Understanding the Universe: Physics Through the Ages

Laboratory 1
Galileo's Inclined Planes and the Laws of Motion

A. Introduction

By the time Galileo wrote his last major treatise, *Two New Sciences* (1638), he had long accepted the "time squared law" for freely falling bodies:

$$\mathbf{d = 1/2at^2} \qquad [1]$$

This law states that the distance travelled by a freely falling body is proportional to the elapsed time of fall squared.

In the *Two New Sciences*, Galileo does not describe how he came to formulate this law. Instead he discusses the law as an example of how to study motion: first, one describes a problem with mathematics, and then one turns to physical experiment to justify the mathematical solution. Here is a summary of his argument:

Galileo wants to claim that the natural motion of a freely falling body is such that it will acquire equal increments of velocity in equal times (so that if times increase as 1, 2, 3, etc. seconds, velocities will increase as 1, 2, 3 units of length/sec). Yet practical problems make it difficult to put such a claim to any experimental test. First, it is nearly impossible to measure a freely falling body because of the quickness of its motion. Second, one cannot measure continuously-changing velocities directly, and in free fall the velocity does change continuously. To deal with the first problem, Galileo replaced a ball in free fall with one rolling down a gently inclined plane. "In order to make use of motions as slow as possible ... I also thought of making moveables descend along an inclined plane not much raised above the horizontal" (TNS, 87). The second problem was trickier. Since he could not measure changing velocities, Galileo demonstrated the mathematical equivalence of a relation involving only **times and distances** (both of which he could measure) to his initial relation involving **times and velocities**. He then performed an experiment with an inclined plane to demonstrate the former, i.e., to confirm the relation we now call the "time squared law" (Eq. [1]).

Note: See Glashow, *From Alchemy to Quarks*, pp. 59-62, for a discussion of the meaning of velocity and acceleration. Velocity is the rate over time at which position changes (m/sec); acceleration the rate over time at which velocity changes (m/sec/sec).

In this lab, you will repeat Galileo's experiment, as he described it in his *Two New Sciences*. Galileo wrote this book in the form of a dialogue. Simplicio (the slightly stupid Aristotelian who loses all the arguments) has managed to follow the proof of Sagredo (the open-minded discussant who usually sides with Galileo) that the time-squared law is mathematically equivalent to saying velocities are proportional to times. But now Simplicio wants to know whether nature actually employs uniform acceleration in free fall. Salviati (who speaks for Galileo) then describes the experiment. Note that Galileo's unit of length, the "braccio," is about 0.6 meters, and that “our Author” refers to Galileo.

Galileo, *Two new sciences*, tranl. Stillman Drake (Madison: University of Wisconsin Press, 1974), pp. 169-70:

"**Simplicio**: Really I have taken more pleasure from this simple and clear reasoning of Sagredo's than from the (for me) more obscure demonstration of the Author, so that I am better able to see why the matter must proceed in this way, once the definition of uniformly accelerated motion has been postulated and accepted. But I am still doubtful whether this is the acceleration employed by nature in the motion of her falling heavy bodies. Hence, for my understanding and for that of other people like me, I think that it would be suitable at this place [for you] to adduce some experiment from those (of which you have said that there are many) that agree in various cases with the demonstrated conclusions.

Salviati: Like a true scientist, you make a very reasonable demand, for this is usual and necessary in those sciences which apply mathematical demonstrations to physical conclusions, as may be seen among writers on optics, astronomers, mechanics, musicians, and others who confirm their principles with sensory experience that are the foundations of all the resulting structure. I do not want to have it appear a waste of time on our part, [as] if we had reasoned at excessive length about this first and chief foundation upon which rests an immense framework of infinitely many conclusions--of which we have only a tiny part put down in this book by the Author, who will have gone far to open the entrance and portal that has until now been closed to speculative minds. Therefore as to the experiments: the Author has not failed to make them, and in order to be assured that the acceleration of heavy bodies falling naturally does follow the ratio expounded above, I have often made the test in the following manner, and in his company.

