Phonon description and the Euler buckling instability of a nanoscopic bar at fixed strain

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The quantum theory of a nanoscopic beam subjected to fixed compressional *strain* is discussed from the perspective of phonons and other related elementary excitations. It is argued that despite the anharmonicity inherent in the buckling instability, at which the classical frequency of small oscillations in the fundamental mode is driven to zero, the concept of well-defined elementary excitations remains intact at low energies. This is possible because of two factors: First, the higher modes remain harmonic through the instability, and second, the quantum vibrational frequency of the fundamental, $(E_2 - E_1)/\hbar$, does not vanish; it is determined by a Landau-like effective potential which is quartic at critical strain. Beyond the buckling instability, phonons recover their harmonic character while a distinct form of excitation evolves associated with the broken symmetry. For a beam of rectangular cross section these are tunneling modes; for a tube of circular cross section they are rotational.