Theory

Consider an ideal liquid, one which is incompressible and which has no internal friction, flowing through a pipe of varying cross section as shown in figure 1. In a time $dt$ a volume $V_1$ of this liquid will enter the pipe through the cross section $A_1$ with a velocity $v_1$. If the liquid has a density $\rho$, the mass flowing into the pipe is given by

$$m_1 = \rho V_1 = \rho A_1 v_1 \, dt$$

In the same time $dt$, a volume $V_2$ of the liquid will flow out of the pipe through the cross section $A_2$ with a velocity $v_2$. The mass flowing out of the pipe is given by

$$m_2 = \rho V_2 = \rho A_2 v_2 \, dt$$

Since the liquid is incompressible, the flow is steady and the liquid cannot flow out through the walls of the pipe, the mass entering the pipe in the time $dt$ must be equal to the mass leaving the pipe in the same time interval. This means

$$\rho A_1 v_1 \, dt = \rho A_2 v_2 \, dt$$

$$A_1 v_1 = A_2 v_2$$

Equation (1) is called the equation of continuity.

The equation of continuity states that, when you are dealing with an ideal liquid in a steady flow state, the product $Av$ is constant. This is a general statement and applies to pipes which are horizontal as well as to those which are not. If the pipe is horizontal and if its cross sectional area decreases, the velocity of the liquid must increase and vice versa. Since the liquid is accelerating or decelerating, it must be acted upon by a net force which in turn means that there must be a pressure difference between the two points of different velocities. If the pipe is not
horizontal, then the pressure difference depends not only on the difference in velocities, but also on the difference in levels. The general expression describing the pressure differences along a pipe was first derived by Daniel Bernoulli in 1738.

To derive Bernoulli's equation we will apply the work-energy relationship to a small element of liquid flowing from one point to another along a pipe of varying cross section. At some initial time, the ends of the element are at points a and c (see figure 2a). The force on the cross section at a is \( p_1 A_1 \) and the force at point c is \( p_2 A_2 \). During a time interval \( \Delta t \), the ends will undergo displacements \( \Delta s_1 \) and \( \Delta s_2 \) respectively (see figure 2b). If \( \Delta t \) is kept small, the pressures and cross sectional areas, and hence the forces on the ends, can be considered to be constant. Therefore, the work done on cross sectional area \( A_1 \) is \( p_1 A_1 \Delta s_1 \) and the work done by cross sectional area \( A_2 \) is \( p_2 A_2 \Delta s_2 \). The net work done on the element during the interval \( \Delta t \) is

\[
W = p_1 A_1 \Delta s_1 - p_2 A_2 \Delta s_2
\]  

(2)
Because of the equation of continuity, the volume that vacated the portion of the pipe bounded by the points $a$ and $b$ must be equal to the volume that moved into the portion of the pipe bounded by the points $c$ and $d$ (see figure 2c),

$$\Delta V = A_1 \Delta s_1 = A_2 \Delta s_2.$$  \hspace{1cm} (3)

Using equation (3), we can rewrite the equation for the net work done on the element as

$$W = p_1 \Delta V - p_2 \Delta V$$

$$W = (p_1 - p_2) \Delta V.$$  \hspace{1cm} (4)

The work done on the element must equal the changes in kinetic energy and potential energy which it undergoes. During the time element $\Delta t$, a volume of liquid $V$ with a mass $\Delta m = \rho \Delta V$ enters the pipe past point $a$ bringing with it a kinetic energy equal to $(1/2) \Delta m v^2 = (1/2) \rho \Delta V v_1^2$ and a potential energy of $\Delta mgh_1 = \rho \Delta V gh_1$. At the same time, a similar volume of liquid is leaving the pipe past point $c$ taking with it a kinetic energy $(1/2) \rho \Delta V v_2^2$ and a potential energy $\rho \Delta V gh_2$. The changes in kinetic energy and potential energy are given by

$$\Delta E_k = \frac{1}{2} \rho \Delta V \left(v_2^2 - v_1^2\right)$$

$$\Delta E_p = \rho \Delta V \left(h_2 - h_1\right).$$  \hspace{1cm} (5a)

Equating the work done on the element to the changes in energy yields

$$\{p_1 - p_2\} \Delta V = \frac{1}{2} \Delta V \rho \left(v_2^2 - v_1^2\right) + \rho \Delta V \left(h_2 - h_1\right)$$

$$p_1 - p_2 = \frac{1}{2} \rho \left(v_2^2 - v_1^2\right) + \rho \left(h_2 - h_1\right).$$  \hspace{1cm} (6)

Equation (6) is Bernoulli’s equation.

