Theory

The term microwave applies to any electromagnetic wave that has a wavelength between 0.1 and 10 cm. This places them on the electromagnetic spectrum between uhf radio and television waves on the one extreme and far infrared waves on the other. In this laboratory session, you will study the interference of microwaves as it applies to the Michelson interferometer and to slit diffraction. In principle these effects are the same as those observed for light; the only difference being that the dimensions have been changed. The change in dimensions allows us to easily observe effects that would be very difficult to observe in the case of light waves. The source of the microwaves is a device called a reflex klystron. An explanation of how it works is contained in an appendix at the end of this write-up. The theory of interference and diffraction was given in detail in the last lab write-up. A summary of the main features of that theory is given below.

Interference and Diffraction

Consider a two slit system illuminated by a coherent source of electromagnetic radiation as shown in figure 1. Let $d$ be the slit separation, $L_1$ be the distance from slit 1 to the point $x$ on the screen, $L_2$ be the distance from slit 2 to point $x$ on the screen and $L$ be the perpendicular distance from the two slit system to point $x = 0$ on the screen. The slits are infinitesimally narrow. If $L >> d$, then $L_1 \sim L_2 \sim L$ and the two angles shown are approximately equal. The intensity pattern on the screen turns out to be a series of areas of high intensity.
separated by areas of low intensity. This pattern can be explained as follows. The path difference of the two waves at point \( x \) on the screen is \( \Delta L = L_1 - L_2 \). If an intensity maximum falls on point \( x \), then \( \Delta L = n \lambda \) where \( \lambda \) is the wavelength of the electromagnetic radiation and \( n \) is an integer, positive or negative. If a node falls at point \( x \), then \( \Delta L = (n + 1/2) \lambda \). From the drawing it can be seen that \( \Delta L = d \sin \theta = dx/L \). Thus, the condition for the location of a maximum on the screen is

\[
n\lambda = d \sin \theta = \frac{dx}{L}
\]
or

\[
x = \frac{n \lambda L}{d}
\]  

(1)

where \( n = 0, \pm 1, \pm 2, \) etc. The central maximum corresponds to \( n = 0 \). Thus, for two infinitesimally narrow slits the pattern on the screen would look like figure 2

![Figure 2](image)

If we actually do the experiment above, however, we see that the pattern formed is different from figure 2. To explain this we need to reexamine the assumptions we have made. In deriving equation (1), we assumed slits of infinitesimal width. The fact that each slit has a finite width alters the interference pattern from the ideal one shown in figure 2. The reason for this is that the light coming from one portion of a finite slit interferes with the light coming from other portions of the same slit. To examine this effect, we consider a single slit of width \( a \), as shown in figure 3a. In this case, each point in the slit is acting as source of spherical waves which interfere with the wavelets from other points. To find the pattern on the screen, we would add up the phase differences at each point. This can be done with the use of calculus and as it turns out the intensity is given by

\[
I = I_m \left( \frac{\sin \alpha}{\alpha} \right)^2
\]  

(2)

where \( \alpha = (\pi a/\lambda) \sin \theta \) , \( a \) is the slit width and \( \theta \) is the angle shown in figure 3a. The general shape of the plot of \( I \) vs \( \theta \) is shown in figure 3b. Minima in the diffraction pattern occur when in equation (2), \( \alpha = m\pi \) (\( m = 1, 2, 3, \) etc). Combining this with our definition for \( \alpha \), leads to

\[
a \sin \theta = m\lambda
\]  

(3)
Now consider the effect of finite slit width on the two-slit interference pattern. For slits of infinitesimally small width, the diffraction maximum is infinitely broad and thus has no effect on the height of the (c) interference maxima. All such maxima are equally intense regardless of n, as shown in figure 2. For slits of finite width, this is no longer true. The interference pattern is modulated by the shape of the diffraction pattern as shown in figure 3c. The interference maxima still occur at the same places, but their amplitudes are affected by the diffraction pattern. The dotted line in the figure is called the diffraction envelope and represents the shape of the single slit diffraction which would result from either slit acting alone.
A detailed theory of the Michelson interferometer is given in Halliday and Resnick's, Physics, (third edition) chapter 45. A brief review of that material is given here.