In a wooden beam or rafter about twelve braccia long, half a braccio wide, and three inches thick, a channel was rabbeted in along the narrowest dimension, a little over an inch wide and made very straight; so that this would be clean and smooth, there was glued within it a piece of vellum, as much smoothed and cleaned as possible. In this there was made to descend a very hard bronze ball, well rounded and polished, the beam having been tilted by elevating one end of it above the horizontal plane from one to two braccia, at will. As I said, the ball was allowed to descend along the said groove, and we noted (in the manner I shall presently tell you) the time that it consumed in running all the way, repeating the same process many times, in order to be quite sure as to the amount of time, in which we never found a difference of even the tenth part of a pulse-beat.

This operation being precisely established, we made the same ball descend only one-quarter the length of this channel, and the time of its descent being measured, this was found to be precisely one-half the other. Next making the experiment for other lengths, examining now the time for the whole length [in comparison] with the time of one-half, or with that of two-thirds, or of three-quarters, and finally with any other division, by experiments repeated a full hundred times, **the spaces were always found to be to one another as the squares of the times**. And this [held] for all inclinations of the plane: that is, of the channel in which the ball was made to descend, where we observed also that the times of descent for diverse inclinations maintained among themselves accurately that ratio that we shall find later assigned and demonstrated by our Author.

As to the measure of time, we had a large pail filled with water and fastened from above, which had a slender tube affixed to its bottom, through which a narrow thread of water ran; this was received in a little beaker during the entire time that the ball descended along the channel or parts of it. The little amounts of water collected in this way were weighed from time to time on a delicate balance, the differences and ratios of the weights giving us the differences and ratios of the times, and with such precision that, as I have said, these operations repeated time and time again never differed by any notable amount.

Simplicio: It would have given me great satisfaction to have been present at these experiments. But being certain of your diligence in making them and your fidelity in relating them, I am content to assume them as most certain and true."

B. Measuring a Falling Body

1) **Goals**:

Galileo claims that the distance (**d**) traveled by a body falling from rest in time (**t**) is proportional to t^2. We now write this relation:

$$d = 1/2at^2$$

Test this first by measuring the time required for a ball to roll various distances down an inclined plane, and determine the value of the acceleration (**a**) at two different degrees of incline. Then test the relation with the modern (electronic) apparatus of the spark timer for an object in free fall.

2) **The inclined plane measurement**:

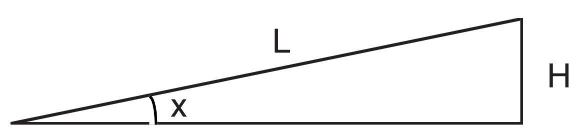
Step 1) As described by Galileo, you will use an inclined grooved track to slow down the motion in free fall, and will use the flow of water to measure time (remember that Galileo had no stop watch or electronic timer at his disposal!). Practice using the timing apparatus several times, by measuring repeatedly the time required for the ball to roll a given length. Open the water valve as you release the ball, and close the value as the ball passes the marked length. After you have weighed your sample of water (in grams), pour it back into the container. **How consistent are your measurements? Why should you pour the water back into the container after each measurement? How does the repeatability of your measurements compare with Galileo's claim for the consistency of his apparatus?**

Step 2) Determine the degree of inclination of your track. Measure the differences in height of each end of the track (**H**) and its total length (**L**). The ratio of **H/L** is the sine of the angle of incline (**x**). Set the inclination to around 5° for your first measurements.

$$SIN(x) = H/L$$

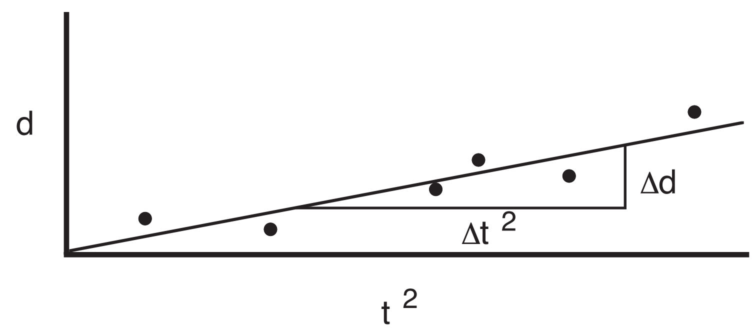
$$x = ASIN(H/L) \qquad [2]$$

You will need a calculator to determine **x**, using the ASIN function.