In this lab you will apply Bernoulli’s equation and the equation of continuity respectively to water exiting from a large vessel through a small hole (an orifice) and through a capillary tube. As in previous labs you will be looking to see how well the theories describe the actual situations by testing predictions based on the theory.

**Torricelli’s Theorem**

The apparatus for this part of the lab is a relatively large bore (1" diameter) glass tube fitted at the base with a copper elbow into which has been placed a rubber stopper fitted with a brass tube capped off with a brass cap in which a small hole has been drilled (see figure 3).
To apply Bernoulli's equation to this situation, we begin by rewriting it in the form

\[ p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2 \]  \hspace{1cm} (7)

The subscripts 1 and 2 refer to any two points in the system. The points that have been chosen are shown in the diagram. Let the reservoir have a cross sectional area \( A_1 = \pi R^2 \) and be filled to a height \( h \) with water at a density \( \rho \). Let the cross sectional area of the orifice be \( A_2 = \pi r^2 \) and let the velocities at the two points be \( v_1 \) and \( v_2 \) respectively. The pressure at point 1 is the pressure of the atmosphere plus the pressure of the water at that depth \( (p_1 = p_a + \rho gh) \). The pressure at point 2 is atmospheric pressure \( (p_2 = p_a) \). Applying Bernoulli's equation gives

\[ p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2 \]

But \( h_1 = h_2 \) so we have

\[ \rho gh + \frac{1}{2} \rho v_1^2 = \rho gh + \frac{1}{2} \rho v_2^2 \]

Simplifying and rearranging terms gives us an expression for the velocity with which the water exits from the orifice

\[ v_2 = \sqrt{2gh + \frac{v_1^2}{2}} \]

If \( A_1 >> A_2 \) then \( v_1 \ll v_2 \) and our expression for \( v_2 \) becomes

\[ v_2 = \sqrt{2gh} \]  \hspace{1cm} (9)
Equation (9) states that the speed with which the liquid exits from the reservoir is the same as that acquired by any body in freely falling through a height \( h \). This is Torricelli's theorem.

If \( A_1 \gg A_2 \) then \( v_2 \gg v_1 \) and \( v_2 \) will be difficult to measure with any accuracy. \( v_1 \) can be much more easily measured; hence, we will test our theory by deriving an expression for \( v_1 \).

From the equation of continuity we have another expression for \( v_2 \)

\[
v_2 = \left( \frac{A_1}{A_2} \right) v_1 = \left( \frac{\pi R_2^2}{\pi r_2^2} \right) v_1
\]

\[
v_2 = \left( \frac{R_2}{r_2} \right) v_1. \quad (10)
\]

Setting the two expressions for \( v_2 \) equal and solving for \( v_1 \) gives

\[
v_1 = \left( \frac{r_2}{R_2} \right) \sqrt{2gh}
\]

or since \( v_1 \) is the rate of change of the height of the water in the large tube, \( \frac{dh}{dt} \), we have

\[
\frac{dh}{dt} = \left( \frac{r_2}{R_2} \right) \sqrt{2g} \frac{1}{h^2}
\]

This equation can be integrated to give

\[
\frac{1}{h_0^{1/2}} - \frac{1}{h^{1/2}} = \left[ \left( \frac{r_2}{R_2} \right) \sqrt{\frac{g}{2}} \right] t \quad (13)
\]

where \( h_0 \) is the height at \( t = 0 \) and \( h \) is the height at time \( t \). If we consider \( h_0^{1/2} - h^{1/2} \) to be a single variable \( y \) and \( t \) to be another variable, then we essentially have an equation of the form \( y = at \) which if plotted would give a straight line plot with slope \( (r_2/R_2)(g/2)^{1/2} \). This is the prediction you will test in this part of the lab.