Figure 4 shows a schematic drawing of the Michelson interferometer as it will be used with your microwave apparatus.

![Figure 4](image)

The partially reflecting mirror divides the beam into two parts which are sent to different mirrors. The beams are reflected back, recombined, and sent to the receiver. Because the distance the two beams travel will not necessarily be the same, the two beams may no longer be in phase when they recombine. If the path difference \( d \) of the two beams is an integral number of
wavelengths the beams are in phase and the intensity is a maximum. If the path difference is a half wavelength (\(\lambda/2\)) or \(n\lambda + \lambda/2\) the intensity will be a minimum. Thus, the intensity as a function of mirror position will look like

References

The following sections in Halliday & Resnick's *Physics* (third edition) are pertinent to this lab. They should be read before coming to lab.

1. Chapter 38 sections 38-1 to 38-5
2. Chapter 41 sections 41-1, 41-2, 41-6 to 41-9
3. Chapter 43 sections 43-1 to 43-4
4. Chapter 45 sections 45-1 to 45-4, 45-7, 45-8
5. Chapter 46 sections 46-1 to 46-6
6. Chapter 47 sections 47-1 to 47-4
7. Chapter 48 section 48-1

Experimental Purpose

To study the interference of electromagnetic radiation in the microwave frequency as it applies to the Michelson interferometer and to slit diffraction. The specific lab activities will be:

1. to measure the wavelength of the microwaves emitted from the reflex klyston in three ways: (a) using a research quality cavity wavemeter, (b) using a Michelson interferometer, and (c) using a diffraction grating;

2. to plot the interference and diffraction patterns of various slit patterns (double slits, single
3. to plot the interference and diffraction patterns of two unknowns and use computer modeling to determine their nature and dimensions; and

4. to observe polarization.

Procedure

Before beginning, a few comments on the equipment and its usage are in order.

a. The power output of the klystrons are on the order of milliwatts. At that level, it is safe to put your hands and arms into the beams. It is not, however, a good idea to look directly into the transmitter while it is emitting microwaves as this could potentially cause damage to your eyes.

b. The klystron tube gets very hot after several minutes of use. Touching it can cause severe burns.

c. Because the microwaves have wavelengths on the order of a few centimeters, they will readily reflect off of most things in the room and could reflect back into a receiver and thereby cause erroneous readings. Thus, people moving around the room could cause changing readings from one apparatus to another or from reading to reading on an individual apparatus. For this reason, please keep your movements around the room to a minimum and try to be in the same position each time you make a measurement at your own station.

d. There is a computer program for use with this lab called DIFFRACT. It is stored in the subcatalog P14 in the public library PHYSLIB***. To use it, sign on to a terminal and type

\[
\text{RUN PHYSLIB***:P14:DIFFRACT}
\]

This program is compiled. To see the documented source code, type

\[
\text{LIST PHYSLIB***:P14:DIFF.DOC}
\]

Given the slit dimensions and the experimental data, the program will: (1) provide tabular output of the theoretical intensity versus azimuth angle, (2) plot (on a gigi terminal) the experimental data and the theoretical curve of intensity versus azimuth angle, and (3) allow computer modeling of the data for the unknowns. All lengths are in millimeters and angles in degrees. The curves are normalized to the input "MAX INTENSITY" to facilitate comparison with the experimental data. "MAX AZIMUTH ANGLE" is the
largest angle for which an intensity is calculated and output.

