Physics 3 Summer 1990
Lab 4 - Energy Losses on an Inclined Air Track

Theory

Consider a car on an air track which is inclined at an angle θ to the horizontal as shown in figure 1. If the car is held at rest a known distance x_o from the bottom of the track and released, it will accelerate down the track, collide with the bumper and rebound up the track to a height x_1 that is smaller than the original distance x_o . The car will then accelerate back down the track to collide with the bumper again. This process will continue with the maximum distance x_n the car

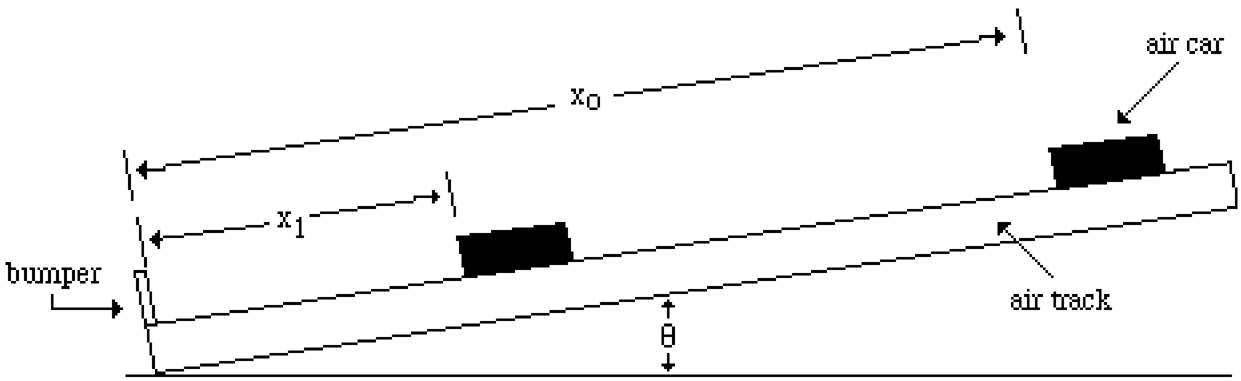

Figure 1

travels up the track growing smaller with each successive rebound until the car eventually comes to rest against the bumper at the bottom of the track. A few simple quantitative measurements shows that the differences between successive x_n's grows smaller in a nonlinear manner.

It is obvious from these observations that there is some mechanism by which energy is removed from the system. One possible mechanism is that the forces which act on the car as it travels up and down the track cause the energy losses. On close inspection, however, this seems unlikely. Since the car is riding on a cushion of air, friction between the car and the surface of the track is essentially eliminated. In addition, if θ is kept small, the speed of the car will be small. This, along with the fact that the car presents very little surface area perpendicular to the direction of motion, means that air resistance is also very small. In essence then, the system is frictionless and the only force acting on the car is gravity. Gravity, however, is a conservative force. Energy cannot be lost by its action.

This leaves only one likely possibility - the energy losses occur in the interaction between the car and the bumper. If that is the case, we could explain the behavior of the system in the following way. When the car is held at rest at x_o , it has potential energy mgh_o (measured with the level of the bumper as the reference level). When released, it is acted upon by gravity and experiences an acceleration down the track of magnitude $g\sin\theta$. As it accelerates, its potential energy is changed into kinetic energy. Since there are no energy losses on the way down, the car's kinetic energy when it reaches the bumper is equal to its initial potential energy, $.5mv^2 = mgh_o$. Up to this point, mechanical energy has been conserved. During the collision, however, a small amount of energy is removed. (How?) The car leaves the bumper and travels back up the track. Its kinetic energy is changed back into potential energy until the car again comes to rest with all its