Poiseuille's Law

In this part of the lab the apparatus is the same as in part 1 except that the orifice has been replaced by a capillary tube as shown in figure 4. Up to now we have considered the
liquid to be incompressible and having no internal friction. For the liquid flowing in a capillary, the incompressible assumption is valid, but the assumption that there is an absence of internal friction is not. Therefore, we cannot apply Bernoulli’s equation; and before we can apply the equation of continuity we must determine how we must modify the theory to account for this internal friction. This internal friction is called viscosity.

For a viscous liquid flowing in a round channel, the internal friction shows up as a shearing force between layers that are moving with different speeds. The outermost layer clings to the walls of the capillary and its velocity is zero. The walls of the capillary exert a backward drag on this layer which in turn drags backward on the next inner layer. This process is continued toward the center of the capillary. The velocity of the liquid is therefore zero at the walls and a maximum at the center of the capillary. This is shown graphically in figure 5a.
Let the radius of the capillary be $R_c$ and consider a cylindrical element of the liquid coaxial with the pipe which has a radius $r$ and length $L$ as shown in figure 5b. The force on the left end is the product of the pressure and the cross sectional area, $p_1 \pi r^2$. The force on the right end is $p_2 \pi r^2$. The net force is then $F = (p_1 - p_2) \pi r^2$. This force must balance the viscous retarding force at the surface of the element if the element is to maintain a steady flow. The viscous force is given by

$$F = \eta A \frac{dv}{dr}$$

where $\eta$ is the coefficient of viscosity and $A$ is the surface area over which the force acts. Since $A = 2\pi RL$ the viscous force is

$$F = \eta 2 \pi r L \frac{dv}{dr}.$$  \hspace{1cm} (14)

Equating the viscous force and the force due to the pressure difference between the ends and rearranging terms yields

$$-\frac{dv}{dr} = \frac{(p_1 - p_2)r}{2 \eta L}.$$  \hspace{1cm} (14)

The negative sign shows that the velocity changes more and more rapidly as we go from the center to the pipe wall. Integrating we get

$$v = \frac{p_1 - p_2}{4 \eta L} \left[ R_c^2 - r^2 \right]. \hspace{1cm} (15)$$

This equation gives the velocity of the liquid across a cross-section. Equation (15) can be used to find the total rate of flow through the capillary. Consider a thin element of the cylinder of liquid flowing through the pipe as shown in figure 6.
The volume of liquid crossing the ends of this element in a time $dt$ is

$$dV = v \, dA \, dt$$

where $v$ is the velocity of the element at the radius $r$ and $dA$ is the shaded area. Substituting the expression for $v$ given in equation (15) and $2\pi r \, dr$ for $dA$ gives

$$dV = \frac{p_1 - p_2}{4\eta L} \left( R_c^2 - r^2 \right) 2\pi r \, dr \, dt .$$

The entire volume flowing through the entire cross section is found by integrating this equation over all the elements from $r = 0$ to $r = R_c$. Integrating gives

$$dV = \left[ \frac{\pi R_c^4}{8\eta} \right] \left( \frac{p_1 - p_2}{L} \right) \, dt . \quad \text{(16)}$$

The total flow per unit time is given by

$$\frac{dV}{dt} = \left[ \frac{\pi R_c^4}{8\eta} \right] \left( \frac{p_1 - p_2}{L} \right) . \quad \text{(17)}$$

This is called Poiseuille's law. With this expression we are now ready to apply the equation of continuity.

