrate equation (P1.1) with respect to time to obtain an expression for the *velocity* of a freely falling object as a function of time. Don't forget that there might be a constant of integration.... Now integrate again to find an expression for the *position* as function of time. Again, don't forget the constant of integration. If you've done everything correctly, you should have an equation that looks something like this:

$$y(t) = y_0 + v_0 t + \frac{at^2}{2}. \tag{P1.3}$$

- Your pre-lab write-up should show a complete and accurate derivation of Eq. (P1.3)!!

Well, now we have our prediction. What we need to do now is use this prediction to test our hypothesis. But it is not in its most useful form for step 3 of our experimental process: comparing our data to the prediction. To understand how to take advantage of some built-in things that humans are good at, and to see how we can really make a good comparison, consider the phony "data" in the following table.

Aside: To understand where we are headed, think about what is constant in the hypothesis: the acceleration. Suppose we could express our hypothesis mathematically as the equation of a straight line. What part of a straight line is constant? The slope. So, it might be a good idea to figure out how to plot our data so that it would appear as a straight line with the acceleration as the slope—*if our hypothesis is correct!* Then, if the data supports the hypothesis, the data will appear as a straight line, and if it doesn't, then it will appear as a curved line. That lets us take advantage of the fact that humans are really, really good at straight lines. Probably has something to do with survival of those who could aim accurately enough to bring home dinner.

t (seconds)	*y (meters)*
3	4
4	15
5	36
6	67
7	108
8	159
9	220
10	291
11	372
12	463

This data has just been made up, but we'll pretend it represents the actual vertical (since we are talking about *y*) position of an object as a function of time. Notice that the data are neatly arranged in columns, and the top of each column is clearly labeled with the *name* of the measured quantity, and *THE UNITS* of the measured quantity. This is **VERY** important. Units are **VERY** important. Just in case you missed that: **UNITS ARE VERY IMPORTANT.**

- Also in your write-up, you should show that every term in Eq. (P1.3) has that same units. You can't add apples and oranges! A subtle fact that will almost always tell you when you are doing something wrong....

- On a full, clean page of your lab book, make a plot of this data. (Yes, that's right--waste a whole, entire page on this one plot. In fact, waste a whole, entire page on *every* plot of data you make in Physics 3! Nothing leads to bad physics and incorrect results faster than a puny little plot scrunched at the bottom of the page because you just can't bear to use a whole sheet.) Your plot should be neat, and the axes should be **CLEARLY LABELED** with both the *name* of the variable being plotted, and the **UNITS**.

Put time on the horizontal axis and position on the vertical axis. Squint at the plot for a while. Does the data agree with the prediction of Eq. (P1.3)?? Kind of tough to tell, isn't it? It sort of curves the right way, but telling the exact dependence on time is impossible from this kind of plot.

As a first step, look back at Eq. (P1.3) and notice what we would have if v_0 happened to be zero: $y(t)$ would be a straight line with t^2 as the "x" variable, $a/2$ as the slope, and y_0 as the intercept. Let's see if that is the case.

• Make a new data table in your lab book. It should have three columns. The first column should be the time, the second column should be the time squared, and the third column should be the position. Carefully label each column with the name of the quantity and the **UNITS**.

• Make a new plot in your lab book. Put time squared on the horizontal axis, and position on the vertical axis. Is this a straight line? It may help to hold the page at an angle and squint down the line of data points. Then any deviations from a straight line will leap out at you. Use your ruler to draw the best straight line you can through the data.

Well, it turns out you've been set up. The data are NOT a straight line. That does not mean that our hypothesis is wrong (we're testing a hypothesis, remember?)--at least not yet. It just means that we can't assume $v_0 = 0$. In order to make a better test, we have to be slightly more sophisticated in our treatment of the data.

Look back at Eq. (P1.3). Subtract off y_0 from both sides, and divide the entire equation by t. The result is a messy object involving $y(t)$ equal to a very simple expression whose general form you should recognize.

• Do this math in your lab book and also identify the form of the equation in your lab book.

Now, how do we apply this to the data? What does the algebra tell us to do? If we translate it into words, it means "take the position at time t, subtract off the position at time zero, and divide the result by the time t." But wait! you say. (Did you say it? You should have.) We don't know the position at time zero! According to that table, that is true, but when was time zero, anyway? At the beginning of the universe? When you were born? It is completely arbitrary! So why not declare the first data point to have occurred at time zero! Then the position at time zero is the position at the first time in the data table. If you are not completely convinced on this, go back to the data that was given and change the time values so that the first one is zero and adjust the others accordingly (0,1,2...). Now make another plot of y vs. t (like the very first plot you made). You should see you have the same curve, except now, it is shifted along the time axis.

