THE SPECTRUM OF HYDROGEN

I. Introduction

Read Eisberg & Resnick, Ch. 4, or Beiser, Ch. 4.

II. Goals

Measure the wavelengths of the visible lines in the spectrum of hydrogen, determine an experimental value for the Rydberg constant, compare it to the predictions of the Bohr theory.

III. Apparatus

The equipment you will use is called a *grating spectrometer*, illustrated below. Light from the source passes through a slit and is collimated into a parallel beam. The beam hits a ruled grating on a glass slide and is diffracted. The viewing telescope is mounted on a rotating arm. You can determine the angle through which a particular wavelength has been diffracted by centering the cross-hair in the telescope on the image of the slit and reading off the angle from the rotating scale. The scale has a vernier which allows you to accurately determine the angle to the nearest arc-minute.

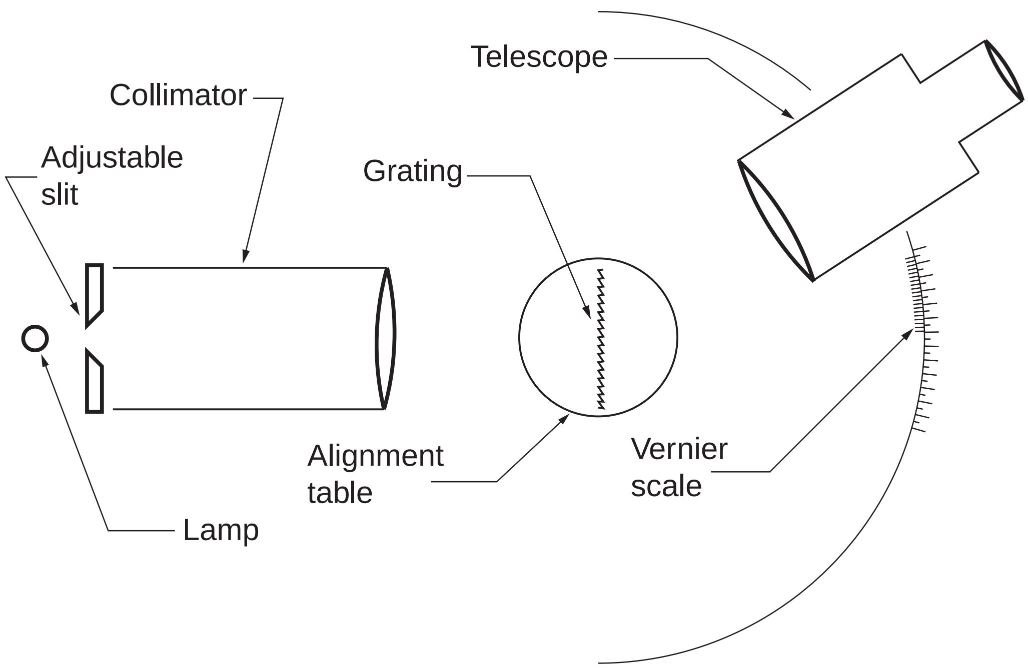


Figure 1. Schematic top view of the grating spectrometer.

It is critically important that your spectrometer be aligned properly. They will have all been aligned prior to lab, so please don't tweak things at random. There is an alignment procedure as an appendix to this hand-out. It is mainly there as emergency back-up, and you would not be able to do all of it yourself since it requires specialized tools.

To see how the alignment affects things, first realize that the diffraction grating equation we derived in P16 relied on the light hitting the diffraction grating at normal incidence. The figure below shows what happens when the light hits the grating at an angle θ_i to the grating normal. For a beam diffracted through an angle θ_L, the condition for constructive interference is

$$d \sin\theta_L + d \sin\theta_i = m\lambda,$$

where m is an integer giving the *order* of the diffraction maxima. On the other hand, for a beam diffracted in the other direction through an angle θ_R, the condition for constructive interference is

$$d \sin\theta_R - d \sin\theta_i = m\lambda.$$

(Why? Are you sure you want to take my word for it?)

