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Development of a high-sensitivity torsional balance for the study of the Casimir force in the 1–10 micrometre range

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Abstract
We discuss a proposal to measure the Casimir force in the parallel plate configuration in the 1–10 μm range via a high-sensitivity torsional balance. This will allow for the measurement of the thermal contribution to the Casimir force thereby discriminating between the various approaches discussed so far. Accurate control of the Casimir force in this length scale is also required to improve the limits to the existence of non-Newtonian forces in the micrometre range predicted by unification models of fundamental interactions.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The search for deviations from Newton's gravitation law has been a recurrent issue for the last three decades. Initially motivated by the possibility of deviations from standard gravity due to new forces with couplings of the order of that of gravity [1], the search has been more recently encouraged by unification models which predict the existence of forces up to 10^5 times stronger than gravity in the 1 μm and 100 μm range [2, 3]. Even if its results have not met initial hopes for the observation of a 'fifth force', the search has greatly improved understanding of the gravitation law, generated an impressive body of new knowledge, and narrowed the remaining open windows for new fundamental forces.

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The hypothetical extra-gravitational force is often parametrized by a Yukawa range λ and a coupling strength α such that the corresponding potential is

$$V_{\mathrm{Newton}}(d) + V_{\mathrm{Yukawa}}(d) = -\frac{GM_aM_b}{d}(1 + \alpha\,\mathrm{e}^{-d/\lambda}). \tag{1}$$

The Newton and Yukawa potentials have been written for two point masses M_a and M_b at a distance d from each other and the coupling strength is defined with respect to Newtonian gravity. The current limits in the (λ, α) plane (see for instance [4]), summarize the considerable progress achieved during the last decades, thanks to a variety of laboratory experiments and solar system observations. At the same time, windows remain open for deviations of standard gravity in the submillimetre range or for scales larger than the size of planetary orbits [5]. In this paper, we focus our attention on the submillimetre window.

The accuracy of short-range tests has recently been much improved for Cavendish experiments performed at small distances. The best limits for distances of the order of $\simeq$100 μm have been obtained by the group of Adelberger at the University of Washington using torsion balances and rotors [6–8]. Difficulties in extending this technique to smaller distances arise from the stringent requirements necessary to maintain the surfaces parallel during the rotation. Experiments in the distance range below 100 μm have been recently performed with microresonators, using dynamical detection techniques first introduced in [9–11], by groups at Boulder [12] and Stanford [13], but still limited to distances $\geqslant$20 μm also due to the presence of conducting shields of non-negligible thickness. For even smaller distances, of the order or smaller than 1 μm, the hypothetical new forces have to be measured against a large background coming from the Casimir force [14]. The latter has been measured with increasing accuracy during the last few years by various groups using atomic force microscopes or microresonators monitored by means of capacitively or optically coupled displacement transducers [15–25].

At this point, it is worth emphasizing that the theoretical predictions are most promising in the 10 μm range [26–28]. It is therefore important to design new experiments aiming at the detection of forces acting in this distance range, where the Casimir force is still the main known background. It is the purpose of this paper to describe a proposal for a high precision study of the Casimir force at distances from a few μm to 10 μm. The challenge is to be able to measure the Casimir force which becomes weaker and weaker when the distance is increased. On the other hand, some of the main corrections known to endanger the accurate measurement of the Casimir force are expected to be more easily controlled at large distances. Meeting this challenge will allow us to bring new information on problems still present in the theory of the Casimir force [29–35], and more generally on the understanding of the properties of quantum vacuum [36–38].