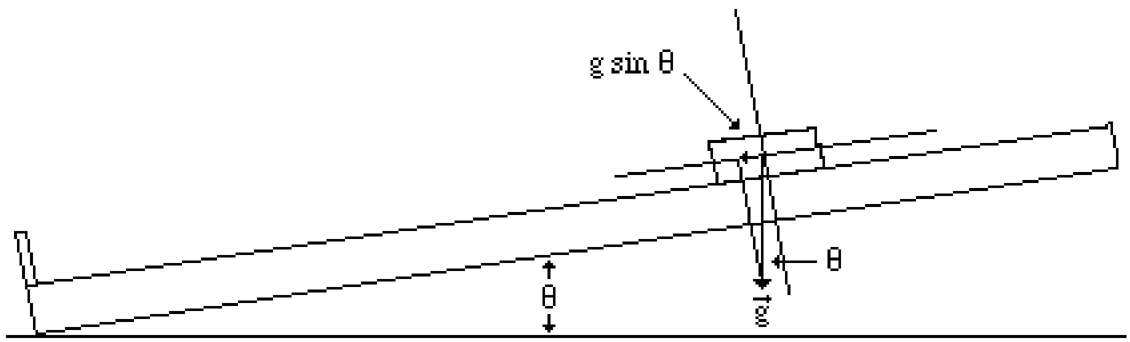

Figure 2

energy being potential. Due to the energy loss in the collision, however, the new potential energy is smaller than the original value and hence the car does not rise as high as the original height. Gravity now begins to accelerate the car back down the track and the process repeats itself until all the initial energy has been removed and the car rests against the bumper. Since the same process is removing the energy during each collision and since the differences between successive x_n's grows smaller in a nonlinear manner, it is probable that the same relative amount of energy is removed during each collision.

At this point we are faced with a situation common to scientific research. Once the behavior of a system is known and think we know why that behavior occurs, how do we find out if we are correct? How do we know if our ideas correspond to reality? One common approach to answering these questions is to develop a mathematical theory based on those ideas that allows us to make predictions about specific details of the behavior of the system. We can then go into the laboratory, make the necessary measurements and check to see if our predictions are accurate. If they are, then we will have good reason to believe that our ideas are correct. If they are not, then we will know that our ideas need to be modified or rejected. We will use this method in this lab.

Let h_o and x_o be respectively the height of the car above the bumper and the distance from the bottom of the track to the car at time $t = 0$ (see figure 3); let h_n be the maximum height the car attains above the bumper after the nth bounce; and similarly, let x_n be the maximum distance up the track from the bumper that the car travels after the nth bounce. From similar triangles we see that

$$\frac{x_n}{x_o} = \frac{h_n}{h_o}$$

This relationship allows us to write heights (and therefore potential energies) in terms of distances up the track. This is important only because it is experimentally easier to measure distances up the track than to measure heights above the bumper. Therefore, in what follows we will express all heights in terms of distances up the track.

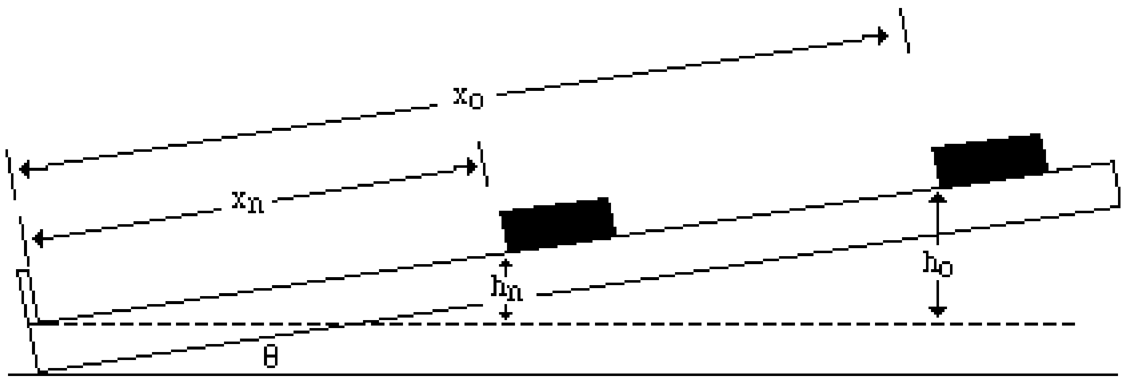

Figure 3

If the energy losses do occur only during the interaction between the car and the bumper, then for each trip up the track we should be able to write

$$\frac{1}{2}mv_n^2 = mgh_n = mgx_n\sin\theta \qquad (1)$$

where v_n is the velocity with which the car leaves the bumper after the nth bounce. Canceling terms and solving for v_n^2, yields

$$v_n^2 = 2x_ng\sin\theta \qquad . \quad (2)$$

If gravity is the only force acting on the car, then the motion of the car up the track is an example of uniformly accelerated motion and we can write

$$0 = v_n + t_ng\sin\theta$$

or

$$0 = v_n + t_ng\sin\theta \qquad (3)$$

where v_n is defined as above and t_n is the time it takes for the car to reach its maximum distance up the track after it leaves the bumper on the nth bounce. Substituting this into equation (2) and solving for t_n^2, yields

$$t_n^2 = \left(\frac{2}{g\sin\theta}\right)x_n \qquad . \quad (4)$$

If we let $t_n^2 = y$, $x_n = x$ and $2/(g\sin\theta) = a$, then we see that equation (4) is a linear equation of the form $y = ax$. If plotted, a graph of t_n^2 vs x_n would be a straight line with slope $2/(g\sin\theta)$ intersecting the y axis at the origin. It should be emphasized that this will be true only if the assumption used in its derivation is true, namely, that the energy losses take place only during the interaction between the car and the bumper. θ, x_n and t_n are easily measurable quantities. Therefore, we have a way to check our first assumption.

