

**Physics 17 Spring 2003**  
**The Stefan-Boltzmann Law**

Theory

The spectrum of radiation emitted by a solid material is a continuous spectrum, unlike the line spectrum emitted by the same material in gaseous form. The peak frequency of the emitted spectrum increases with the temperature of the solid body, as does the total power radiated. According to the Stefan-Boltzmann law, the total power radiated by an ideal emitter is proportional to the fourth power of the absolute temperature.

Consider an enclosure whose walls are maintained at a constant temperature  $T_2$ . If an object is suspended in the enclosure as shown in figure 1, regardless of its initial temperature  $T_1$ , it eventually comes to equilibrium at the same temperature as the walls  $T_2$ . If  $T_1 > T_2$ ,

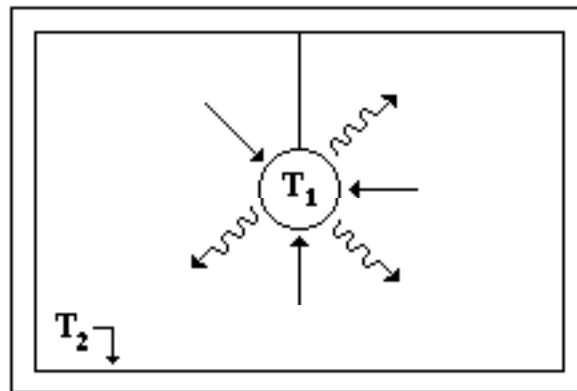


Figure 1

the suspended body must radiate energy in order to lower its temperature to  $T_2$ ; and if  $T_1 < T_2$ , the object must absorb energy in order to raise its temperature to  $T_2$ . Experimental measurements have shown that the rate at which energy is emitted depends on the material of the object, the condition of its surface, and on its temperature. Quantitatively,

$$R_e = e\sigma T_1^4 \quad , \quad (1)$$

where  $R_e$  is the rate at which energy is emitted per unit area,  $e$  is called the emissivity (a number between 0 and 1 depending on the material of which the object is made and on the temperature),  $\sigma$  is Stefan's constant ( $= 5.67 \times 10^{-8}$  watts/m<sup>2</sup>K<sup>4</sup>) and  $T_1$  is the Kelvin temperature of the body.

Equation (1) was first suggested by Josef Stefan and is called the Stefan-Boltzmann law. The rate of absorption also depends on the nature of the object and on the temperature of its surroundings,

$$R_a = e\sigma T_2^4 \quad , \quad (2)$$

where  $R_a$  is the rate at which energy is absorbed per unit area and  $T_2$  is the Kelvin temperature of

the surroundings. Thus

$$R_{\text{net}} = R_e - R_a = e\sigma T_1^4 - e\sigma T_2^4 = e\sigma(T_1^4 - T_2^4) \quad . \quad (3)$$

If  $T_1 > T_2$ , then the suspended body will be emitting energy faster than it is absorbing and its temperature will drop. If  $T_1 < T_2$ , the body will be absorbing faster than it is emitting and its temperature will rise. If  $T_1 = T_2$ , the body is absorbing and emitting at the same rate and it is in equilibrium with the enclosure. (Note that this requires the coefficients in equations (1) and (2) to be the same).

From equations (1), (2) and (3), we can see that a poor emitter is a poor absorber and that a good emitter is a good absorber. The best emitter then will be the best absorber, a body that absorbs all of the energy that reaches it. Since no incident energy is reflected from an ideal absorber, it would appear black (at  $T_1 = 0$ ) and hence is called a blackbody. For such a body  $e = 1$  and the rate at which it emits energy per unit area is given by

$$R_e = \sigma T_1^4 \quad . \quad (4)$$

Equation (4) is the Stefan-Boltzmann law for blackbodies. The study of radiation from a blackbody played an important role in the historical development of Planck's quantum theory.

No actual surface is ideally black, but a close approach is lampblack which reflects about 1% of the radiant energy reaching it. However, at very high temperatures, the rate of emission per unit area for many bodies approximates that of a blackbody and can be described by equation (4). In this experiment, we shall measure the rate at which energy is radiated from incandescent tungsten and compare it with the Stefan-Boltzmann law to see how closely it approximates a blackbody. Before beginning, two points should be noted:

1. Equation (4) gives the rate at which energy is radiated per unit area from the surface of a body. In this experiment, the incandescent tungsten is the tungsten filament in a small flashlight bulb. For such a bulb, we can measure the rate at which energy is radiated from the entire surface of the filament, but in order to determine  $R_e$  we would need to know the surface area of the tiny filament, which we cannot measure with the equipment available. Therefore, we rewrite equation (4) by multiplying both sides by the surface area:

$$P = A\sigma T^4 \quad , \quad (5)$$

where  $P$  is the rate at which energy is radiated from the entire surface (the power),  $A$  is the surface area of the filament and  $\sigma$  and  $T$  are defined as before. This is the form of the Stefan-Boltzmann law we will be using in this lab.

