The Acceleration Due To Gravity

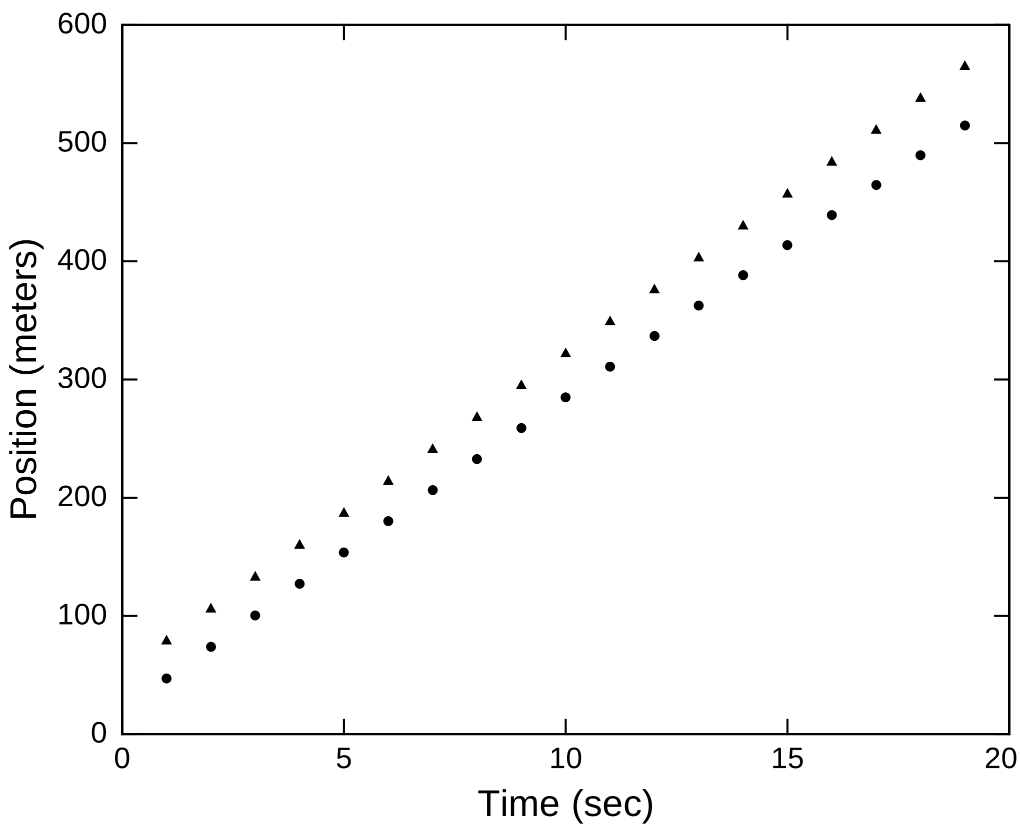
I. Introduction

At its most basic level, the testing of a scientific theory or hypothesis consists of three steps:

1. Use the hypothesis to make a prediction about the outcome of an experiment.
2. Perform the actual experiment.
3. Compare the predictions with the data and decide if the hypothesis is correct.

