Rotational Motion

§1 Purpose

The main purpose of this laboratory is to familiarize you with the use of the Torsional Harmonic Oscillator (THO) that will be the subject of the final lab of the course on damped and driven harmonic oscillators. This exercise should also reinforce your understanding of rotational kinematics and dynamics.

Some parts of this lab (in particular, some of the calculations) should be done outside of lab. Those parts have been marked “Homework” in the write-up below. Read through those parts anyway, as you will sometimes need the results of the calculations during the lab itself.

![Figure 1: Torsional harmonic oscillator. The locations of the rotor and fiber are indicated.](image)

§2 Introduction to Torsional Harmonic Motion

While a “torsional harmonic oscillator” may sound like something very complex, it is really nothing more than the rotational analog of a mass on a spring. For a mass/spring system, the relationship between the displacement $\Delta x$ of the mass from equilibrium and the force $F$ exerted on it by the spring is given by Hooke’s law

$$F = -k \Delta x$$
where $k$ is the spring constant with units of N/m.

When the mass is displaced a distance $x_A$ from equilibrium and released, it will oscillate around its equilibrium position. To perform this lab, you will need to know just a little of the mathematical description of this motion. We will discuss these results in more detail soon; in the meantime, we give them without proof. These basic results will be familiar to many of you in any case.

Mathematically, this oscillation of the mass about equilibrium can be described by a cosine function

$$\Delta x(t) = x_A \cos \omega_0 t$$

where $\omega_0$ is the characteristic angular frequency of the mass/spring system and $t = 0$ is chosen to be the moment when the mass is released. The characteristic frequency $\omega_0$ is given in terms of the mass $m$ and spring constant $k$ by

$$\omega_0 = \sqrt{\frac{k}{m}}$$

A torsional oscillator is completely analogous to the mass/spring oscillator, except that instead of looking at the linear displacement $x$ of a mass we look at the angular displacement $\Delta \theta$ of a mass that can rotate around a fixed axis. This is best seen in the picture of the THO you will use in this laboratory shown in Fig. 1.

The mass in your THO consists primarily of a copper rotor (the flat, thin copper disk with the scale bar on it). There are also an aluminum spindle, some magnets, and a few other things that are not so important dynamically. The analog of the spring is a torsional fiber (really just a piece of steel piano wire) that runs from top to bottom in your THO. The rotor is clamped to the fiber so that when you rotate the rotor an angle $\Delta \theta$ from its equilibrium position, the fiber will exert a restoring torque $\tau$ that opposes the angular displacement. The relationship between displacement $\Delta \theta$ and torque $\tau$ is given by the angular version of Hooke’s law

$$\tau = -\kappa \Delta \theta$$

where the torsional constant $\kappa$ (with units N · m) plays the role of the spring constant $k$.

If you give the rotor an angular displacement $\theta_A$ from equilibrium and release it, it will oscillate around equilibrium by rotating first one way and then the other. Its angular position is described by a cosine function

$$\Delta \theta(t) = \theta_A \cos \omega_0 t.$$  

The characteristic frequency $\omega_0$ is given in terms of the moment of inertia $I_0$ (in kg · m²) of the rotor/spindle assembly and torsional constant $\kappa$ of the fiber by

$$\omega_0 = \sqrt{\frac{\kappa}{I_0}},$$

in exact analogy to the expression for $\omega_0$ in a mass-spring system.

§ 3 Overview of the Experiment

For this experiment you need three main pieces of equipment, as shown in Fig. 2.

1. A torsional harmonic oscillator and its accessories.

2. A digital storage oscilloscope. This write up assumes you are using a Tektronix 2012B scope; a picture of the scope and its controls is shown in Fig. 5 in the Appendix.
3. A digital multimeter.

The angular displacement output of your THO (leftmost coaxial output on the front panel) will be connected either to the digital multimeter or to the CH 1 input of your scope. The angular velocity output of your THO (rightmost coaxial output on the front panel) will be connected to the CH2 input of your scope.

![Image of experiment setup](image)

Figure 2: Setup for the rotational motion lab. The scope is used to monitor the angular displacement and angular velocity outputs. The multimeter is used to obtain a digital readout of the angular displacement sensor.

The experiment will consist of two parts:

1. Learning about the parts and operation of the THO and calibration of the sensors; discussed in §4.

2. Static measurement of the $\tau$-$\Delta \theta$ curve; discussed in §5.

Next week, we’ll let the rotor oscillate, measure the frequency, and explore what happens when we apply a controlled drag force.

