

The “Unfriending” Problem

The Consequences of Homophily in Friendship Retention for Causal Estimates of Social Influence

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Abstract

An increasing number of scholars are using longitudinal social network data to try to obtain estimates of peer or social influence effects. These data may provide additional statistical leverage, but they can introduce new inferential problems. In particular, while the confounding effects of homophily in friendship formation are widely appreciated, homophily in friendship *retention* may also confound causal estimates of social influence in longitudinal network data. We provide evidence for this claim in a Monte Carlo analysis of the statistical model used by Christakis, Fowler, and their colleagues in numerous articles estimating “contagion” effects in social networks. Our results indicate that homophily in friendship retention induces significant upward bias and decreased coverage levels in the Christakis and Fowler model if there is non-negligible friendship attrition over time.

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1 Introduction

Until recently, scholars have given relatively little attention to the influence of personal relationships on human behavior, instead studying people largely as atomistic individuals ripped from the social context in which they live. Thankfully, this impoverished approach has started to give way to an interdisciplinary movement seeking to understand the influence of social networks in domains ranging from health to politics. Results from cases in which peers were randomly or quasi-randomly assigned such as college roommates have provided credible evidence of such effects (e.g., Sacerdote 2001; Zimmerman 2003; Carrell, Hoekstra and West 2011; for a review, see Kremer and Levy 2008).¹

However, when random assignment of peers is not feasible, researchers must use observational data, which creates serious inferential problems (Manski 1993). In particular, peers may behave similarly as a result of “correlated effects” such as common environmental shocks or shared characteristics rather than social influence. Given the likelihood that peers will be similar on a range of characteristics due to homophily (McPherson, Smith-Lovin and Cook 2001), distinguishing between homophily and peer effects has proven to be a very difficult challenge.²

Many scholars have therefore turned to use longitudinal network data to try to separate homophily from social influence effects. In principle, observing dyads over multiple periods seems as though it could help separate homophily in tie formation from subsequent peer influence. However, homophily may also affect whether social ties are *maintained* over time, confounding estimates of peer influence effects. We call this the “unfriending” problem in honor of the Facebook practice of removing a person from one’s list of friends on the online social network site.

We illustrate the potential inferential consequences of this problem below in an analysis of the statistical model used to estimate “contagion” ef-

¹Related experimental studies by Nickerson (2008) and Fowler and Christakis (2010) provide suggestive evidence that such effects can extend two or more degrees, though the applicability of their results to non-experimental contexts is unclear.

²For a review of the literature, see Soetevent (2006).

fects in a series of widely publicized studies by Christakis, Fowler, and their colleagues (hereafter CF). Our Monte Carlo simulations, which are adapted from those of CF, indicate that their model’s estimates of social influence effects are unbiased and have accurate coverage levels when homophily in friendship retention is not present. However, when non-negligible attrition is present, estimates from the CF model show substantial upward bias and decreased coverage levels as homophily in friendship retention increases. In short, the “unfriending” problem can create spurious evidence of social influence when none exists.

2 Leveraging dynamic networks: A solution?

The CF studies, which use longitudinal social network data from the Framingham Heart Study (FHS) and National Longitudinal Study of Adolescent Health (Add Health), make strong claims about the effects of one’s friends³ on a wide range of dependent variables: obesity (Christakis and Fowler 2007; Fowler and Christakis 2008b), smoking (Christakis and Fowler 2008), happiness (Fowler and Christakis 2008a), loneliness (Cacioppo, Fowler and Christakis 2009), depression (Rosenquist et al. 2010), alcohol consumption (Rosenquist, Fowler and Christakis 2010), sleep loss (Mednick, Christakis and Fowler 2010), and divorce (McDermott, Fowler and Christakis N.d.). Each CF paper uses the same approach, estimating versions of the following model for ego i and alter j observed at times t_0 and t_1 :

$$Y_{i,t_1} = f(Y_{i,t_0}, Y_{j,t_0}, Y_{j,t_1}, \text{controls}) \quad (1)$$

These models are typically estimated using generalized estimating equations (GEE) with an independent correlation structure to account for repeated observations of the same ego (specifically, those who name or are named by more than one friend) and dyad (those who name each other and are thus included twice, one each as the ego and once as the alter). The

³The studies typically also estimate social influence effects among family members; we do not consider the validity of those estimates here.

functional form of the model varies depending on the distribution of the dependent variable (logistic regression if the dependent variable is binary; linear regression if it is continuous or quasi-continuous).⁴

CF argue that this specification controls for initial homophily (i.e., the likely similarity between Y_{i,t_0} and Y_{j,t_0}), allowing us to identify the causal effect of *changes* in the alter's trait from t_0 to t_1 by estimating the effect of Y_{j,t_1} controlling for Y_{j,t_0} . In Christakis and Fowler (2007), they write that including alters' lagged obesity as a covariate "controlled for homophily" (373). In later work, the language is somewhat more hedged—for instance, they write in Christakis and Fowler (2008, 2251) that a lagged measure of alter smoking "*helped* to account for homophily" (our emphasis)—but the suggestion that the coefficient for Y_{j,t_1} is a causal estimate of peer effects remains. Christakis and Fowler (2009) expands on these claims, stating that observed clustering at up to three degrees of separation reflects "Three Degrees of Influence" for happiness (51), obesity (108), and smoking (116) and asserting that we "now know that obesity is contagious" (111).