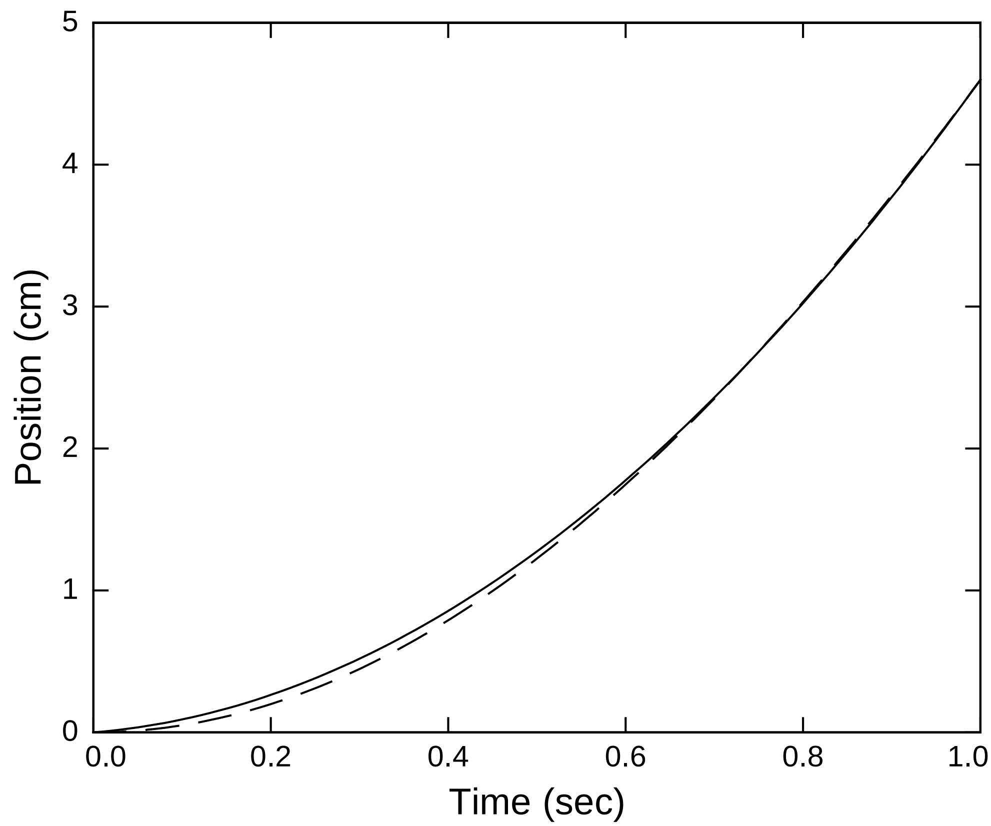
Certain experiments can be more complicated than this, or sometimes simpler, but these three steps are pretty universal. The success or failure of an experiment depends on how well they are carried out. (The success or failure of the hypothesis is a separate matter. A successful experiment can prove that a hypothesis is wrong.) You may think that all the action is in step 2, but actually it is step 1 that is really important. How you phrase the hypothesis can guide how the experiment is done. Some thought about what a hypothesis means can make the difference between a good experiment and a pointless one.

In this lab you will learn how to *linearize* a hypothesis. Why? Because people work well with straight lines—it is very easy for us to tell when a line is straight, and when it is not. Consider this graph:



Both lines represent (made up) position vs. time data for cars going about 60 mph. The speed of one of these cars is changing by about .2% per second. Even though the difference is subtle, you should have no trouble telling which one it is. Try holding the page at an angle and looking directly along the lines. (Is the car slowing or speeding up?) An evolutionary biologist might tell us that this has something to do with the survival of those who could aim accurately enough to bring home dinner...

Here is an even more interesting question: for the line that is not straight, what is the exact functional form? In the figure below, one of the lines is a cosine, and the other is square law. Are you good enough to tell which is which?



It is even worse than that. You have been lied to. One of the curves *is* a cosine, but the other curve is not a pure square law. It is a parabola which follows the equation $y = 4.1x^2 + .5x$. So you can see that it is practically impossible to guess the exact mathematical form of the curve just by looking. So, whenever possible, you should analyze your data so that if it does what you expect (meaning: agrees with your hypothesis) it will appear as a straight line. That way, the difference between your predictions and your experimental results will be easy to spot—your line will be curved instead of straight.

II. Pre-Lab Exercises: Linearizing a Hypothesis

The following exercises will show you to take a hypothesis, express it mathematically, linearize it, and test it against a set of data. You must have answers to these questions written in your lab notebook at the start of lab. It would probably be a good idea to answer them together with your lab partner (or in even larger groups). The questions are all numbered. Please number your answers correspondingly.

HYPOTHESIS: A freely falling object accelerates at a constant rate.