Note to the TA: Both inputs on the scope should be DC coupled. The scope should be set to straight sampling mode (no averaging), triggering off CH 1, rising edge, auto. The scope measurement functions should be set to amplitude and frequency for both CH 1 and CH 2.
§4 Introduction to the THO

A detailed view of the most important components of the THO is shown in Fig. 3. It has lots of bits and pieces, but isn’t too hard to understand. The following steps will walk you through operation of the instrument.

1. You’ll be working with the same apparatus next week, and you’ll get better results if you use the exact same setup as you use this week. So, before doing anything else, write down the serial number of the box you’re using. It’s printed on a label located at the back of the wooden box that holds the apparatus.

2. First, take a look at the copper rotor. On either side of the rotor are a set of magnetic dampers (brakes) which can be moved in and out from around your rotor. For now they should be all the way out. Note also the scale bar on the rotor; this can be used to measure the rotor’s angular displacement in radians. The equilibrium position should be at a position of about 3.0 on your rotor’s scale bar. Finally, lower down on the fiber is a clamp; this provides a convenient means of using your hand to provide a torque to the fiber and rotor.

3. Using the clamp, try displacing the rotor through some fixed angle (about 1.0 radian would do nicely) and letting go. The rotor should begin to oscillate smoothly around its equilibrium position, with an amplitude that decays only very slowly in time.

4. You can also pump the rotor by twisting the clamp at the natural frequency of the rotor. You’ll find that you do this without thinking, much like you automatically push a person on a swing at the right frequency. Note: Be careful when pumping the rotor to keep the
amplitude to around 1.0 radians to avoid damage to the fiber. It’s easy to pump the rotor, so be gentle!

5. The output of the angular position sensor should be connected to the CH1 input of your scope. If it’s not, attach it now; see Figs. 2 and 5. Look at the output of the angular position sensor on the oscilloscope. You should see a clean sinusoidal pattern on the scope. (For this measurement you can turn the scope’s CH 2 input off by pressing the blue CH 2 Menu button a couple of times.) Try playing around with the CH 1 voltage knob and the time scale knob to find reasonable settings. Typical values are 200 mV to 1 V/div for CH1 and 0.5 to 2.5 s/div for the time base.

6. Now look at the output of the angular velocity sensor. (Attach this output to CH 2 on the scope if it’s not already connected; see Fig. 2 and 5.) You can turn CH 2 back on by pressing the CH 2 menu button, and turn CH 1 off by pressing its menu button. Play around with the voltage scale knobs for CH 2 to find a convenient scale; 50 to 200 mV/div should be about right.

7. For fun, look at both the displacement and velocity signals at once by turning CH 1 and 2 on simultaneously. Take a look at the phase relationship between the two signals. We’ll return to this point later.

8. Now try out the magnetic dampers. These can be moved over the rotor by means of two knobs on the outside of the cabinet. Put the dampers partway in and try pumping the rotor. You should feel significant resistance, and the oscillations should die away much more rapidly. Try a variety of different positions for the dampers, and see how they affect the motion.

Finally, put the dampers all the way in. It will be very difficult to move the rotor quickly, and any oscillations will die away almost instantly. Leave the dampers in this position for the next part.

Note: the dampers are powerful magnets that affect the motion of the rotor through a phenomenon called eddy current damping. You’ll hear more about this effect in the next lab and even more in P14.

9. Let the rotor settle down, and connect the output of the angular displacement sensor to the digital multimeter. With the rotor at equilibrium, use the zero adjust knob (see Fig. 3) to minimize the output of the sensor. The sensor itself is actually a capacitive sensor on a circular piece of printed circuit board below the main rotor assembly. The next step is to calibrate the sensor.

10. Using the clamp, rotate the rotor to different angular positions, as measured by the scale on the rotor itself. You should see the voltage from the position sensor change as you do so.

11. To calibrate the position sensor, turn the rotor to a series of different angles $\theta$ in the range $2.0 < \theta < 4.0$ and measure the output voltage $V_\theta$. Ten or so evenly spaced values of theta is sufficient. Record your observations in the space below.
12. Using DataStudio on one of the lab computers, enter your data in a table and plot it. Be sure to use $\theta$ as the independent ($x$) variable. Using a linear fit of the form $y = mx + b$, find the slope $m$ and intercept $b$ for your data. Finally, treat your data as being of the form

$$V_{\theta} = s_{\theta}\Delta\theta = s_{\theta}(\theta - \theta_e)$$

where $s_{\theta}$ is a scaling factor and $\theta_e$ is the equilibrium angle of your THO. Find $s_{\theta}$ and $\theta_e$ in terms of the slope $m$ and intercept $b$ from your curve fit. How accurately does a linear function describe the output of the sensor?