e. The unknowns are made of two pieces of fiber board glued together. Between the fiber board is a sheet of aluminum foil in which a slit pattern has been cut. The fiber board is transparent to microwaves, but the aluminum foil is not. The pattern in the aluminum foil is either a single slit, a double slit or a bar. Please do not try to take apart the fiber board. Your TA will identify the unknown and its dimensions when he/she returns your lab report. The computer program will be particularly helpful in determining the nature and dimensions of the unknown. A suggested procedure would be to enter your experimental data for an unknown, take a guess at its dimensions, and then have the computer plot the experimental points and the theoretical curve based on those dimensions. If the theoretical curve is not a good fit to the data, change one of the slit parameters and plot again. Keep doing this until a good fit is obtained.
Figure 6
f. The apparatus is crude. Good data is possible, however, with careful alignment of the system. Take your time when setting up any of the experimental systems. When changing the horn or handling the transmitter or receiver please use care. The transmitters and receivers cost several hundred dollars apiece and the horns if bent, or otherwise damaged, will cause inaccurate readings during any future use. Please do not write on the wooden experimental boards.

g. Do not run the receiver with a gain high enough to give a reading greater than 100. In adjusting the gain for each part, the object is to get the highest reading possible without going over 100. If the receiver suddenly ceases to work, try tightening the contact between the diode and the meter located on the top of the receiver. If after doing this, the meter still does not work, see your TA.

1. Put the klystron and detector into operation in the following way (see figure 8 for pictures identifying the various pieces of apparatus):

a. Align the transmitter and receiver as shown in figure 7. The azimuth angle should be 0 degrees and the receiver should be just behind the hole on the movable ruler. Set the gain on the receiver to 2.

![Figure 7](image)

b. Turn the power knob on the transmitter until it just clicks on. Allow the transmitter several minutes to warm up before proceeding further.

c. Slowly turn the power knob on the transmitter to increasingly higher settings and watch the needle on the receiver. It should rise, peak and fall three or four times (most often three times) with each peak reading being larger than the proceeding one. Adjust the power knob until the needle on the receiver indicates the reading of the peak with the highest output. This will occur for a setting of the power knob on the transmitter of between 3 and 4. This setting should be checked periodically throughout the lab.

d. Adjust the gain on the receiver to give a reading of between 95 and 100.

2. Use the cavity wavemeter to measure the frequency of the microwaves. Your TA will instruct you in its use. Convert the frequency to a wavelength. This value of the wavelength should
be used as the accepted value in all calculations. (Note: There is only one wavemeter which your TA will move from station to station. If it is not available at this time, go on to the next step.)

3. Measure the dimensions (slit width, slit spacing, etc.) of the half reflecting plate, the double slit plates, the single slit plates and the bar.

4. Set up your apparatus as shown in figure 4 in the introduction and do the following to optimize the positioning of the various pieces:

   a. Remove the two full reflectors and maximize the reading on the receiver by slightly changing the positions of the transmitter and receiver. (Note: It may be necessary to increase the gain on the receiver. When doing so, be sure that the needle reading does not exceed 100.)

   b. Replace the two full reflectors and position the 270° full reflector for a minimum reading on the receiver. Sharpen this null by adjusting the position of the 180° full reflector. Adjustment of the 180° full reflector will now give deep nulls.

   Vary the position of the 180° full reflecting plate and record the position of a minimum of five nulls. From that data, compute an average value for the wavelength of the microwaves.

5. Set up your apparatus as shown in figure 8. Move the receiver clockwise about the axis of rotation until first a minimum and then the maximum reading (order number = 1) is obtained (approximately 105°). This locates the center of the beam after it is bent by the diffraction grating. Calculate from this the angle of incidence and the angle of diffraction as described in the introduction. Compute the wavelength using equation (5).