Another important motivation is coming from open questions in cavity quantum electrodynamics, in particular the issue of the finite temperature corrections to the Casimir force. This correction is particularly significant at large distances, since only then can the cavity formed by the two mirrors sustain electromagnetic field modes filled with a non-negligible number of thermal photons [39–42]. The only experiment which has investigated the Casimir force at distances up to about 8 μm in a plane–spherical configuration [15], has shown no evidence of the thermal contribution which was yet expected to be visible at such distances. The discrepancy can be interpreted in various manners, going from an overestimation of the accuracy claimed in the first letter [43–46] to the much debated possibility that previous calculations have mistreated the thermal contribution for dissipative mirrors [47–55]. Therefore, the interplay between thermal and quantum fluctuations needs to be studied more closely. Obtaining an unambiguous experimental output for the value of the Casimir force at large distances could decide between the contradictory models used by different authors.

Another experiment, performed at the University of Padova, has explored the Casimir force at room temperature in a parallel plate configuration [23] with distances up to 3 μm. In this experiment, the accuracy of 15% was not sufficient to measure the amplitude of the thermal corrections, weaker at 3 μm than at 8 μm. Let us emphasize that it is important to test the Casimir force in a parallel plane configuration for the following reasons [56]. First, the parallel plane configuration, originally considered by Casimir, is the only one for which an exact analytical expression of the force exists. For other geometries considered so far, predictions always rely on the proximity force approximation based on analogies with the treatment of additive forces [57, 58]. Second, for an apparatus of a given size the parallel plane configuration leads to the largest possible signal, because the entire surface fully contributes to the Casimir force. This is to be contrasted with the obvious loss of signal in the plane–sphere geometry due to the smaller size of the closest approach region between the two surfaces. This argument favours the parallel plane configuration for the purpose of testing thermal contribution at large distances, since one important issue of this distance range is the rapid decrease of the amplitude of the Casimir force. Finally, the purely gravitational force does not depend on distance, in a parallel plane configuration, apart from modelizable border effects. It is thus easier to disentangle standard gravity from other possible forces.

The main difficulty of the parallel plane configuration is the need for a careful parallelization of the two surfaces. In the experiment carried out at the University of Padua [23], this was the major factor limiting the accuracy to a level of 15%. The solution proposed in the present paper is to take advantage of the recent progress in parallelization techniques obtained in a neutron experiment performed at the Institute Laue-Langevin (ILL) in Grenoble [59–63]. This technique, described in more detail below, should allow us to overcome the limitation associated with parallelization of the plates.

2. Estimates of signals and backgrounds

In this section, we briefly sketch the estimates of signals and backgrounds expected in our configuration.

When evaluated for a null temperature, the Casimir force depends only on the separation between the two mirrors d, their surface S, and the two fundamental constants of relativistic quantum mechanics, the Planck constant $\hbar$ and the speed of light c, as

$$F_{\mathrm{Cas}} = \frac{\pi^2 \hbar c}{240} \frac{S}{d^4}. \tag{2}$$

Throughout the paper, we consider the case of a rectangular mirror of size 10 cm $\times$ 12 cm. We then find a Casimir force $\simeq$25 nN at $d = 5$ μm or $\simeq$1.5 nN at $d = 10\,\mu$m. In fact, at this distance the force is significantly enhanced by the presence of room-temperature blackbody photons. Assuming the mirrors to be perfectly reflecting at all frequencies and both polarizations, we get a simple expression which gives a correct value valid for distances larger than 5 μm

$$F_{\mathrm{thermal}} = \frac{\zeta(3) k_B T}{4\pi} \frac{S}{d^3}. \tag{3}$$

At $T = 300$ K, this force evaluates to $\simeq$38 nN at $d = 5$ μm and $\simeq$5 nN at $d = 10$ μm. More accurate expressions for the thermal Casimir force can be found in [64, 65], which also treat the case of mirrors described by a lossless plasma model.

The real mirrors used in the experiments are actually better described by a Drude model taking into account dissipation of the optical response of electrons. For this situation, various models used for calculating the thermal Casimir force predict values lying between the preceding expression of F_{thermal} and a value smaller by about 50%. This comes from the fact that some of these models predict the two polarizations, transverse electric (TE) and transverse magnetic (TM), to contribute equally to the force whereas others predict the contribution of the TE polarization to vanish for dissipative mirrors [47–55]. Therefore, at distances between 5 and 10 μm, even an accuracy of a few per cent would be enough to bring new interesting information about the temperature corrections to the Casimir force. This point is discussed in more detail in [66, 67] (see in particular figure 2 in [66]) where it is shown that the discrepancy between the two families of models is particularly emphasized in the plane–plane geometry.