Now, we want to turn that hypothesis into an actual prediction.. In order to do that, we need to get specific. What is acceleration? It is the second derivative with respect to time of the position of an object. So, we'd better have a variable to describe the position of an object. Let's choose y as this variable. Now we'll choose a variable to describe the acceleration of the object: a. The instantaneous acceleration is *defined* by the relation

$$a(t) = \frac{d^2y(t)}{dt^2}. \tag{1}$$

Our hypothesis, on the other hand, says that objects should accelerate at a *constant* rate, which means that a should not change with time. To make that explicit, we could re-write Eq. 1 to read

$$a = \frac{d^2y(t)}{dt^2}. \tag{2}$$

An important part of experimental design is knowing how to match your hypothesis to the apparatus that you have or can build. In this particular case, you will have equipment that will let you measure *position* and *time*, but nothing that directly measures acceleration.

1. Take the expression for uniformly accelerated motion,

$$y(t) = y_0 + v_0 t + \frac{at^2}{2}, \tag{3}$$

   and take its time derivative twice to show that you recover Eq. 2. Explain clearly what all the constants in this equation are and where they came from. If you're comfortable with integration, follow the discussion on p. 102 of your text and derive Eq. 3 directly from Eq. 2.

Where is this heading? We know that you are good at telling if a line is straight or not. What part of a straight line is constant? The *slope.* What part of our hypothesis is constant? The *acceleration.* So, it might be a good idea to figure out how to plot our data so that it would appear as a straight line with the acceleration as the slope—*if our hypothisis is correct!* Then, if the data supports the hypothesis, it will appear as a straight line, and if it doesn't, it will appear as a curved line.

2. Consider the data in the following table:

   (Notice that the data are neatly arranged in columns, and the top of each column is clearly labelled with the name of the measured quantity, and the *units* of the measured quantity. This is **very** important. Never put any data in your lab book without the units to go with it.)

t (sec)	y (m)
3	4
4	15
5	36
6	67
7	108
8	159
9	220
10	291
11	372
12	463

Read Appendix B of this lab.

On a full, clean page of your lab book, make a plot of the above data. (Yes, that's right–waste a whole, entire page on this one plot. In fact, waste a whole, entire page on every plot of data you make in lab! Nothing leads to bad physics and incorrect results faster than a puny little plot scrunched at the bottom of the page because you just can't bear to use a whole sheet. You will NOT run out of pages in either of your lab notebooks over the course of the term.) Your plot should be neat, and the axes should be clearly labelled with both the name of the variable being plotted, and the units. Put time on the horizontal axis and position on the vertical axis.

Squint at the plot for a while. Does the data agree with the predictions of Eq. 3? Kind of tough to tell, isn't it? It sort of curves the right way, but telling the exact dependence on time is impossible from this kind of plot.

3. Notice what would happen to Eq. 3 if v_0 happened to be zero: $y(t)$ would be a straight line with t^2 as the "x" variable, $a/2$ as the slope, and y_0 as the intercept. See if that is the case. Make a new data table in your lab book. It should have three columns. The first column should be the time, the second column should be the time squared, and the third column should be the position. Carefully label each column with the name of the quantity and the units. Make a new plot in your lab book. Put time squared on the horizontal axis, and position on the vertical axis.

   Is this a straight line? It may help to hold the page at an angle and squint down the line of data points. Then any deviations from a straight line will leap out at you. Use your ruler to draw the best straight line you can through the data.

It turns out, of course, that you've been set up. The data are NOT a straight line. That does not mean that the original hypothesis is wrong (we're testing a hypothesis, remember?)–at least not yet. It just means that we can't assume $v_0 = 0$. In order to make a better test, we have to be slightly more sophisticated in our treatment of the data.

4. Look back at Eq. 3. Subtract off y_0 from both sides, and divide the entire equation by t. The result is a messy object involving $y(t)$ equal to a very simple expression whose

general form you should recognize. Do this math in your lab book and also identify the form of the equation in your lab book.