The equation of continuity says that the rate of volume flow in the large tube is equal to the rate of volume flow in the capillary tube. The rate of volume flow in the large tube is

$$\frac{dV}{dt} = -A \left( \frac{dh}{dt} \right) = -\pi R^2 \left( \frac{dh}{dt} \right) .$$

The rate of volume flow in the capillary tube is given by Poiseuille's law

$$\frac{dV}{dt} = \frac{\pi R_c^4}{8\eta L} \left( p_1 - p_2 \right) . \quad \text{(18)}$$

Equating these two expressions, solving for the rate of fall of the height in the large tube and
substituting $gh$ for $(p_1 - p_2)$ gives

$$\frac{dh}{dt} = \left(\frac{R_c^4 \rho g}{8 \eta L R^2}\right) h.$$  \hspace{1cm} (19)

Combining the seven quantities in the parentheses into one constant gives

$$\frac{dh}{dt} = -\beta h.$$  \hspace{1cm} (20)

The negative sign accounts for the fact that the fluid is falling. Integrating equation (20) gives

$$h = h_0 e^{-\beta t}$$  \hspace{1cm} (21)

where $h_0$ is the height when $t = 0$. Taking the natural log of both sides of equation (15) gives

$$\ln(h) = -\beta t + \ln(h_0)$$  \hspace{1cm} (22)

If we consider $\ln(h)$ to be equal to $y$ and $t$ equal to $x$, we again have an equation of the form $y = ax$. If plotted it will yield a straight line with slope $\beta$. Knowing this slope and the geometry of the system, we can calculate a value for $\eta$. This theory can be tested by comparing the experimental value for $\eta$ with the theoretical value as given in an appropriate handbook.

References

The sections in Halliday and Resnick's *Fundamentals of Physics* 3rd edition which are pertinent to this lab are Chapter 16 sections 16-1 to 16-5 and 16-8 to 16-12. They should be read before coming to lab.

Experimental Purpose

Part 1: Large reservoir emptying through an orifice.

In the introduction an expression was derived relating the height of water in a large reservoir emptying through an orifice as a function of time (equation (13)) starting from the equation of continuity and Bernoulli's equation. This expression predicts that a plot of $h_0^{1/2} - h^{1/2}$ vs $t$ will yield a straight line with a slope given by $(r^2/R^2) (g/2)^{1/2}$. The purpose of the first part of this lab is to test that prediction by making the appropriate measurements, plotting the data and computing the slope of the resulting curve.

Part 2: Large reservoir emptying through a capillary.

In the introduction an expression was derived relating the height of water in a large reservoir emptying through a capillary as a function of time (equation (22)) starting from Poiseuille's law and the equation of continuity. This expression predicts that a plot of $\ln(h)$ vs $t$
will be a straight line with a slope equal to \( \beta \) where \( \beta = \frac{R_c^4 rg}{(8\eta LR^2)} \). From \( \beta \) the viscosity of water at the given temperature can be measured. The purpose of the second part of this lab is to test that prediction by making the appropriate measurements, plotting the data, computing \( \beta \) and \( \eta \) and comparing their values to theoretical values.

**Procedure**

**Part 1: Large reservoir emptying through an orifice.**

1. Measure the pertinent dimensions of the apparatus. The inside diameter of the large tube is measured by the following steps.
   
a. Plug the elbow with a solid rubber stopper.
   
b. Pour water into the large tube until the water reaches a height of approximately 10 cm. Record the exact position of the top of the water using the meter stick attached next to the large tube.
   
c. Using the large graduated cylinder, pour exactly 200 ml of water into the large tube. Record the position of the top of the water.
   
d. Compute the height the water rose due to the addition of the 200 ml of water. Knowing this value, the volume of water added and the formula for the volume of a cylinder, the radius of the large tube can be calculated.
   
e. Repeat steps b-d two more times and use the average of the three values for the radius of the large tube in all future calculations.

   The radius of the orifice can be measured using the provided calibrated eyepiece. Use the most precise scale available on the eyepiece. One important detail must be added here. The water streaming out of the orifice pulls together and narrows to parallel streamlines in a stream that is narrower than the orifice. It is the radius of this narrowest point that should be used as the value of the radius in equation (13). Experiment shows that the narrowest point of the stream has an area of approximately 65% of the area of the orifice. Therefore, multiply the computed area by 0.65 and use that value as the area of the orifice.