Cohen-Cole and Fletcher (2008*a,b*) and Halliday and Kwak (2009) question whether CF's model adequately controls for homophily, which has been shown to be significant for weight status (Trogon, Nonnemakera and Paisa 2008; Halliday and Kwak 2009; Valente et al. 2009)⁵, and suggest that their model may generate spurious inferences (see also Shalizi and Thomas 2011, Lyons N.d., and Ellen 2009).⁶ In response, CF describe Monte Carlo simulation results "documenting that homophily (ranging from no homophily to complete homophily) does not result in bias in the estimates of induction in this model specification" (Fowler and Christakis 2008*b*, 1404).

⁴For other approaches to obtaining causal estimates of peer effects in longitudinal or repeatedly sampled network data, see Anagnostopoulos and Mahdian (2008), Bramoullé, Djebbaria and Fortin (2009), Aral, Muchnika and Sundararajana (2009), and Lazer et al. (2010).

⁵de la Haye et al. (2010) finds homophily in obesity-related behaviors as well. For further explorations of possible social transmission of obesity or weight status, see Anderson (N.d.), Barnes, Smith and Yoder (N.d.), Brown, Hole and Roberts (N.d.), McFerran et al. (2010*a*), McFerran et al. (2010*b*), and Campbell and Mohr (N.d.).

⁶There are other concerns with this statistical model such as possible simultaneity bias and environmental confounding that we do not discuss here.

CF's Monte Carlo results, which are presented in Fowler and Christakis (2008b) and in a very similar form in Fowler et al. (2011), are derived from a stylized model in which a population of individuals with a randomly chosen value on some characteristic of interest form friendships and then influence each other or not (we discuss the procedure in more detail below). CF find that estimates of this influence coefficient are approximately unbiased across varying levels of homophily when the true peer effect is 0 and have a slight downward bias when the true peer effect is 0.1. On this basis, they conclude that "This simulation evidence suggests that the [Cohen-Cole and Fletcher] assertion that homophily causes us to overestimate the size of the induction effect is false." However, as we discuss below, their simulation does not incorporate friendship *attrition* and thus fails to fully account for the effects of homophily.

3 The "unfriending" problem in longitudinal data

Due to the prevalence of cross-sectional data and interest in fixed characteristics such as race and sex, scholars of social networks have tended to think about homophily in relatively static terms and to analyze it as a propensity to form ties with others who share similar characteristics. However, social networks are actually the result of a dynamic process of friendship *formation* and *dissolution*.

As a result, while relatively few longitudinal network studies have been conducted, most report substantial levels of friendship dissolution between survey waves. For instance, Mollenhorst (2009) finds that about half of adult friendships were replaced over the seven years between the two waves of his survey. For the adolescents in Add Health, Moody (N.d.) found approximately half of the friends named by respondents during in-school interviews were named again during in-home interviews six to eight months later. Studies of social networks among younger children have found rates of attrition that are even higher still (Hallinan and Williams 1987; Cairns and Cairns 1995). A partial exception is the FHS data used by CF, which was conducted in a relatively stable community. O'Malley and Christakis

(N.d.) report that 82% of friendships were maintained between waves in FHS, which could be a result of asking for “close friends” who could help the researchers contact the participant in the future.⁷

When friendship attrition is present, homophily is likely to be a factor. Just as people who are similar are more likely to be friends, friends who are less similar are more likely to *stop* being friends. Most of us have had friends from whom we have grown apart in this way. As we have less in common with those people, we stop spending time with them and eventually fall out of touch. In some cases, one person may deliberately end the relationship as a result of differences in political views, alcohol consumption, or other behaviors or characteristics.

Numerous examples of homophily in tie dissolution have been documented in the sociology literature (see Burt 2000 and McPherson, Smith-Lovin and Cook 2001 for reviews). One well-known example is a two-wave study of adolescent friendships by Kandel (1978). She describes homophily in friendship retention based on both initial characteristics and subsequent behavior (430):

At time 1, prior to any subsequent change, pairs that will remain stable over time are much more similar in their behaviors and values [marijuana use, educational goals, political views, and delinquency] than the subsequently unstable pairs... At time 2, homophily among former friends is lower than among new friendship pairs or stable pairs.