When considering the role of possible backgrounds, it first appears that spurious electrostatic charges could be a bottleneck. For instance, a constant stray electric potential of 0.1 V would give rise to a force of $\simeq$25 μN at $d = 5$ μm or $\simeq$5 μN at $d = 10$ μm. This value is larger by roughly three orders of magnitude than the force we wish to measure. This challenge has been faced in all previous experiments through the application of counterbias voltages to cancel the stray voltage as closely as possible. The solution of this problem is more demanding at larger gaps, requiring a control of the stray voltage at a level better than one part in 10^3. Local fields must also be controlled with the greatest care but this issue is less important at larger gaps if the typical correlation length of patch potentials remains constant [68]. These issues will be taken care of at ILL in Grenoble where the surfaces will be precisely characterized thanks to locally available surface analysis facilities. The spurious electrostatic effects will then be corrected either by applying counterbias voltages or via off-line data analysis. The optimal distance for the measurement will be determined by a kind of balance between the control of bias and locally varying spurious fields. Concerning border effects related to the finite surface area of the plates, we can estimate their influence using recent numerical results [69] according to which the change in the Casimir energy can be taken into account by introducing an effective surface area. In the case of a scalar field, the effective area is related to the geometrical area S and its perimeter C by the relation $S_{\text{eff}} \simeq S + 0.36dC$. This leads to a relative correction $\Delta F_{\text{Cas}}/F_{\text{Cas}} \simeq 0.12Cd/S$, directly proportional to the plate separation d. For square mirrors with a surface area of the order 10×10 cm^2 at a distance of 1 μm we get $\Delta F_{\text{Cas}}/F_{\text{Cas}} \simeq 4.8 \times 10^{-6}$. For the electromagnetic field the correction should be similar, with the prefactor 0.36 in the scalar case replaced by unity [70]. Even at the largest separations, corrections due to the border effects are too small to be detectable.

Finally, it is important to evaluate the contribution expected from Newtonian gravity and hypothetical Yukawa forces

$$F_{\text{Newton}}(d) + F_{\text{Yukawa}}(d) = \frac{GM_a M_b}{d^2}\left[1 + \alpha\left(1 + \frac{d}{\lambda}\right)\exp\left(-d/\lambda\right)\right]. \tag{4}$$

When integrated over two plane parallel plates with densities ρ and thicknesses τ, this leads to [71]

$$\begin{aligned} F_{\text{Newton}}(d) &= 2\pi G\rho_a\rho_b S\tau_a\tau_b \\ F_{\text{Yukawa}}(d) &= 2\pi G\rho_a\rho_b S\lambda^2\alpha\exp(-d/\lambda)[1 - \exp(-\tau_a/\lambda))(1 - \exp(-\tau_b/\lambda)]. \end{aligned} \tag{5}$$

For glass plates with the density $\rho_a = \rho_b = 3\times10^3$ kg m^{-3} and the thickness $\tau_a = \tau_b = 15$ mm, we find a Newtonian force $\simeq$10 nN which, as already stated, does not depend on distance. If this force is measured with a torsion balance, it could be subtracted by compensating its action