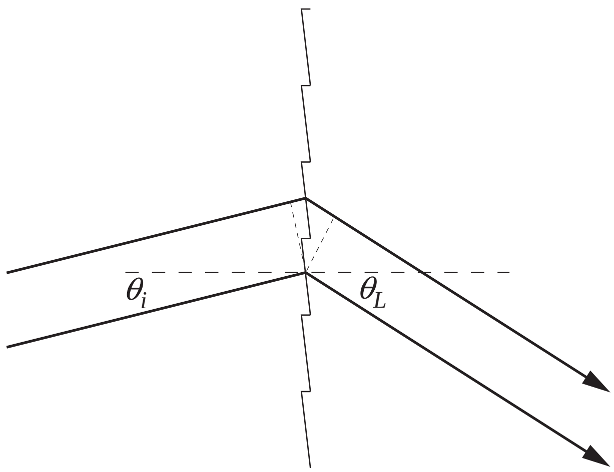


Figure 2

The problem is that you don't know θ_i. The solution is to add these two equations to find

$$\lambda = \frac{d}{m}\frac{(\sin\theta_R + \sin\theta_L)}{2}.$$

In other words, you can compensate for $\theta_i \neq 0$ by measuring $\sin\theta$ for the left and right diffracted beams and using the average.

This is all well and good, but it assumes you are measuring the diffracted beam with respect to the grating normal. If you are unable to perfectly align the incoming beam with the grating normal, how can you magically align your measuring scale with the grating normal? Fig. 3 shows the same situation as Fig. 2, but with the added (unknown) angle α between the normal to the grating and the zero of the angle measuring scale. The angles you measure are β_R and β_L. Clearly $\beta_R + \beta_L = \theta_R + \theta_L$. Unfortunately, you are not interested in the sum of the angles, but in the sum of *sines* of the angles. These quantites are related by

$$\sin\beta_R + \sin\beta_L = (\sin\theta_R + \sin\theta_L)\cos\alpha + sin\alpha(\cos\theta_R - \cos\theta_L).$$

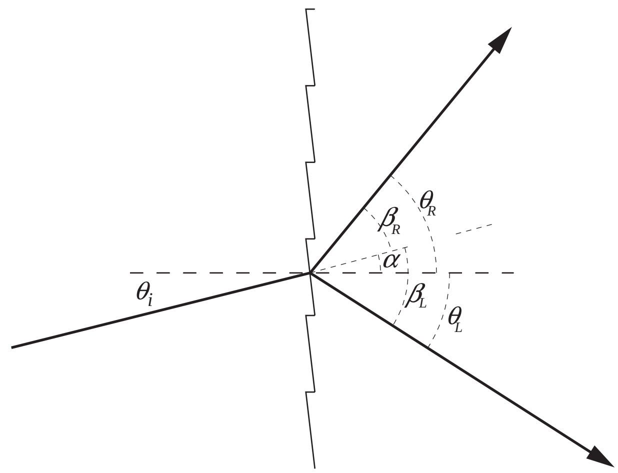


Figure 3

One equation and three unknowns. Pretty much impossible. The only solution is to align the spectrometer as carefully as possible and make both α and θ_i small. Practically speaking, if β_R and β_L are within a degree of each other, then

$$\sin\beta_R + \sin\beta_L = \sin\theta_R + \sin\theta_L$$

to with a few parts in ten thousand.

IV. Procedure

Unlike much of what we use for labs, these spectrometers are precision instruments. Play nice.

The following two parts can be done in either order, which is a good thing, since we don't have very many sodium lamps.

A. Determining the Grating Pitch

The yellow light from the sodium consists of two lines (a doublet) with wavelengths of 5889.953 Å and 5895.923 Å. The wavelengths of these lines are so well determined that they are considered a spectroscopic standard, and it is common practice to determine pitch (spacing between lines) on a diffraction grating in just this way.

Put the sodium lamp in front of the spectrometer entrance slit and turn it on. It will take three or four minutes to warm up completely. When fully warmed up, it will be bright enough for you to do this part of the lab with the lights on.

While you are waiting for the sodium lamp to warm up, check out the spectrometer: Look at the rotation scale and practice reading the vernier. Look through the eyepiece of the telescope at the crosshair. If you cannot get it into focus, try *gently* sliding the eyepiece in and out. When everything is aligned properly, *both* the crosshair and the entrance slit should be in focus at once. (If you are an eyeglass wearer, try this without your glasses on! Glasses and eyepieces are a bad mix, since the eyepiece is designed to be used with your eye quite close to it. If you are unable to get both the crosshair and the slit in focus, but a non-eyeglass wearer can, then you just have to try it with your glasses on. But unless your need for glasses is extremely severe, you will have better luck with them off.)

Determine the positions of sodium doublet lines in first and second order and from this determine the pitch d of your grating. (If you see one line instead of two, the width of your spectrometer's entrance slit is too wide. Try closing it down a bit.)

B. Determining the Rydberg Constant

Measure the wavelengths of the visible hydrogen lines. If you eyes are fully dark adapted, you *might* be able to see some or all in second order. Try waving your flashlight around to get just enough stray light in to see the crosshairs but not so much that you can't see the lines.