To get a similar expression for the assumption that the same relative amount of energy is removed during each collision we reason as follows. The maximum potential energy after the nth bounce is

$$E_n = mgh_n = mgx_n \sin\theta \quad . \qquad (5)$$

If the same relative amount of energy is removed during each collision, then the ratio E_{n+1}/E_n should have a constant value and should be equal to

$$\frac{E_{n+1}}{E_n} = \frac{mgx_{n+1}\sin\theta}{mgx_n \sin\theta} = \frac{x_{n+1}}{x_n} \quad . \qquad (6)$$

Letting this ratio equal α gives

$$\frac{x_{n+1}}{x_n} = \alpha \quad , \ (\alpha \leq 1) \quad . \qquad (7)$$

Taking the logarithm of both sides of equation (7), yields

$$\log\left(\frac{x_{n+1}}{x_n}\right) = \log(\alpha)$$

or

$$\log(x_{n+1}) - \log(x_n) = \log(\alpha) \quad .$$

Since α is constant, the difference between successive x's is constant and we can write

$$\log(x_n) - \log(x_o) = n \log(\alpha)$$

or

$$\log(x_n) = \log(\alpha)\, n + \log(x_o) \quad . \qquad (8)$$

If we consider $\log(x_n)$ and n to be variables and $\log(\alpha)$ and $\log(x_o)$ constants, we see we have another linear equation with the general form $y = ax + b$. If our second assumption is correct, a plot of $\log(x_n)$ vs n should be a straight line.

References

The following are the sections in Halliday and Resnick's Fundamentals of Physics 3rd edition which are pertinent to this lab. This material should be read before coming to lab.

1. Chapter 7 sections 7-1 through 7-5
2. Chapter 8 sections 8-1 through 8-5 and 8-8

Experimental Purpose

The specific experimental reasons for doing this lab are to test the following two assumptions about the mechanism of energy losses in our air track-car system.

1. The air track-car system is essentially a frictionless system. All of the energy losses occur in the interaction between the car and the bumper.

2. The same relative amount of energy is removed from the car during each interaction between the car and the bumper.

If the first assumption is valid, then a plot of t_n^2 vs x_n should be a straight line with slope equal to $2/(g \sin \theta)$ intersecting the y axis at the origin. If the second assumption is valid, then a plot of $\log(x_n)$ vs n should be a straight line with a y intercept equal to $\log(x_o)$.

The more general (and primary) purpose is to illustrate the relationship between theory and experiment in scientific research and the process that defines that relationship.

Procedure

Air tracks are delicate, expensive pieces of equipment. Therefore, before beginning the lab a few words on their care and use is appropriate. From the operating manual supplied with the air tracks:

> "Caution: Air tracks and cars operate best if they are clean and smooth. Use care in handling them. They will dent easily. Avoid high velocity impacts."

Some further suggestions:

a. keep your hands off the track as much as possible (skin oil traps dust);

b. make certain there are no cars on the track before turning on or off the air supply; and

c. get a feeling for the way in which the car behaves by trying a number of simple experiments such as:

    1. noting the effects of the track's air jets on the ends of the air car; and

    2. noting that an individual car may "float" better when facing one way on the track than when facing the other way, etc.

1. First the track must be leveled. This is done by adjusting the three thumbscrews on the bottom of the supporting beam, the three that support the beam on the bench. (Do NOT loosen the

machine screw and nut combinations which hold the track itself and the beam together. These have been adjusted so that the beam is straight. If you believe they need adjustment, ask your instructor for assistance.) The car and track together form a self-leveling device. Turn on the air supply and adjust the thumbscrews until the car stays (nearly) still at all points along the track. The system is so sensitive that it is difficult not to have the cars accelerate somewhat. It is only important that the acceleration be negligible compared to when the track is deliberately tilted, as explained in the next paragraph.