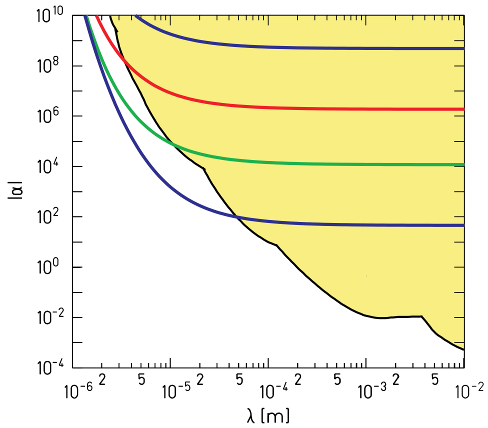

Figure 1. Constraints on Yukawa parameters α versus λ (measured in m) deduced from analysis of the expected sensitivity of the experiment proposed in the present paper. It is assumed that the resolution reaches the level of 1 pN for a force measurement at 5 μm, with all systematic effects mastered at the same level. The four curves correspond to different values of the thickness of the gold layer, respectively, $\tau = 0.3$, 1, 3, 10 μm from the top curve to the bottom one. The shaded region represents the exclusion domain for any hypothetical forces deduced from previous measurements (see figure 5 in [3]).

on both arms of the balance, by comparing measurements performed at different distances, or by using dynamical detection techniques sensitive only to the spatial gradient of the force. The Yukawa force (5) is then determined by the parameters of the metal layers deposited on the glass plates and by the unknown parameters λ and α. To give an estimate, we consider the case of gold layers with $\rho_a = \rho_b = 19.3 \times 10^3$ kg m^{-3}. The thickness $\tau_a = \tau_b = \tau$ should be chosen with care since it plays a key role in the determination of expression (5): the thicker the layer, the larger the Yukawa force. Note that the estimate of the Yukawa force might be enlarged by adding the contribution of the glass plate to that of the gold layer. However, the metal layer would have to be changed in order to investigate the material dependence of the expected Yukawa signal, so we prefer to give conservative numbers corresponding to the metal layer alone.

If we consider that we can reach a resolution of the order of 1 pN for the force measurement, with all systematic effects mastered at the same level, we can translate expression (5) into the following limit on the relative Yukawa amplitude α versus λ

$$\alpha \simeq 532 \times \exp\left(\frac{d}{\lambda}\right)\lambda^{-2}\left[1 - \exp\left(-\frac{\tau}{\lambda}\right)\right]^{-2}. \tag{6}$$

The corresponding limit in the (λ, α) plane is shown in figure 1 for four different values for the layer thickness, with λ spanning from 10^{-6} m to 10^{-2} m. We compare our limits to the bounds already known at short distances, as represented by the black solid line (corresponding to figure 5 in [3]). To fix ideas, the bound obtained for $\lambda = 10$ μm, $d = 5$ μm and $\tau = 10$ μm is $\alpha \sim 10^3$ which would improve the current limits by a factor of 100. Larger values for τ lead to still better bounds, allowing one either to rule out theoretical models discussed in [26] or to bring new information supporting one of them. Note that smaller values of τ, such as a few units of 0.1 μm, can be used in the first stage of the experiment aimed at measuring the thermal contribution to the Casimir force.