2. Replace the solid stopper with a stopper containing an orifice. Adjust the apparatus so that the zero of the meterstick is at the level of the output. Make certain the large tube is vertical and the section of the large tube containing the orifice is horizontal. Fill the large tube with water and let the water run out of the tube without taking measurements. This will give you an idea of how long you will have to take measurements. For the orifice it would be useful to
note the times at which the liquid passes the points 100 cm = 10^2 cm, 90.3 cm = 9.5^2, 81 cm = 9^2 cm and so forth (ie., h = n^2 or h = (n + 1/2)^2, where n is an integer). The reason for doing this will become clearer when you consider the data analysis. It is also helpful to make up a data table in advance of taking the measurements.

3. Measure h as a function of time, starting soon after you fill it, for points between h = 100 cm and h = 10 cm. It is helpful to choose a starting height h₀ such as 100 cm, fill the large tube above 100 cm and wait to start taking measurements until the water height reaches the 100 cm mark.

4. Repeat the appropriate parts of steps 1-3 for an orifice with a different radius.

Part II: Large reservoir emptying through a capillary.

1. Measure the length of the capillary and its radius. The radius can again be measured with the calibrated eye piece and again you should use the most precise scale on the eyepiece. This is especially important with the capillary. Why? Note: There are three capillaries to choose from. Two are made of non-precision bore capillary tubing. The other is made of precision with an inside diameter of 1.016 mm. The precision bore tubing is marked with masking tape. Please do not remove the tape as it is the only way of distinguishing the precision bore tubing from the non-precision bore tubing.

2. Insert the capillary into the large tube. Adjust the apparatus so that the zero of the meter stick is at the level of the output. Make certain the large tube is vertical and the section containing the capillary is horizontal. Fill the large tube with water and let the water run out of the tube without taking measurements. Estimate the time you have to make measurements. For the capillary divide the time into 20 intervals and have one partner call "read" at the predetermined times so that the person watching the level can note it.

3. Measure h as a function of time for points between h = 100 cm and h = 10 cm. As before it will be helpful to choose an initial height h₀ such as 100 cm, fill the large tube above the 100 cm mark and wait to start taking measurements until the water height reaches the 100 cm mark. Record the temperature of the water.

4. Repeat steps 1-3 for a capillary with a different radius. Be sure that one of the capillaries you use is the precision bore tube.

Lab Report

Follow the usual lab notebook format. Your lab report should include the answers to all of
the questions asked in the introduction or procedure, all raw and derived data, and an estimate of the magnitude and sources of error in any data taken. When answering any question or when giving any comparison or explanation, always refer to specific data to support your statements. (Note: There is a computer program called POISE stored in the public library PHYLIBLE*** to aid you in doing some of the tedious calculations involved in this lab. To use it, type OLD PHYLIBLE***:P3:POISE . The program will prompt you for the information that it needs. As with all previous labs, show a sample calculation of each calculation done by the program to show that you know what the program is doing for you.)

Part 1: Large reservoir emptying through an orifice.

1. a plot of $h_o^{1/2} - h^{1/2}$ vs t for each orifice with the computed value of the slopes of your experimental curves;

2. a discussion of how well your experimental data agrees with the theoretical predictions outlined in the experimental purpose section.

Part II: Large reservoir emptying through a capillary.

1. a plot of $\ln(h)$ vs t for each capillary with the computed value of the slope of your experimental curve and your experimental values of $\eta$. This is most easily done on semilog paper (h plotted on the log scale).

2. a discussion of how well the predictions of our theory agree with your experimental results. In the appendix is a table of $\eta$ values for various temperatures. Your experimental values for $\eta$ should be compared to these values. In your discussion consider these three points:

   a. The theory we have developed in this lab gives us two different equations for h as a function of time, depending on whether we call the hole out through which the water is flowing a capillary or an orifice. The question arises as to when does one apply one law and when the other? At what point does a capillary become short enough to be considered an orifice?

An answer to this can be seen if we consider work and energy relations during the decay of the pressure head. In an instant of time $\Delta t$, let the height fall $\Delta h$ and let a stream of length $\Delta t$ emerge from the hole (capillary or orifice) at the bottom. We can set the changes in total energy of the fluid, kinetic and potential, equal to the work done by friction during efflux.