How would you apply this new form of the hypothesis to the data? Do exactly what the algebra tells you: "take the position at time t, subtract off the position at time zero, and divide the result by the time t."

But wait! you say. (Did you say it? You should have.) We don't know the position at time zero! According to that table, that is true, but when was time zero, anyway? At the begining of the universe? When you were born? It is completely arbitrary! So why not declare the first data point to have occurred at time zero! Then the position at time zero is the position at the first time in the data table.

(If you are not completely convinced on this, go back to the data that was given and change the time values so that the first one is zero and adjust the others accordingly (0,1,2...). Now make another plot of y vs. t (like the very first plot you made). You should see you have the same curve, shifted along the time axis.)

5. Make a new data table in your lab book. Make the first (meaning left-hand) column time. The first time should be 0 seconds, and all the rest of the times should be adjusted accordingly. Put your position data in the second (right-hand) column. You should also have new y values in the table because you should "take the position at time t, and subtract off the position at time zero." Be sure to label the headings of the columns so that another person (for example, the person grading your write-up!!) can tell *exactly* what you've done. Now add a third column to the data table you just made. This column should be the adjusted y values divided by the adjusted t values. In other words, each entry in this new column should contain the result of dividing an entry in the second (now middle) column by an entry in the first (left-hand) column. You clearly can't do this for the for the first data point, since it would mean dividing by zero! That is because we have "used up" that point in adjusting the y values for the rest of the data.

   Make a plot of this third column versus time. Again, be neat! Clearly and completely label both axes.

Is this magic, or what? Actually, there is absolutely no magic here at all. But there is some really important stuff going on, so let's review what we've done. We started with a hypothesis: "a freely falling object accelerates at a constant rate." We then found that this same hypothesis could be expressed mathematically as

$$y(t) = y_0 + \mathrm{v}_0 t + \frac{at^2}{2}.$$

We then found that this mathematical statement could be re-arranged to read

$$\frac{y(t) - y_0}{t} = \mathrm{v}_0 + \frac{a}{2}t. \tag{4}$$

The right-hand side of this equation describes a straight line. To test our hypothesis, we take data on a freely falling object, and manipulate this data according to the "instructions" in the left-hand side of Eq. 4. If our data fall on a straight line (a big if!!) then our measurements support the hypothesis! (Which is different from proving the hypothesis to be "true." We can never truly prove such statements about nature. We can only say that all the evidence supports them.) In this case, if you did everything right, you should get resounding agreement, since the data were artificially manufactured in the first place.

What if they didn't agree? Deciding what to do in this case requires sophistication and good judgement. Getting really good at this sort of thing really good at it will take will take all of your college career and beyond. Over a lifetime, you should continue to get better and better at it. We'll take the first steps in this learning process in the actual lab. But for now...

6. Use your last plot to determine the initial velocity of the freely falling object, and the value of its constant acceleration. Report your results and your method for obtaining them in your lab book.

III. Procedure

This lab has two parts. In the first part, you will have a chance to test for yourself the hypothesis you worked over in the pre-lab exercises. By now you should have a pretty good idea about what sort of data it would be nice to have and how to analyze it.

Little g.

Test the following hypothesis experimentally: *A freely falling object accelerates at a constant rate.*

Use the following three step program:

1. Use the hypothesis to make a prediction about the outcome of an experiment.
2. Perform the actual experiment.
3. Compare the predictions with the data and decide if the hypothesis is correct.

Since you did the pre-lab carefully and completely(!) you know how to express this hypothesis in a convenient mathematical form that makes a prediction about how the data might come out. In other words, you have already done Step 1. So, you can move right on to Step 2.

But first a few words about the apparatus. The instruments we are going to give you for this part include the following:

objects of different mass
a meter stick
a spark timer

Meter Stick—This is a piece of wood with markings every millimeter (10^{-3} m). The other side has markings left over from a strange and ancient system of measure. When you use the meter stick, watch out for two things: the ends are not protected from damage, and probably look like someone's dog has been chewing on them. How well can you estimate where the true end of the stick used to be? The second thing to watch out for is "parallax" error which occurs of you don't look straight across the stick when using it to make a measurement.