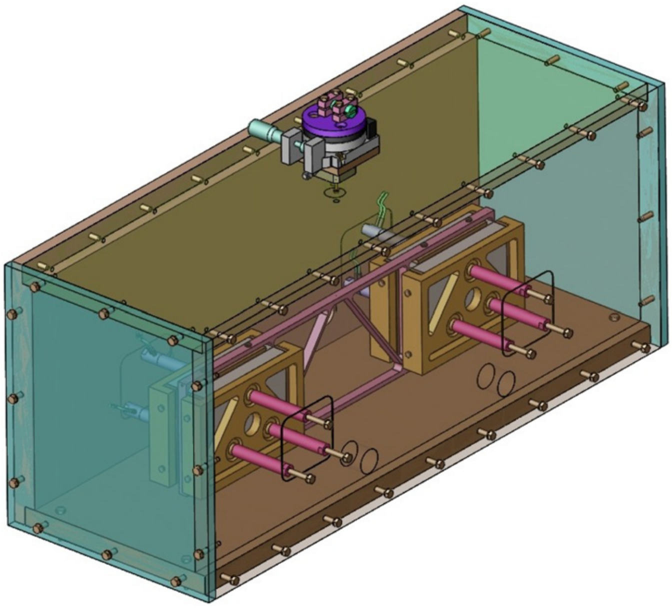

Figure 2. Torsional balance under construction at ILL, Grenoble. The positioning of the flat surfaces is assured through three piezoelectric actuators on each plate. A system of capacitors on the other side of the balance, located at larger gaps, allow for measurement of the force necessary to keep the distance of the balance plates from the flat surfaces constant. The use of high-precision piezoelectric actuators and the closed-loop option available through their controllers allow us to push the accuracy in the parallelism down to a few times 10^{-7} radians. A four-mirror symmetric scheme allows for numerous cross-checks on parallelism, absolute values of the gaps, and control over many systematic effects. The absolute value of the gap can also be determined by using wire spacers with diameters known within 0.2 μm.

3. Torsional balance: design and sensitivity

For the measurement of the force with large surfaces, we intend to use a torsional balance similarly to those used in experiments exploring the equivalence principle with the Eötvos technique (see [72, 73] for updated discussions of torsional balances). The torsional balance has been designed at ILL and its construction is in progress (see figure 2 for a schematic layout of the apparatus). The two pairs of plates are installed on opposite arms of the assembly, one moving and the other static. The plates for the Casimir force measurement and for the electrostatic calibration/actuation have a surface area of 120 cm^2 for measurements at the largest gaps of $\simeq$10 μm. The area can be reduced to 15 cm^2 for measurements at smaller gaps; the larger mirror size increases the force magnitude but the larger mass of the plates/frame assembly and thicker suspension wire result in a decreased absolute sensitivity of the force measurement. The optimal parameters will be chosen experimentally by maximizing the measurement accuracy while minimizing systematic effects.

Based on the measurement of the free oscillation of the balance with tungsten or quartz wires of diameter 50–150 μm, the torque sensitivity is estimated in the 1–100 μN m rad^{-1} range. Considering the leverage of the balance of $R = 30$ cm, the force δF_{min} which, if other sources of noise are kept under control (most notably the seismic noise, with proper attenuation stages, and the electronics noise, using commercial low-noise operational amplifiers with voltage noise spectral density corresponding to $S_v^{1/2} \simeq 10$ nV Hz$^{-1/2}$), is limited by the

Brownian noise with force noise spectral density $S_{F_{min}} = 2\gamma K_B T/R^2$, where γ is the effective damping factor of the torsional oscillator and T is the environmental temperature. By using realistic values in our configuration for the relevant parameters ($\gamma = 10^{-6}$ N m s rad^{-1}, corresponding to the estimated damping including the effect of the electric feedback circuit, and $T = 300$ K) we estimate the force noise spectral density as $S^{1/2}_{F_{min}} = 3 \times 10^{-13}$ N Hz$^{-1/2}$, corresponding to a minimum detectable force (with unitary signal-to-noise ratio) smaller than 1 pN having measurement times of the order of 1 s. Considering the quoted torque sensitivity, this is equivalent to a minimum detectable displacement of $\simeq$1 nm.

The position of each side plate is controlled using three high-precision piezoelectric actuators switched in the dynamic positioning mode in such a way that the distance between the side plate and the corresponding balance plate is kept constant during the force measurement, similarly to the procedure described in [15]. The absolute distance between them can be controlled with an accuracy of about 0.2 μm. A much more precise read-out of variations of the distance will be performed by means of capacitors symmetrically located on the other side of the torsional balance with respect to the active surfaces. Their design, for a gap of about 100 μm, has been carefully chosen to make their influence on the response of the torsional balance negligible and ensure at the same time enough force sensitivity. The read-out will be based on the voltage signal required to keep the distance constant, and will be measured capacitively, using a feedback servo circuit. The balance will be placed in a vacuum chamber mounted on an antivibration table with an active levelling system. Noise induced by the tilt should be kept under control by optimizing the wire length, and can be further minimized by locating the centre of force measurement at the same height as the suspension point of the wire, and by properly shaping the plates, as discussed in more detail in [74]. Small tilts in the relative position will not affect to the leading order the resulting Casimir force signal.

