Analytic Solution for Finite Linear Chain with Two General Impurities under General Boundary Condition

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A finite chain of masses and springs with two impurities is studied by solving its dynamical equation analytically under general boundary condition. The six parameters of impurities for their locations, mass changes, force-constant changes are explicitly taken into account. Not using the usual space transform, it was treated as a difference equation on the particle index. Three possible boundary conditions are (1) specifying the displacement of one end particle and the applied force on the other as functions of time, (2) the applied forces on both, and (3) the displacements. Together with the reported result for (1) [H. S. Lee et al., J. Korean Phys. Soc. 29, 190 (1996)], the present result for (2) and (3) completes the analytical study of this complex system with four scattering centers (at chain ends and impurities). Simple closed-form expressions for the frequencies are obtained, and they show explicit dependence on the impurity locations as well as the inter-impurity distance, in addition to the usual dependence on the mass-defect and force-constant-change parameters. No complexity of computation arises with increasing system size and generalization to more impurities seems easy so the result is a first step to better structural understanding of impurity concentration dependence of measurable properties. Computational experiment and actual measurement of resonance frequencies on an equivalent LC delay line confirm correctness of the obtained closed-form expressions, as summarized in Fig.: "two frequencies of a 13-mass chain under (1) are not perturbed by introduction of one (or two) heavy isotopic impurity (or impurities) at the site(s) 5 and/or 10," in agreement with what is expected on simple and elementary physical grounds.