V. Analysis

Compare your measured wavelengths to the accpted values. Compute the Rydberg constant from your data and compare it to what you calculate from the Bohr theory. If you've been careful, you should be able to see that the correction for the finite nuclear mass is required.

Be sure to give estimates of your uncertainties and a discussion of how you arrived at them.

VI. Appendix: Aligning the Spectrometer

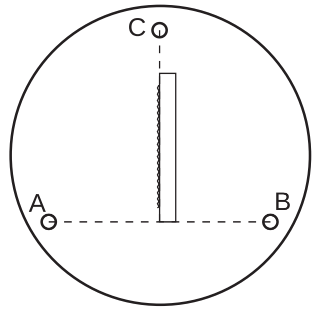
1. **Focus the telescope.** Move the eyepiece in and out until the crosshair is in crisp focus. Try to keep your eye relaxed when you do this. Aim the telescope perpendicular to the grating and turn on the reticle illuminator. Rotate the grating until the light from the illuminator is reflected back into the eyepiece. You should also see a "return" image of the crosshairs. They should be in focus too. If not, loosen the screw on the right side of the telescope body and slide the tube in and out until the return image is in focus and tighten the screw. Make sure the cross-hair is vertical.

   Can't get both things in focus? The crosshairs need to be moved. Turn the telescope so it points at an object as far away as possible. Focus on that object, ignoring the crosshairs. When the focus is as good as you can get it, adjust the eyepiece until the crosshairs are in focus again. If you had to pull the eyepice out, the crosshairs need to be moved towards the eyepiece. If you had to push it in, the crosshairs need to be moved away. You can push the crosshairs by taking off the objective or the eyepiece and gently tapping with some kind of tube with a large diameter. It needs to push on the ring that supports the crosshairs and not the crosshairs themselves (they will break).

2. **Focus the collimator** Illuminate the entrance slit to the spectrometer with e.g. a hydrogen spectral lamp. Set the telescope to $180^\circ\ 0'$ and lock it securely in position. If the slit is not in focus, loosen the small screw on the left of the collimator and move the tube in or out.

   If you can't get the slit in focus by this procedure, you need to move the whole collimator. There are two allen-type set screws on either side of the collimator, at the front. Loosen them until the whole tube can be moved in and out. Move the tube until the slit is in pretty good focus with the regular adjustment in the middle of its range. Then tighten the set screws so the slit is about vertical and roughly centered (we will come back to this). Make final focus adjustments with the sliding tube and tighten its little screw.

3. **Leveling the telescope** This is the most complicated step. The grating is mounted on a little table that rests on 3 screw points. Move the table around on the screw points until the grating is positioned as shown here:



The grating should be perpendicular to the line connecting two of the screws, and should lie along the perpendicular line out to the third screw.

Loosen the knurled set screw that allows the whole grating platform to rotate. Look through the telescope with the reticle illuminator on and rotate the grating platform until the return image of the crosshair is seen. (There may be two return images, one from each side of the glass plate. Try to guess which one corresponds to the side the grating is on.) Note the height of the horizontal crosshair in the return image relative to the crosshair itself. Now flip the grating table 180° until you see the return image again. Note the new height of the horizontal crosshair.

Pick screw A or B in the figure above and stick with that one only. Using only that screw, and continually flipping the grating by 180°, make adjustments until the return image of the horizontal crosshair is always at the same height relative to the crosshair itself. It may help to use the telescope leveling screw (located under the eyepiece) to bring the crosshair and its image nearer each other.

When you have got the grating adjusted so that crosshair image is always at the same height, the face of the grating is parallel to the rotation axis. That means the normal to the grating surface lies in the same plane as the telescope swings around in. Use the telescope leveling screw to actually align the horizontal crosshair and its retrun image.

4. **Setting the Grating Normal** Rotate the grating platform until the return image of the *vertical* crosshair lines up with the vertical crosshair and lock down the little knurled screw. (There may be more than one return image, as above.)

5. **Final slit adjustments** Using the two allen-head set screws at the front end of the collimator, center the slit in the field of view of the telescope and align it with the vertical crosshair. The collimator leveling screw (located under the slit) can also help here.

6. **Adjusting the Grating Tilt** This step works better with a bright spectral source, like a sodium lamp. Look at the slit directly with the telescope. Find some irregularity (dirt) along the edge of the slit and note its vertical distance from the horizontal crosshair. Unlock the telescope arm, swing around until the diffracted spectrum is visible in first or (better) second order. Adjust screw C on the grating platform until that irregularity is the same distance from the crosshair as it was in zeroth order.