• Make a new data table in your lab book. Make the first (meaning left-hand) column *time*. The first time should be 0 seconds, and all the rest of the times should be adjusted accordingly. Make the second (right-hand) column *position*. You should also have new y values in the table because you should "take the position at time t, and subtract off the position at time zero." Be sure to label the headings of the columns so that another person (for example, the person grading your write-up!!) can tell *EXACTLY* what you've done.

• Now add a third column to the data table you just made. This column should be the adjusted y values divided by the adjusted t values. In other words, each entry in this new column should contain the result of dividing an entry in the second (now middle) column by an entry in the first (left-hand) column. You clearly can't do this for the for the first data point, since it would mean dividing by zero! That is because we have "used up" that point in adjusting the *y* values for the rest of the data.

- Make a plot of this third column versus time. Again, be neat! and clearly and completely label both axes.

Now I ask you, is this magic, or what?

Actually, there is absolutely no magic here at all. But there is some really important stuff going on, so let's review what we've done. We started with a hypothesis: "a freely falling object accelerates at a constant rate." We then found that this same hypothesis could be expressed mathematically as

$$y(t) = y_0 + v_0 t + \frac{at^2}{2}. \qquad \text{(P1.4)}$$

We then found that this mathematical statement could be re-arranged to read

$$\frac{y(t) - y_0}{t} = v_0 + \frac{a}{2} t. \qquad \text{(P1.5)}$$

The right-hand side of this equation describes a straight line. To test our hypothesis, we take data on a freely falling object, and manipulate this data according to the "instructions" in the left-hand side of Eq. (P1.5). If our data fall on a straight line (a big "if!!") then our measurements support the hypothesis! In this case, if you did everything right, you should get resounding agreement, since the data were artificially manufactured in the first place.

What if they didn't agree? Deciding what to do in this case requires sophistication and good judgment. Doing it well is what separates good scientists and engineers from the turkeys. Part of what you will learn in Physics 3 is how to make these judgment calls. Getting really good at it will take will take all of your college career and beyond. Over a lifetime, you should continue to get better and better at it. We'll take the first steps in this learning process in the actual lab.

- For now, use your last plot to determine the initial velocity of the freely falling object, and the value of its constant acceleration. Report your results and your method for obtaining them in your lab book.

That's it for now. Hopefully, you have learned how the process of *linearizing the data* (which is what you've just done) can provide a powerful tool for checking your experiment against a hypothesis.

LABORATORY 1: KINEMATICS

December, 1994

Apparatus: meter stick
spark timer
objects to drop

The purpose of this lab is two-fold: to reinforce what you have learned about kinematics in class, and to introduce you to the fine art of performing physics experiments. You absolutely mush have read through these pages **BEFORE** coming to lab so you are prepared to go right to work once you are in the lab.

In this lab we will test the following hypothesis:

A freely falling object accelerates at a constant rate.

Read this hypothesis carefully, and bear in mind exactly what it says. There are lot's of similar and related hypotheses we could be testing, but this is the one we are after. And we will test this hypothesis using the following three step program:

1) Use the hypothesis to make a prediction about the outcome of an experiment.
2) Perform the actual experiment.
3) Compare the predictions with the data and decide if the hypothesis is correct.

Since you did the pre-lab carefully and completely (!) you know how to express this hypothesis in a convenient mathematical form that makes a prediction about how the data might come out. In other words, you have already done Step 1. So, you can move right on to.....

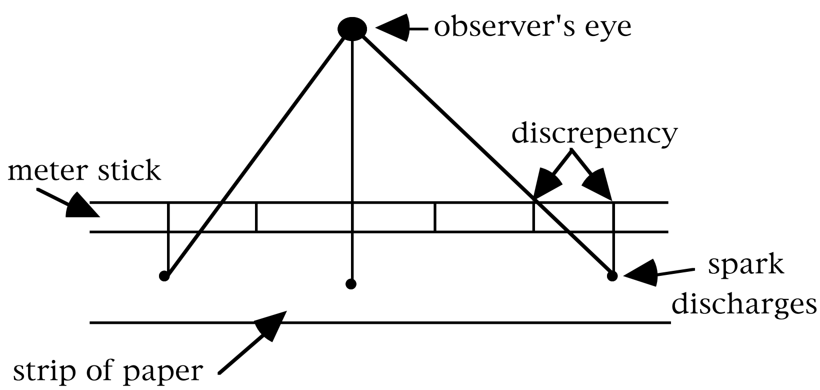
STEP 2: ACQUIRING DATA

The instruments we are going to give you for this part include the following:

- objects of different mass
- a meter stick
- a spark timer

METER STICK- This is a piece of wood with markings every millimeter (1 millimeter is 1/1000th of a meter). The other side of the meter stick has markings left over from a strange and ancient system of measure. When you use the meter stick, watch out for two things: the ends are not protected from damage, and probably look like someone's dog has been chewing on them. How well can you estimate where the true end of the stick used to be? The second watch out for is "parallax" error which occurs of you don't look straight across the stick when using it to make a measurement. The diagram below illustrates how this can lead to errors in your measurements.