The potential energy change in the system takes place only in the large tube and is given by

$$\Delta E_p = \Delta m g h = -\rho \Delta V g h$$
As potential energy is lost in the large tube it appears as kinetic energy input to the capillary. At the same time the stream exiting the hole takes with it kinetic energy. The change in kinetic energy is therefore given by

\[ \Delta E_k = \frac{1}{2} \Delta m v^2 - \frac{1}{2} \Delta m \left( \frac{dh}{dt} \right)^2 \]

\[ \Delta E_k = \frac{1}{2} \rho \Delta V v^2 - \frac{1}{2} \rho \Delta V \left( \frac{dh}{dt} \right)^2 \]

\[ \Delta E_k = \frac{1}{2} \rho \Delta V \left( v^2 - \left( \frac{dh}{dt} \right)^2 \right) \]

The work done by frictional forces in the capillary is given by

\[ \Delta W = -\Delta p \Delta V \]

where \( \Delta p = 8\eta v L / r^2 \) as derived from Poiseuille's law (equation 17). Equating the sum of the changes in energy with the work done yields

\[ -\rho \Delta V \rho h + \frac{1}{2} \rho \Delta V \left( v^2 - \left( \frac{dh}{dt} \right)^2 \right) = -\rho \Delta V \]

Assuming \( \Delta h/\Delta t \ll v \) and dividing through by \( \Delta v \) yields

\[ -\rho g h + \frac{1}{2} \rho v^2 = -\Delta p = -8 \eta v L / r^2 \quad (23) \]

From this equation and equation 15, we can see that depending on the experimental conditions (values of \( L \) and \( r \) primarily) either the \( + (1/2) \rho v^2 \) term or the \(-8\eta v L / r^2 \) term will be negligible. If the former is the case, it is viscous flow and Poiseuille's law is applicable. If it is the latter, it is essentially a nonviscous flow and Bernoulli's equation is applicable. Is this needed to explain the data?

b. Equation (23) also tells us that in the case of viscous flow it may be necessary to subtract the \( (1/2) \rho v^2 \) from \( \rho gh \) to obtain the pressure head actually working against viscosity. (Remember when deriving equation (22) we assumed the pressure difference to be \( \rho gh \)). To do this, \( v \) is calculated from \( dh/dt \) and a factor \( \rho v^2 / 2g \) is added to the experimental data. If your plot of \( h \) vs \( t \) is not a straight line, try plotting the data after making this correction. Is this needed to explain the data?

c. In applying Poiseuille's law to the flow through a long, thin capillary we have been assuming that the flow was laminar. Laminar flow means that the streamlines are smooth and layered, that the water never swirls or doubles back on itself. In many flow systems, if the fluid moves fast enough, laminar flow breaks down and the swirling, turbulent flow mentioned does develop. Turbulent flow can be seen in fast moving streams, water jets, and smoke columns rising in air.
The transition from laminar to turbulent flow is controlled by many factors, and theories of turbulence are complicated. The main parameters for a round tube are the fluid density ($\rho$), viscosity ($\eta$), velocity ($v$), and the tube diameter ($D$). These can be combined into $N_R$, with

$$N_R = \frac{\rho v D}{\eta}.$$  

The empirical findings for round channels are that flow will be laminar for $N_R < 1500$ and turbulent for $N_R > 3000$. In the intermediate range it can be either laminar or turbulent.

In this experiment the initial velocity of the water in one size of capillary may be sufficient to exceed the critical Reynolds number. Therefore, the flow is turbulent until the pressure head has decayed to the point that the velocity (and the Reynolds number) is sub-critical.

Compute the Reynold's number for the capillaries used in this lab. Does this help explain the data?

3. Optional: In the discussion in 2a above, the question was asked as to when a capillary becomes short enough to be considered an orifice. Modify POISE so that it will model the situation and use that modeling capability to answer that question. Be sure to justify your conclusions.
### Appendix I - Table of Temperature vs Viscosity of Water

Viscosity of Water (0 - 100°C)

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