Spark Timer—This little device will emit a little spark every 1/10 or 1/60 of a second, depending on how you have its switch set. The positions of the switch are labelled "10 Hz" and "60 Hz," from which you can figure out that "Hz" (pronounced Hertz after the Dead European White Male physicist, not car rental company, of that name) means "times per second." In the lab you will find rolls of a special paper that turns black if a spark happens to zap through it. If a strip of this paper is pulled quickly through the timer (while it is sparking), little black marks will be left on the paper which tell both time and position. (There is friction between the paper and the timer. Is it negligible?) When using the timer, make sure it is off when feeding in the paper, and that you feed in the paper in the direction of the arrow on the machine. Leave about a centimeter of paper exposed at the bottom, and make sure the paper is not tangled. Two things to be aware of: sometimes the spark timer will fail to leave a mark on the paper when it should have. Also, when you look at your paper, there will probably be a big, fuzzy black dot where the timer was sparking but the paper was not moving. Do you know the exact time interval between this dot and the dot next to it?

1. Obtain "raw" data: Use the apparatus to obtain data on a falling object. Remember you will want to plot the data you obtain in a meaningful way, so think about the kind of data you'll need. You might want to think about the following questions: How much data do you need? Is friction in the spark timer important? How could you look for the effects of friction? How could you reduce the effects of friction? (Hint: you have been given several masses. You don't need to take data on all of them, but you might make a choice among them that will help with this friction business.) The end result of this step should be a very legible table of data, carefully labeled with units and all. You should also have an estimate of the uncertainty in each measurement, and a brief paragraph describing how you estimated these uncertainties.

2. Make a quick plot of your data in such a way that you can tell if it supports the hypothesis you are testing. You can do a more careful one later. The point now is to look for problems. Are you in a hurry? Then ask yourself: what is the simplest unit of time to use? Are you constrained by some ancient Babylonian's choice of a time unit?

There will often be a missing point where the timer failed to spark. If you don't catch this, all your subsequent time assignments will be incorrect. This will be obvious when you plot the data. You can fix this by correcting the time assignments—no need to re-take all the data. On the other hand, you may find that things are so hopelessly confused that you have no choice but to throw out all the old data and repeat the experiment.

B. Pendulums

In the second part of the lab, we'll just play with pendulums a little bit. We'll treat these much later in the course when we talk about oscillations – for now, I'll just tell you the answers and let you do some measurements which may be interesting.

We'll be focusing on the *period* of the pendulum. This is simply the time it takes the pendulum to make one full trip back and forth.

You'll have a stopwatch to measure the period of the pendulum. Let's think a little bit about how to do this measurement best. There's some error associated with your measurement, caused by your reaction time, mostly. Now, if each cycle takes the same amount of time, your precision can be much better if you *time how long it takes for the pendulum to oscillate a known number of times* rather than just timing one swing (or timing one swing many times). For instance, suppose the pendulum takes 1 second to swing, and your error is 0.1 second. That's a 10 per cent error! If you time 10 swings, the error in your timing is still 0.1 second, *but* you divide by 10 to find the time for a single swing. So now you know the time for a single swing to ± 0.01 second! The only trick in doing this is to be sure to count *ZERO*, not one, when you start the stopwatch.

Although we're not in a position to analyze the motion of a pendulum yet, we can get the right answer for its period without actually doing the analysis, using *dimensional analysis*. This is amazingly sleazy but – it works! Let's write down the various factors which *might* affect the period of a pendulum, and their units:

$$\begin{array}{rl} \text{length} & \text{m} \\ \text{mass} & \text{kg} \\ \text{acceleration of gravity} & \text{m s}^{-2} \end{array}$$

Now just play with these until you get something with the units of seconds. Note that the mass must not matter, since you need to cancel the kilograms and there's nothing to cancel it with. After a little playing you find that the period must be something like:

$$P \sim \sqrt{\frac{\text{length}}{g}}.$$

(I've cheated here – what about the distance the pendulum swings? In principle, that might matter. But it doesn't, actually). A full analysis shows that the period is actually larger than this, by a factor of 2π, that is

$$P = 2\pi\sqrt{\frac{L}{g}},$$

where I'm using L for the length.