The Problem of Parallax



SPARK TIMER- This little device will emit a little spark every $^{1}/_{10}$ or $^{1}/_{60}$ of a second, depending on how you have its switch set. The positions of the switch are labeled "10 Hz" and "60 Hz," from which you can figure out that "Hz" (pronounced Hertz after the physicist, not car rental company, of that name) means "times per second." In the lab you will find rolls of a special "spark paper" that turns black if a spark happens to zap through it. When one pulls a little streamer of this spark paper through the timer, the little black marks on the paper appear at each fraction of a second (the marks tell both time and position). (There is friction between the paper and the timer. Is it negligible?) When using the timer, make sure it is off when feeding in the paper, and that you feed in the paper in the direction of the arrow on the machine. Leave about a centimeter of paper exposed at the bottom, and make sure the paper is not tangled. Two things to be aware of: sometimes the spark timer will fail to leave a mark on the paper when it should have, and there will probably be a big, fuzzy black dot where the timer was sparking but the paper was not moving. Do you know the exact time interval to use between this dot and the dot next to it?

PROCEDURE

(A) OBTAINING "RAW" DATA: Use the apparatus to obtain data on a falling object. Remember you will want to plot the data you obtain in a meaningful way, so think about the kind of data you'll need. You might want to think about the following questions: How much data do you need? Is friction in the spark timer important? How could you look for the effects of friction?

(B) REDUCING THE DATA: "Cook" your raw data a bit - in an honest and well-defined way! - and make it into a table of information to be plotted.

Once you have all the data recorded in a nice, neat table all carefully labeled with units and all so that your mother would be proud, what is the best way to plot it? Do it now. Label both axes clearly and with units. If you accidentally skipped a point, or if the timer failed to spark (it sometimes does that) and you didn't notice, you should be able to tell now--there will be a big jump in your data. If so, fix the data table and the plot. If you can't figure out how to fix it because things have gotten hopelessly confused, throw out all the old data and repeat the experiment.

(C) ANALYZING YOUR DATA: In this step, we will actually test the hypothesis! Think about how you did the prelab and what you can do to prove the hypothesis.

(D) THINKING ABOUT YOUR DATA: This is the most important step of all. You should write answers to all the following questions in your lab book. Have pity on the TA. Number your answers so she or he has some hope of matching your responses with the questions.

1. Does your data support the hypothesis? Explain what has to happen for the data to support the hypothesis. Discuss the *quality* of the support that your data gives: is it super strong? kinda, sorta OK? really weak? Does your data suggest an entirely *different* hypothesis? If so, what is that hypothesis, and how might you test it? If your data do not give you what you expected, try re-analyzing your data assuming that the *second* point in your data table occurred at $t = 0$ (just ignore the first point). If that better supports the hypothesis, explain why.

2. As an extra in this experiment, we can get out the acceleration of the object, and its initial velocity. The initial velocity does not tell us anything profound or universal. It is just a function of how you started your experiment, so we won't bother with it. But let's find a value for the acceleration. NOTE: this really is an extra. Our hypothesis said nothing about the value of the acceleration of a freely falling body, only that it was constant. So...take your clear plastic ruler, and draw the "best" straight line that you can through the data. What "best" means takes some judgment. Your line should be as close as you can make it to as many of the points as possible. If one of the points is way away from the rest, and the rest make a reasonable line, then maybe you should ignore that point entirely. What is your value for a?

3. What is the uncertainty in your value of *a*? There are several ways to calculate this uncertainty, some are very sophisticated, and some are simple. Try this simple method: use your ruler to make two more lines on your data plot. You already made a line with a "best" slope. Now make two with "OK" slopes. One should have a slope greater than the best line, and one should have a slope less than the best line. Drawing these lines also takes judgment. You would like to avoid lines with slopes that are "terrible" or "ridiculous." In general, you want to draw the "worst" line that still goes inside the region of the plot covered by the data points. Suppose the slope of your best line is m, then report your slope as $m \pm \Delta m$, where Δm is the average difference between your best line and your "worst "lines.

4. Consider the following hypothesis: "*all* freely falling objects accelerate at the *same* constant rate." Explain how you would test this hypothesis. (You don't have to do it!)

5. There is a generally accepted value for the acceleration you have measured: 9.8 m/sec^2. Compare your result with this value. Also compare your result with the other groups in the lab. Is *your* result within *your* estimated error of the accepted value? Are the results of the entire lab section all on one side of the accepted value? Or do they fall on all sides of it? Discuss how using the results of the entire lab can *and* cannot be used to test the hypothesis in question 4.

6. Discuss the possible sources of error in this experiment. Point out how each source will or will not support the hypothesis of this experiment (the main one, not the one in question 4), and how each one might or might not affect the particular value of the acceleration that you find.

At this point you are done. Be sure to write the name of your lab partner *at the start* of your lab write-up and hand your lab book in to your TA.