6. Carefully replace the large horn on the receiver with the small horn. Be careful not to drop, bend or damage the horns in the process. Such damage would adversely affect the results of any experiments done with the damaged horn from then on. Set up your apparatus as shown.
in figure 9. The single slit plate should be perpendicular to the direction of propagation of the microwaves, vertical with respect to the wooden experimental platform and exactly centered on the horn. (Note: The plates come in a variety of thicknesses and the their stands come with several slit widths. Try to match the thickness of the slit plate to the thickness of the stand slits. If an exact match is not possible, small slips of paper inserted between the slit plate and the slit in the stand can be used to make a tight fit and to make the slit plate vertical. Also note that the slit stands have one corner cut off. They should be attached to the slit plate so that the angles

![Figure 9](image)

through which the movable ruler can move without moving the slit plate are maximized.) The transmitter should be within one centimeter of the slit plate. Attach the positioning attachment to the receiver by pulling back on the washer and the spring on the end of the screw on the attachment, inserting the exposed stop at the end of the screw in the slot at the back of the receiver and releasing the washer and spring. Put the receiver and the positioning attachment onto the movable meterstick so that it is just behind the hole used to measure the azimuth angles. Position the receiver on the zero azimuth angle and adjust the gain in the receiver for a maximum reading. Adjust the positions of the transmitter and receiver slightly from their positions until you maximize the intensity readings on the receiver and then using a small screwdriver tighten the screws on the position attachment just enough so that the receiver will not move along the meterstick when it is moved from angle to angle. Be sure not to cover the hole through which the angle measurements are to be made. (Why is it necessary that the receiver not move along the movable meter stick while you are taking data?) Do NOT tighten the screws so as to make deep holes in the movable meterstick. Move the receiver from that position - first to the left and then to the right - and note what happens to the reading on the receiver. When the single slit is exactly centered on the transmitter, the central maximum of the diffraction pattern should occur at 0 degrees. If the reading on the meter went up rather than down when you moved the receiver to the left or to the right, it means that the slit plate is not centered on the transmitter. Adjust the position of the slit plate until the central maximum occurs at the 0 degree mark and the readings on the receiver go down symmetrically when you move the receiver to the left and right off the 0 degree mark. (Note: This adjustment procedure will need to be done each time the slit plates are changed.
7. Measure the intensity as a function of azimuth angle for as many angles as possible at 2.5 degree intervals. Be sure to make measurements on both sides of the central maximum. Also record the angular locations of all maximums and minimums. (Note: These maximums and minimums do not always occur on exactly 2.5 degree marks. Therefore, you may have to locate and record them separately.) Repeat step 6 and 7 for the second single slit.

8. Replace the single slit plate with the double slit plate. Repeat steps 6 and 7.

9. Replace the double slit plate with the bar. Repeat steps 6 and 7.

10. Replace the bar with one of the unknowns. Be sure to record the letter code of the unknown. Repeat steps 6 and 7.

11. Carefully replace the small horn on the receiver with the large horn and setup the system shown in figure 10. Before putting in the polarization grid adjust the receiver gain for a reading between 95 and 100. Put in the polarization grid with the slits horizontal and record the intensity reading. Rotate the polarization grid so that the slits are vertical and record the reading. Rotate the grid so that the slits are at approximately a 45° angle with respect to the wooden experimental platform and record the intensity reading.

12. When you are done - turn off the transmitter, set the gain on the receiver to its lowest value, remove the position attachment from the receiver, and put the equipment back into the arrangement in which you found it when you came to lab.

**Lab Report**

Follow the usual format for your lab report. Your lab report should include the answers to all of the questions asked in the introduction or procedure, all raw and derived data, an estimate of the magnitude and sources of error in any data recorded, and the following information:
1. the computed values of the wavelength of the microwaves used from the data taken with the Michelson interferometer and from the data taken with the diffraction grating; a comparison of these values with the value given by the cavity wavemeter;

2. the plots of intensity vs. azimuth angles for the two single slits and the two double slits; a comparison of these experimental plots to the theoretically expected plots and an explanation of any discrepancies noted;

3. the plots of the intensity vs. azimuth angles for the two unknowns; a description of the nature and dimensions of the two unknowns and an explanation of how you arrived at those conclusions;

4. the plot of the intensity vs. azimuth angle for the bar; a comparison of the plot for the bar with those for the single and double slits; an indication of which slit pattern the plot of the bar most closely resembles and an explanation as to why this is so; and

5. an explanation of the observations made in step 11.

