Fractal Structures in Multi-Soliton Collisions

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Rapidly growing interest in the study of the solitary-wave interaction in nonintegrable nonlinear models is explained by the possibility to observe many of the predicted effects experimentally [G.I. Stegeman and M. Segev, Science 286, 1518 (1999)], including the soliton energy and momentum exchange. One of the most intriguing properties of the soliton interaction in nonintegrable models is the observation of *the fractal nature of their scattering*, first discussed for the kink-antikink collisions in the so-called ϕ^4 model [P. Anninos, S. Oliveira, and R.A. Matzner, Phys. Rev. D44, 1147 (1991)]. The main features of the fractal soliton scattering are usually explained by the excitation of *the soliton internal mode*, that is an important property of solitary waves of many nonintegrable soliton-bearing models [Yu. S. Kivshar *et al.*, Phys. Rev. Lett. 80, 5032 (1998)]. Thus, the physics of the fractal soliton scattering can be understood as a resonant energy exchange between the soliton translational motion and its internal mode [D.K. Campbell, J.F. Schonfeld, and C.A. Wingate, Physica D9, 1 (1983)]. Recently, the similar mechanism was proposed to explain the fractal properties of soliton collisions in [J. Yang and Yu Tan, Phys. Rev. Lett. 85, 3624 (2000)].

In the present study, we discuss *a novel physical mechanism of the fractal soliton scattering* and consider, for simplicity and historical traditions, the well-known model described by the sine-Gordon (SG) equation weakly perturbed by discreteness effect. The perturbation is so weak that the role of the soliton internal mode is negligible, and the fractal structures observed in the soliton scattering should be explained by a qualitatively different mechanism. In particular, we study the breather scattering in such a model, and describe several interesting phenomena that can be understood as a manifestation of the multi-particle effects in the soliton collisions, due to a resonant coupling between the "atomic" and "molecular" degrees of freedom of the colliding composite solitons.

We demonstrate numerically that, due to the discreteness that breaks the integrability, there exist (i) a weak attraction force between two breathers and (ii) the energy and momentum exchange between breathers when they collide [S. V. Dmitriev *et al.*, Phys. Lett. A246, 129 (1998); Phys. Rev. E61, 5880 (2000)]. With these two properties, the fractal nature of the breather collisions can have a simple physical explanation. When two breathers collide with a small relative velocity, there is a finite probability that they gain a very small velocity after collision and subsequent splitting. With such a small initial velocity, the breathers cannot overcome their mutual attraction and collide again. In the second collision, due to the momentum exchange, the breathers can acquire an amount of the kinetic energy sufficient to escape each other, but, again, there exists a finite probability to gain the kinetic energy below the escape limit. In the latter case, the breathers will collide for the third time, and so on. Thus, a series of collisions leads to a resonant energy exchange between the "atomic" (kink's translational) and "molecular" (relative oscillatory) breather's degrees of freedom, and to the fractal scattering.

To study the fractal nature of the breather collisions in more detail, we propose and analyze a simplified model where the breathers are replaced by the classical particles. The particles interact via exponential attraction potential and we assume the possibility of energy exchange between them. The nonlinear mapping derived for the model shows the properties very similar to that observed in the numerical experiments for the breathers.