2. From equations (1) and (4), it might appear that  $R_e$  for a non-blackbody varies simply with the fourth power of the temperature as well as for a blackbody, the only difference being in the

coefficients. It must be emphasized that this is not true. The factor  $\epsilon$  in equation (1) is not a constant, but varies with temperature (since it depends on the frequency of the radiation). Only for a blackbody is there a simple direct proportion between  $R_e$  and  $T^4$ .

## References

The sections in Beiser's Concepts of Modern Physics which are pertinent to this lab are Chapter 9 sections 9.5 - 9.7. They should be read before coming to lab. Also read Halliday and Resnick's Physics sections 47-1 to 47.3 .

## Experimental Purpose

For a blackbody the relationship between the total power radiated and the Kelvin temperature is given by equation (5). If we take the log of both sides of that equation, we get

$$\log P = 4 \log T + \log A \quad . \quad (6)$$

Letting  $\log P = y$  ,  $\log T = x$  ,  $4 = a$  and  $\log A = b$  , we see that this is a linear equation of the form  $y = ax + b$ . Therefore, for a blackbody, a plot of  $\log P$  vs  $\log T$  will be a straight line with a slope of 4 . The purpose of this experiment is to see how closely the tungsten filament approximates a blackbody by measuring  $P$  and  $T$  for a number of voltage and current settings, plotting  $\log P$  vs  $\log T$  for those settings, and comparing the shape and slope of the experimental curve to that predicted above.

## Procedure

### Part A

In this part of the experiment, you will make the power vs temperature measurements for the tungsten filament bulb. The power radiated by the filament will be determined from the power input to the bulb by measuring the voltage and current at the terminals. Assuming that the small amount of heat loss down the filament supports may be neglected at high temperatures, the power radiated will equal the power supplied and we have

$$P = VI = A \epsilon T^4 \quad . \quad (7)$$

The temperature will be obtained by computing the resistance of the filament at the current-voltage readings used to determine the power radiated and then using a calibration table which lists resistivity vs temperature for tungsten.

1. Measure the room temperature resistance of the bulb on the General Radio impedance bridge.
- Your instructor will show you how to use the bridge. This is more accurate than the multimeter

and the measuring current is small enough not to heat the filament. (Why is it important not to heat the filament?)

- Using the two Simpson multimeters, the Hewlett-Packard 30 V dc power supply, and the bulb socket fixture provided, construct the circuit shown in figure 2. Be sure to connect the voltmeter and ammeter as shown since the voltmeter is more nearly ideal than the ammeter and a substantial voltage drop occurs across the ammeter.

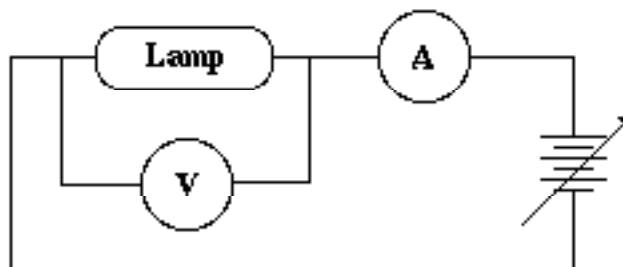


Figure 2

- Measure the voltage and current into the lamp at 0.1 V intervals starting at 0.1 V and going to 5.0 V. Note the current-voltage readings where the bulb begins to glow and where it is red, orange, yellow and white. [**Caution:** Do not exceed 5.0 V. Doing so may burn out the bulb. All of your measurements must be made on the same bulb, and so burning out a bulb midway through the experiment will necessitate starting over again.]
- Using equation (7), compute the power radiated by the filament for each of the current- voltage settings taken in step 3. Note: There is a computer program called STEFAN which can be used to do this and other calculations in this lab. It is stored on D1 in the P23 subcatalog of the public catalog PHYSLIB\*\*\* and can be accessed by typing:

RUN PHYSLIB\*\*\*:P23:STEFAN

- Determine the Kelvin temperature for each current-voltage reading using the calibration table on the next page. To use the table, you must know the resistivity of the bulb at the