§5 Static Measurement of $\tau$ Versus $\Delta\theta$

Figure 4: (a) Hanging weights, brass quadrants, and ball bearings used for static and dynamic measurements of the torsional constant $\kappa$. (b) Schematic diagram shown use of strings (red) attached to hanging weights to apply a torque to the Al spindle.

1. You will now make immediate use of your measurement of the scaling parameter $s_{\theta}$ to measure the torsional constant $\kappa$ of your fiber. You’ll do this by applying a series of different static torques $\tau$ to the rotor assembly and measuring the resulting angular displacements $\Delta\theta$. You will make use of the hanging weights shown in Fig. 4(a).
2. First, put the magnetic dampers all the way back in. This will damp the motion of the oscillator and make the static measurements much easier.

3. To apply a static torque, attach the small screw with two strings provided with your THO to the Al spindle above the rotor. Thread one string directly though the side of the THO (near one of the dampers), over the flywheel, and down. Wrap the other string behind the spindle (see Fig. 4(b)) and then thread it through the other side of the THO. Note You can apply a torque with the opposite sign by changing which string runs behind the Al spindle.

4. The amount of weight you can attach to each line can be increased from 50 to 400 g in increments of 50 g. Attach one of the 50 g weights with the hook to one of the strings on either side of the THO. The rotor should now have some angular deflection $\Delta \theta$.

5. With two identical hanging weights attached to the Al spindle, what is the net torque $\tau$ applied to the rotor assembly, in terms of the mass $m$ of the weights and the radius $r$ of the Al spindle?

6. Attach the output of the angular position sensor to the multimeter.

7. Measure the position sensor output $V_\theta$ versus applied torque $\tau$ of both signs, for masses of each weight varying from 50 to 400 g. The fiber has a little bit of ‘memory’ of its last position (called hysteresis). Because of this, you’ll improve your results by (1) starting with 400 g, (2) working your way down to 50 g, (3) swapping the two strings so that you apply torque in the opposite direction (don’t forget to record the number for zero torque!), and (4) working up to 400 g in the new direction. Record your results here in tabular form. Be sure to calculate $\tau$ in units of N · m. Note: the radius of the Al spindle is $r = 0.5$ in. (Note: 1.0 in = 0.0254 m.) Using the scaling factor $s_\theta$ you found in §4, convert your values of $V_\theta$ to angular displacement $\Delta \theta$, and add those values to your table.
8. Enter your data for $\tau$ and $\Delta \theta$ in DataStudio. Even though $\tau$ is really your independent variable, be sure to enter your $\Delta \theta$ data as the $x$ variable and $\tau$ as the $y$ variable.

9. Plot $\tau$ versus $\Delta \theta$. Give a sketch of what you see below, and comment on it. Do you see the expected $\tau = -\kappa \Delta \theta$ dependence over the entire range of $\Delta$? If not, over what range of $\Delta \theta$ does linear behavior occur?

10. Do a polynomial curve fit of the form $y = A + Bx$ to your $\tau$ versus $\Delta \theta$ data. Since the torque is a restoring torque, what should happen to $\tau$ when $\Delta \theta$ changes sign? For small $\Delta \theta$, your results should be well approximated by a linear relationship $\tau = -\kappa \Delta \theta$. What value do you obtain for $\kappa$? Reminder: you need to apply a torque of both signs during your measurement; having both signs of $\tau$ will make a difference now.

11. **Homework:** Estimate the moment of inertia $I_0$ of the rotor assembly by neglecting everything but the copper rotor itself. The rotor is a hollow cylinder with inner radius $R_1 = 0.51$ in, outer radius $R_2 = 2.475$ in and mass $M = 962$ g. Its moment of inertia is given by $I = \frac{1}{2}M(R_1^2 + R_2^2)$. What rotational frequency do you expect from this estimate of $I_0$ and your measured value of $\kappa$? How does this compare to the frequency you measured in §4?
§6 Appendix

For reference, here is an image of the scope, indicating the location of various controls. The following page contains various dimensions and masses for the THO rotor assembly.

Figure 5: Image of the oscilloscope indicating the location of various controls.