Now let's check it.

1. Rig up a pendulum, about as high as you can make it with the equipment provided. Carefully measure the distance from the top of the string to the *middle* of the weight.
2. Pull the pendulum out a couple of inches and let it go; time 10 swings (remember to start at 0)! Now pull the pendulum out twice as far and time 10 swings. How close to the same are your answers? Is the difference significant, do you think?
3. Try it again with a different mass. Any difference?
4. Let's look how the period depends on the length. Try three more shorter lengths, maybe down to 1/2 meter or so. Measure the length carefully as before, and the period.
5. If you have time left over, you might want to follow through the analysis while you can still check the results.

IV. Analysis

1. If you have not yet done so, make a nice, full page plot of your data in a form that will easily allow you to determine if your falling object accelerated at a constant rate. (Note: you are still not required to use seconds as the unit of time.) Do your data support the hypothesis? Explain what has to happen for the data to support the hypothesis. Discuss the quality of the support that your data gives: is it super strong? kinda, sorta OK? really weak? Does your data suggest an entirely different hypothesis? If so, what is that hypothesis, and how might you test it?

   If your data do not give you what you expected, try re-analyzing assuming that the second point in your data table occured at $t = 0$ (just ignore the first point). If that better supports the hypothesis, explain why.
2. Use your plot to get a value for the local acceleration due to gravity. See Appendix B for an idea on how to estimate the uncertainty in the slope of a line. (If you use a calculator's program to fit the line, it may return an uncertainty in the slope for you. If you do this, please draw that uncertainty on your plot.) Is there a *discrepancy* between your result and the accepted local value of 9.805 m/sec^2?
3. Discuss the sources of random and systematic error in your measurement. Be sure to clearly indicate which is which. Explain how each error would affect your final result, both in the overall test of the hypothesis about falling objects and in your determination of g. Can you devise an experimental procedure that would allow you to reduce or compensate for any of these errors?

4. Part of the point of the pendulum part is to check to see if the period really does depend on the square root of the length. This is a perfect opportunity to use your linearization tricks again. Show that if our formula is correct, a plot of P^2 vs. L should be a straight line through the origin. Now do the plot, and comment on how straight it appears.

5. Compute g using the results from your longest pendulum. How close is it to the accepted value? Do you think this is a better or worse way of finding g than with the spark timer?

Appendix A

The following equipment is supplied for each lab station:

50 g mass
100 g mass
200 g mass
meter stick
spark timer and stand
spark paper
paper punch
string
stopwatch

The following equipment is to be shared among all lab stations:

spring scale
8 kg mass

Appendix B: Graphing Data

This topic could fill a book (see, for example, E. R. Tufte, *The Visual Display of Quantitative Information*) but here a a few pointers using the graph on the next page as an example.

1. The axes are clearly labeled, with units given for the phyical quantities. There are enough numbers, but not so many that the graph is hard to read.

2. The uncertainty in the data is expressed by vertical and horizontal bars (known as "error bars"). In this particular case, the mass was known very well and the horizontal error bars are too small to draw.

3. The graph also shows an example of a "best" line (solid) and two "worst" lines (dashed). You will note that the so-called worst lines are still pretty reasonable. From these lines I got a slope of $2.6 \pm .1$ cm/kg. Drawing these lines is matter of judgement and practice. Expect to get better at it with time. (At some point you will graduate to doing this numerically with computers and/or calculators. But for now, it is best to do it by hand to get a feel for the meaning of things.)

4. Note that there is one point that appears to be way off the line. Since there is only one such point, it was mostly ignored in the process of determining the lines.