Temperature (°K)	Resistivity (microhm-cm)
-----	-----
300	5.05
400	8.06
500	10.56
600	13.23
700	16.09
800	19.00
900	21.04

1000	24.93
1100	27.94
1200	30.98
1300	34.08
1400	37.19
1500	40.36
1600	43.55
1700	46.78
1800	50.05
1900	53.35
2000	56.67
2100	60.06
2200	63.48
2300	66.91
2400	70.39
2500	73.91
2600	77.49
2700	81.04
2800	84.70
2900	88.33
3000	92.04
3100	95.76
3200	99.54
3300	103.30
3400	107.20
3500	111.10
3600	115.00
3655	117.10

various current-voltage readings. The resistivity is related to the resistance by the relation

$$R = \frac{\rho l}{A} \quad , \quad (8)$$

where  $\rho$  is the resistivity,  $A$  is the cross-sectional area of the filament and  $l$  is the length of the filament. If we take the ratio of the resistance of the filament at a given current-voltage reading to the resistance of the bulb at room temperature and set it equal to the corresponding resistivity ratio, we get an expression from which the resistivity at a given current-voltage reading can be calculated:

$$\frac{R}{R_{RT}} = \frac{\frac{\sigma T^4}{A}}{\frac{\sigma_{RT} T_{RT}^4}{A}},$$

$$\sigma = \frac{\sigma_{RT} R}{R_{RT}} \quad (9)$$

$R$  can be calculated from the current-voltage reading using Ohm's law,  $R_{RT}$  was measured in step 1 and, assuming room temperature to be 300° K,  $\sigma_{RT}$  can be read directly off the table. Once  $\sigma$  is known,  $T$  can be obtained from the table, by interpolation.

- Plot  $P$  vs  $T$  on log-log paper. Note that plotting  $P$  vs  $T$  on log-log paper is the same as plotting  $\log P$  vs  $\log T$  on linear graph paper, but it eliminates the need to look up the logarithms. Make this plot using all the data points and repeat using only the last ten data points on an expanded scale. Compute the slope of each line.

## Part B

In part A we made the assumptions that

- the power radiated by the bulb equals the power input to the bulb; and
- the temperatures as determined from the table are correct.

In this part we will check those assumptions by using different methods to measure the power radiated and the temperature and then checking agreement between the values obtained in this part with those obtained in part A. If we obtain the same values using independent methods, then we can have more confidence that our assumptions are valid. Note that these procedures can be done in any order.

- Take your bulb to the setup provided with the optical pyrometer and measure the temperature of the bulb under three current-voltage readings used in Part A. Use data points near the maximum values used in part A. Your instructor will explain how the optical pyrometer works and how to use it. Compare these temperatures with the temperatures obtained from the resistances and calibration table.
- Take your bulb to the system consisting of the standard bulb, thermopile and Keithley nullmeter (see figure 3). The standard bulb has been calibrated by the National Bureau of Standards and it provides a known radiation intensity at a specified distance when operated at a given voltage. The calibration chart is taped to the table next to the nullmeter. The thermopile is a thermocouple device whose output is a dc signal proportional to the intensity of the radiant energy incident on it. It's output is fed into the Keithley nullmeter. The system is aligned so that the only source of radiation in line with the thermopile is the standard bulb. With the standard bulb off the Keithley can be zeroed so that background radiation is not registered.

Hence when the standard bulb is on, the thermopile only "sees" the standard bulb; and the meter reading on the nullmeter is proportional to the intensity of radiation from the standard bulb.