Step 3) Using a ball released from rest, construct a table of distances traveled (**d**, measured in cm) and corresponding values of elapsed times (**t,** measured in grams of water). Measure at least half a dozen pairs of values, for distances ranging over the full extent of your track. Then add a third column to your table for values of t^2. Construct a graph of d (on the vertical scale) versus t^2 (on the horizontal scale), and draw the straight line which most nearly passes through the resulting points (and not necessarily through the origin). Label the graph with the slope of the track you measured in Step 2 above.

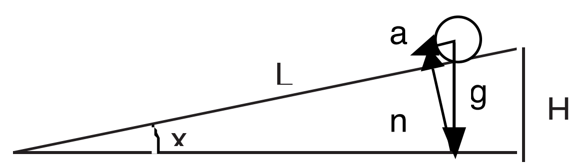
Step 4) Determine the acceleration down the track, **a**, obtained in your measurement. Using your graph from Step 3, determine the slope of your line by dividing an arbitrary change in the vertical component (**Δd**, in cm) by the corresponding change in the horizontal component (**Δt²**, in "gr of water"²). The acceleration **a** is twice the slope, as seen from the time-squared law above [1]. (Remember that for y=mx + b, m gives the slope and b the y-intercept.)



Your value for **a** will be in the arbitrary units of cm/(gr of water)².

Step 5) Repeat Steps 2-4 for another angle of inclination for your plane, i.e., with the elevated end of your track about twice as high.

Step 6) Determine the theoretical value for the acceleration due to gravity (**g**), i.e., the vertical acceleration due to gravity. The acceleration **a** along the inclined plane should equal the component (or projection) of **g** along the plane. The normal force **n** pushes up on the ball at right angles to the surface down which the ball is rolling.



Since the triangle containing **a** and **g** is similar (i.e., has the same shape) to that containing **H** and **L** **(do you see why?),** it follows that:

$$\mathbf{a/g = H/L}$$

or

$$\mathbf{g = a(L/H)} \qquad [4]$$

Use [4] to determine a theoretical value for the vertical acceleration due to gravity, **g**, for each of your two sets of data (**a**, **L**, **H**) as determined in Steps 2-5. **How well do your values of g agree with each other?**

3) **The spark-timer experiment**:

This device measures time by emitting a spark at regularly-spaced intervals. Unlike Galileo's technology, this device can mark such short intervals (60 sparks/sec) that it can be used to measure free fall directly; no inclined plane is required to slow down the motion.

Step 7) Place about 80 cm of tape through the timer, and let it fall. Each marked interval records the distance traveled in the same time (as mentioned in the lecture, Galileo used music and an inclined plane to mark such values). Measure the total distance traveled (since the beginning) at each spark, and record your results in a table of **d**, **t** and $\mathbf{t^2}$, as above (Step 3).

Step 8) Compute the acceleration, **a**, as above (Step 4). **How should this value a compare with the value of g you computed above (Step 6)?**

Step 9) **How might you be able to convert your value for g, measured with the water clock, into the units obtained in your measurement with the spark timer?**

4) **Reporting your results**

Step 10) Write up the results of your measurements. Be sure to include all the tables and graphs of your results, your responses to the questions posed above in bold font, and a discussion of any particular problems you encountered. **What are the differences between your apparatus and procedures and those described by Galileo? Do you think those differences make your results less or more accurate than Galileo's?**

[end]