6. a derivation of equation (4)
Appendix - The Reflex Klystron

The source of microwaves which you will use is a device called a reflex klystron, shown in figure 1.

![Diagram of a reflex klystron showing the cathode, grid, cavity, and reflector.](image)

Figure 1

Its main feature is an LC resonant circuit which consists of two grids (which acts as the plates of the capacitor) and a doughnut shaped cavity (which acts as a one turn inductor). When the power supply,\( V_B \), is turned on, the instantaneous voltage increase will start the LC circuit oscillating at its resonant frequency. Charge will flow back and forth between the grids through the outside of the cavity. The oscillating charge on the grid will create an oscillating electric field between the grids and the oscillating current on the outside of the cavity will create an oscillating magnetic field inside the cavity. Since the charge and current are 90° out of phase, the electric and magnetic fields will be a reproduction of the oscillations of the charge and current. In our case, the charge and current oscillate sinusodially and, therefore, so will the electric and magnetic fields. Figure 2 shows the electric and magnetic fields during various parts of a cycle.

A small coupling loop inserted in the cavity is connected to a coaxial output cable which in turn is connected to a transmitting antenna. The oscillating magnetic field in the cavity induces an oscillating voltage in the coupling loop. The resulting oscillating current is fed out through the output cable to the antenna. The microwaves radiate from the antenna and are sent through a waveguide into an output horn. At sufficient distances, the microwaves are in the form of plane waves.

As the charges surge back and forth energy is lost from the system in two ways: (1) through heating due to resistance in the cavity walls and grid and (2) through the microwaves being
radiated.

Figure 2

If this energy is not replaced, the oscillations would eventually be damped out and the microwave emission would cease. The cathode and reflector electrode give us the means by which energy can be fed back into the system.

The cathode is continuously heated and as a result electrons are continuously emitted. The emitted electrons are accelerated into the oscillating field between the grids by the electric field between the cathode and the cavity-grid assembly. If they enter the area between the grids when the electric field is pointing towards the reflector, they will be retarded. If they enter the area between the grids when the electric field is pointing towards the cathode, they will be accelerated. The result is a bunching of electrons at certain positions along the beam.

After passing through the grid, they are brought to a stop by the negative potential of the reflector in the area between the reflector electrode and the top grid. They are then accelerated back towards the grid. The time it takes for an electron to leave the top grid, come to a stop, and return to the top grid is determined (for a constant beam and reflecting potentials) by whether the electron was accelerated or retarded as it passed between the grids. The accelerated electrons will have more kinetic energy than the retarded electrons and will travel farther before losing all of their energy. Hence, the accelerated electrons will take longer than the retarded electrons to return to the top grid. The result is that the bunched beams returns in a series of pulses which arrive at the same frequency as the oscillating fields. If the beam and reflector voltages are adjusted correctly, the returning electrons will arrive at a point in the cycle where they will be
decelerated. During this deceleration they lose energy which is absorbed by the LC system thus sustaining its oscillations.

The microwave detector consists of a receiving antenna which is connected to a semiconducting diode which in turn is connected to a voltage meter. The electric fields of the incoming microwave induce an alternating current in the antenna. The semiconducting diode changes this into a d-c voltage at its terminals the magnitude of which depends on the amplitude of the microwave signal. At small amplitudes, the diode voltages is approximately proportional to the microwave intensity which in turn is proportional to the square of the electric field amplitude. At higher amplitudes, the diode voltage becomes more nearly proportional to the electric field itself.