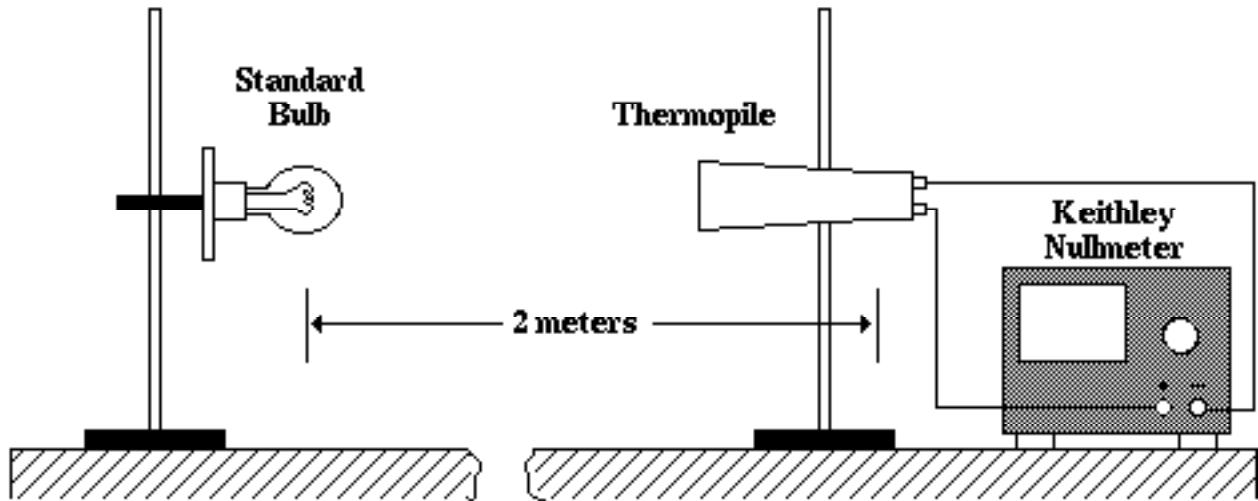


Figure 3

- a. Make sure the thermopile and standard bulb are in line and at the same height. The center of the standard bulb should be 2 meters from the receiving surface of the thermopile. Turn on the Keithley and set the range to 100-300 microvolts full scale. Zero the meter. Run the standard bulb at the highest voltage given on the calibration chart taped to the table beside the nullmeter. Allow about five minutes for the meter reading to stabilize. Record. Repeat for the other two voltages on the calibration chart. Using the radiation intensity information for the voltages given on the calibration chart and the meter readings you just took for those voltages, construct a graph of radiation flux vs meter readings. With this graph, the radiation intensity from any source can be found once the meter reading it causes is known.
- b. Replace the standard bulb with your bulb. Place your bulb 20-25 cm from the thermopile. Record the exact distance between your bulb and the receiving surface of the thermopile. Using the first current-voltage pair used with the optical pyrometer, record the nullmeter meter reading. Using the calibration graph you just made, determine the radiation intensity from your bulb. Assuming that the radiation intensity is proportional to  $1/r^2$ , compute the total power radiated by your bulb at that current-voltage reading. Compare this with input power  $P = VI$ .
- c. Repeat step b using the other two current-voltage pairs used with the optical pyrometer.

### Part C (optional)

1. Attached to the end of this writeup is an article written by John W. Dewdney in which he describes the heat loss from a hot filament. According to Dewdney, the actual power vs temperature relation for a hot filament is

$$P = aT^4 + bT + c$$

where  $a$ ,  $b$ , and  $c$  are constants ( $c < 0$ ). Using the program STEFAN, determine the simplest equation describing the power output of your bulb as a function of temperature. Specifically, see if you can find the coefficients  $a$ ,  $b$ , and  $c$  which best fits your experimental data.

### Lab Report

Follow the usual lab notebook format. Your lab report should include the answers to all of the questions asked in the introduction or procedure, all raw and derived data, and an estimate of the magnitude and sources of error in any data recorded. When answering any question or when giving any comparison or explanation, always refer to specific data to support your statements. (Remember that if you use the computer program to do your calculations, then you must show a sample calculation of each computation done by the program to show that you know what the program is doing for you.) Be sure to include the following items in your report.

#### 1. Part A

- a. a table containing the current-voltage measurements with comments on the temperatures at which the filament had various colors;
- b. samples of all computations done;
- c. a table summarizing the final results - power radiated vs temperature;
- d. the  $\log P$  vs  $\log T$  plots with the slopes computed;

#### 2. Part B

- a. a table containing the current-voltage pairs used with the optical pyrometer and the filament temperatures as measured by the optical pyrometer and the resistivity table;
- b. the calibration graph made with the thermopile-standard bulb system;
- c. a table containing the current-voltage pairs used with the thermopile nullmeter system, the corresponding nullmeter readings, the radiation intensity obtained from the calibration graph, the total power radiated as computed using the amplifier readings and the power input to the bulb;

- d. samples of all calculations done;

#### 3. Part C (optional)

- a. the computer generated plots showing the best fit between Dewdney's equation and your

experimental data;

- b. the values of the coefficients  $a$ ,  $b$  and  $c$  in Dewdney's equation as determined by your computer modeling; and
  - c. a discussion of the physical significance of the coefficients and a justification of the values you chose;
4. a discussion of how closely the incandescent tungsten filament approximates a blackbody.
  5. a listing of the sources of error in the lab with an estimate of their magnitude and effect on the final results.