To incline the track at a known angle, select a brass spacer and measure its thickness with a vernier caliper. Measure the perpendicular distance between the single support at one end of the beam and the line joining the two supports at the other end of the beam. The angle formed when the spacer is placed under the single support can be calculated from the ratio of the spacer thickness to this distance. In this experiment, measurements will be done for two angles using spacers between 0.35 cm and 1.75 cm. Measure the thicknesses of all of the brass spacers.

The black ruled band along the front of the air track is movable. Where should the zero mark on the band be positioned with respect to the bumper? Position the the black ruled band to the appropriate spot.

2. Before beginning the actual measurements, get a feeling for the timing aspect of this experiment by doing the following.

   a. Start the stopwatch and then, as rapidly as you can, stop it. How much time elapsed between the starting and stopping of the stopwatch? This should give you some idea of the error introduced into the experiment by your reaction time.

   b. For each of the following, place the car at the far end of the track and release. Use the large spacer for this part.

      1. Measure (t_3, x_3). Use equation (4) to compute a value for g .

      2. Measure ($2t_3$, x_3) where $2t_3$ is the time for the round trip for the third bounce. Use equation (4) to compute a value for g .

      3. Measure ($2t_1$, x_1) and ($2t_{10}$, x_{10}). Use equation (4) to compute a value for g for each of these data pairs.

   Compare the two values for g computed in steps b1 and b2 above . Which method do you think is more accurate? Why? Compare the two values for g computed in step b3. What conclusions regarding the error introduced by timing do you draw from this comparison?

3. Keep the large spacer under the single support. Place the car at the far end of the track and release. Using the timing method you feel to be more accurate, measure t_n and x_n for $n = 1$, 3, 5, 7 and 9. Since there may be some random variations in t_n and x_n , repeat the process a

minimum of 5 times. Compute average values for t_n and x_n. Using these average values, plot t_n^2 vs x_n and compute the slope. From the slope, compute a value for g. (Note: There is a computer program called E_LOSSES in the P3 subcatalog of PHYSLIB*** which will graph the data and do a number of the required calculations. As usual, if you use the program, you are required to include a sample calculation of each computation done to show that you understand the physics of the situation and to show that you understand what the computer program is doing.)

4. Using the average values of x_n, plot x_n vs n on semilog paper. (What is the purpose of the semilog paper? Why don't we plot $\log(x_n)$ vs n on the semilog paper? Could the data be plotted on rectangular coordinate graph paper? If so, how?) Compute α. Using α, estimate the number of bounces it should take for the car to loose half its initial energy. Experimentally check your estimates.

5. Repeat procedures 3 and 4 using the small spacer.

6. Repeat procedures 3 and 4 using the large spacer and the two small spacers.

7. Optional: Devise some way to further test the theory. For example, tape some large flat object to the front of the air car to increase the air friction, and make some predictions as to how the data will be altered. Test your ideas.

Lab Report

Follow the usual lab notebook format. Your lab report should include the answers to all of the questions asked in the introduction or procedure, all raw and derived data, and an estimate of the magnitude and sources of error in any data recorded. When answering any question or when giving any comparison or explanation, always refer to specific data to support your statements. For this lab also include the following:

1. the t_n^2 vs x_n plots for the three angles with the values of g computed from their slopes - one of the plots must be done by hand, the other two may be computer generated;

2. the semi-log plots of x_n vs n with a computation of α for the three angles - one of the plots must be done by hand, the other two may be computer generated;

3. the estimate for the number of bounces needed to remove one half the energy from the car with an explanation of how you came to that estimate and the experimental number of bounces needed to remove half the energy;

4. a discussion of the agreement between theory and experiment including:

   a. an explanation of whether or not theory needs to be modified or rejected with reasons to support your explanation;

b. explanations for any deviations from theory; and

c. suggestions for a new theory (if necessary) and a way it might be tested;

5. Optional: Write a short computer program to model the action of the car on the air track; its output might be a table listing the information pertinent to the experiment such as x_n , the energy ratio, the energy lost during each collision etc, or it might graphically represent the theoretical and experimental motion of the car.

6. Optional: One possible energy conversion in the interaction of the car with the bumper is from kinetic energy to heat; if this is the case, compute the amount of heat that would be generated in the bumper from the time the car first strikes the bumper until it finally comes to rest against the bumper; how many degrees would this raise the temperature of one cc of water? ; assuming the bumper to be made of steel, how many degrees would the temperature of the steel rise if it absorbed all of the energy from the car without losing any to its surroundings?