The key feature of the project is the use of parallelization procedures in part already tested for an experiment going on at ILL [59–63]. The experiment is aimed at observing the discretized quantum states of ultracold neutrons in a combination of the Earth's gravitational field and a reflecting barrier of 10 cm size. These states are identified through the measurement of the neutron transmission between a horizontal mirror and an absorber. The surfaces are at a distance of few micrometres in order to scan the interesting region where no neutrons are expected if they have quantized energy levels. The quantized states have a spectrum starting with the lowest eigenvalue at 1.4 peV, which has the vertical position probability density located at about 15 μm. A negligible counting rate is therefore expected for smaller separation between the mirror and the absorber. In order to rule out systematic shifts due to distance offset, stringent requirements on the parallelism and the control of the separation are necessary.

In a preliminary report [60], the ground state was clearly identified, later [63] parameters of the first and second quantum state were measured as well. The experiment is progressing with the aim of performing precision measurements of parameters of these and higher quantum states in order to get better constraints for extra short-range forces [62], and this requires more demanding parallelism control systems. The parallelism for sizes of the order of 10 cm is obtained by using so-called *inclinometers*, which have a sensitivity to deviations from parallelism smaller than 10^{-6} radians. Piezoelectric actuators in a closed loop with precision inclinometers allow for maintaining horizontal surfaces at the same level of precision. This accuracy is a routine result of the most recent runs in Grenoble on the experiment involving quantum states of the neutron. In comparison, the estimate of the parallelism for the Padova Casimir force experiment was about 3×10^{-5} radians.

This extremely good control of deviations from parallelism makes it also necessary to perform a precise analysis of the deviation from the flatness of the plates [75]. We will use standard optically polished flat surfaces already utilized as the reflecting surface in the neutron experiment. These surfaces are routinely studied using small angle x-ray scattering, and the root-mean square roughness can be as low as $\simeq$ 1–2 nm for the plates to be used in the first stage of this experiment. The search for hypothetical Yukawa forces will necessitate the use of metal layers with a thickness larger than 1 μm (see the discussion in the preceding section). This may lead to a degradation of the quality of their surface state, a problem which will be addressed by using already available techniques. First, the surface state can be studied for each sample by means of x-ray scattering, allowing one to obtain the Fourier spectrum of the deviations from perfect flatness [75]. Then, these data can be transformed into force corrections via nonspecular scattering models recently developed for the purpose of evaluating roughness effects [76, 77]. Overall, the availability of surface analysis facilities providing *in situ* characterization of the apparatus allow for more accurate assessments of this limitation than in previous experiments, and the remaining concern may be the preservation of the mirror flatness during and after their mounting.

4. Conclusions

We have discussed a proposal to study Casimir forces in the distance range from a few μm to 10 μm, which is the distance range where the theoretical predictions look the most promising. At the same time, it is a 'no experiment's land' where scarce progress has been reported so far, in contrast to the more widely explored regions of either shorter distances, with Casimir experiments up to $\simeq$1 μm, or larger distances, with Cavendish-type experiments down to $\simeq$100 μm.

The program will consist in assembling the apparatus, checking the plate parallelism, calibrating the instrument with known signals like electrostatic forces and studying the background and systematic effects, in particular the residual electrostatic effects. A complete characterization of the sensitivity of the apparatus and noise sources will be performed. The apparatus may also be calibrated with a physical signal such as the gravitational force, by adding controllable external masses as sources in a Cavendish-like experiment. This physical calibration will bypass the current controversies on the actual sensitivity of some of the current experiments on Casimir forces. The measurement will first aim at testing the conflicting models of thermal corrections to the Casimir force. In particular, these results should contribute to settling the controversy in the literature by bringing into the debate experimental information. On a longer term, the goal will be to improve the constraints on non-Newtonian gravity by analysing the residuals between experimental measurements and theoretical predictions.

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