1

2

ball #1 will carry excess charge
ball #2 will be neutral, and
ball #3 will carry excess + charge

It is critical that the separations of the balls occur exactly simultaneously. Otherwise, ball #2 may end up with a charge, too.

3

The total self energy goes up by a factor of nine if all charges are tripled

4

\[ E_x = 0 \] (no change in \( V \) along the \( x \)-direction)

\[ E_z = -\frac{\partial V}{\partial z} = -\frac{V_0}{H} e^{\frac{z}{\lambda}} \]
SAMPLE
EXAM
Solutions

\( \text{IIa) } r < a \) choose Gaussian cylinder

\[ \Phi_c = 2 \pi r \ell E = \frac{\pi r^2 \rho_1}{\varepsilon_0} \]

\[ \Rightarrow E = \frac{\rho_1 r}{2 \varepsilon_0} \]

\( r > a, r < b \)

\[ \Phi_c = 2 \pi r \ell E = \frac{\pi a^2 \rho_1 + \pi (r^2 - a^2) \ell \rho_2}{\varepsilon_0} \]

\[ \Rightarrow E = \frac{1}{2 \varepsilon_0 r} \left[ \frac{a^2 \rho_1 + (r^2 - a^2)}{\rho_2} \right] \]

\( r > b \)

\[ \Phi_c = 2 \pi r \ell E = \frac{\pi a^2 \rho_1 + \pi (b^2 - a^2) \ell \rho_2}{\varepsilon_0} \]

\[ \Rightarrow E = \frac{1}{2 \varepsilon_0 r} \left[ \frac{a^2 \rho_1 + (b^2 - a^2)}{\rho_2} \right] \]

\( \text{IIb) } \) in order to have \( E = 0 \) inside, must have charge/length on inside surface equal to charge/length of cylinder:

\[ \lambda_{\text{in}} = -\left( \pi a^2 \rho_1 + \pi (b^2 - a^2) \rho_2 \right) \]

Since conductor is overall neutral,

\[ \lambda_{\text{at}} = -\lambda_{\text{in}} \]

field is unchanged except in the conductor where \( E = 0 \)
(2) negative (since negative portion of the rod is closer)

\[
V(x) = \int_{x' = 0}^{x' = \frac{L}{2}} \frac{1}{4\pi \varepsilon_0} \frac{\lambda \, dx'}{(x - x')^2} + \int_{x' = \frac{L}{2}}^{x' = L} \frac{1}{4\pi \varepsilon_0} \frac{-\lambda \, dx'}{(x - x')^2}
\]

change variables: \( u = x - x' \)

\[
V(x) = \int_{x = -\frac{L}{2}}^{x = \frac{L}{2}} \frac{du}{u} \left( \frac{-\lambda}{4\pi \varepsilon_0} \right) + \int_{x = -L}^{x = \frac{L}{2}} \frac{du}{u} \left( \frac{-\lambda}{4\pi \varepsilon_0} \right)
\]

\[
= -\frac{\lambda}{4\pi \varepsilon_0} \ln \left( \frac{x - \frac{L}{2}}{x} \right) + \frac{\lambda}{4\pi \varepsilon_0} \ln \left( \frac{x - \frac{L}{2}}{x} \right)
\]

\[
= \frac{\lambda}{4\pi \varepsilon_0} \ln \left( \frac{x(x - L)}{(x - \frac{L}{2})^2} \right)
\]

\[
= -\frac{\lambda}{4\pi \varepsilon_0} \ln \left[ \frac{(x - \frac{L}{2})^2}{x(x - L)} \right]
\]

note that the sign comes out right (e.g., try \( x = 2L \) and find \( V(x) < 0 \) as expected)

(C) set \( \frac{1}{2} m v^2 = (-e) V(x) \)

\[
\Rightarrow V = \sqrt{\frac{2V(x)(-e)}{me}}
\]

recall \( V(x) < 0 \). If you use \( |V(x)| \), then

\[
V = \sqrt{\frac{2e |V(x)|}{me}}
\]