She interprets these results as a combination of selection (choosing to become and stay friends with those who are like you) and socialization (acting more like your friends in those relationships you maintain) (433–435):

The results support the general conclusion that adolescents coordinate their choices of friends and their behaviors, in particu-

⁷ CF report that they treated friendship ties as maintained when a friend as named at t_1 and t_3 but tie information was missing at t_2 (personal communication). Under this definition, 96% of friendship ties were maintained between waves. However, since missing tie information was often the result of missing an exam altogether, friendship retention in their statistical analyses is likely to be lower.

lar the use of marijuana, so as to maximize congruency within the friendship pair. If there is a state of unbalance such that the friend's attitude or behavior is incongruent with the adolescent's, the adolescent will either break off the friendship and seek another friend or will keep the friend and modify his own drug behavior.

Childhood and adolescent friendships have also been found to be more stable when students are more alike by gender (Tuma and Hallinan 1979; Degirmencioglu et al. 1998; Moody N.d.), race (Moody N.d.), grade (Moody N.d.), achievement/competence (Tuma and Hallinan 1979; Newcomb, Bukowski and Bagwell 1999), and aggression (Newcomb, Bukowski and Bagwell 1999).

Similar patterns have been observed among adults. For instance, Kossinets and Watts (2009) find that dyads who are more similar demographically are less likely to break ties in the email network of a large American university (433–434); Popielarz and McPherson (1995) show that members of voluntary groups who are dissimilar from other members are more likely to leave; and Burt (2000) documents homophily in the maintenance and dissolution of bankers' collegial relationships along several dimensions. Most notably, O'Malley and Christakis (N.d.) document homophily in friendship retention within the FHS data used in almost all of CF's studies. They find that differences in BMI, smoking, and measures of body type are significantly related to the dissolution of friendship ties.

These patterns of homophily in unfriending are potentially a problem for any statistical analysis of peer effects in observational data. Social network scholars have been concerned for some time about the difficulty of separating homophily in friendship formation from contextual influences and peer effects. However, homophily in friendship retention is an equally difficult problem. In particular, unfriending is a significant concern for the CF approach. The specification of their generalized estimating equation models requires an ego to name an alter as a friend for two or more consecutive waves (although see footnote 7 above). In this way, they attempt to leverage longitudinal network data to control for past friendship

ties. However, what happens when some of the dyads at t_0 are no longer friends at t_1 ? Fowler and Christakis argue that including such friendship pairs in their data will bias the results against finding an effect because “it essentially adds ‘random’ non-friend relationships (i.e., people who are no longer friends) to the pool of friends” (Fowler and Christakis 2008*b*, 1401). This is a legitimate issue; non-friends presumably can no longer influence the person in question.

However, the friendships that have been terminated may not have been “random.” Relationships often end for a reason. If the reason for friendship termination is related to or is correlated with changes between t_0 and t_1 in the underlying trait we are examining, it will induce an association between Y_{i,t_1} and Y_{j,t_1} that is not captured by the lagged values of the variables in question.⁸ In the CF model, the coefficient for Y_{j,t_1} is interpreted as a causal effect. As such, the association induced by homophily in the unfrnding process could create the appearance of an influence effect even if none exists.⁹

4 Monte Carlo simulation procedure

To determine the extent to which homophily in friendship retention might lead to spurious inferences under CF’s model, we conduct Monte Carlo simulations in which we do not allow for social influence of alters on egos.¹⁰

⁸The threat of non-random unfrnding can be considered a specific example of the problem noted by Nickerson (Fowler et al. 2011) in his discussion of the value of conducting experiments on static networks: “it is always possible that measurement of the network post-treatment could be correlated with the provision of the treatment. If treatments cause certain relationships to become more salient or networks to change composition, then many strategies for defining networks ... may cease to be equivalent for treatment and control groups.”

⁹In principle, one might attempt to model the selection process by which friendships are maintained in order to recover the true value of the influence coefficient. However, it seems impossible to obtain data that is granular enough to separate stochastic changes in the trait of interest from t_0 to t_1 from peer effects. In the absence of such data, accurately modeling the friendship retention process requires knowing the value of Y_{i,t_1} that would have been observed if no influence had taken place—an unobserved counterfactual.

¹⁰A broader question that we do not engage here is whether statistical models of such effects are formally identified. Shalizi and Thomas (2011) presents a graphical causal model

In reality, of course, such influence seems likely to be common. However, we follow standard practice in Monte Carlo evaluation of statistical models in assuming that the coefficient in question is zero and estimating the bias and coverage of the model. (The simulations reported in Fowler and Christakis (2008b) and Fowler et al. (2011) also include estimates of bias in the model where the true influence effect is equal to 0.) By working within this framework, which offers well-defined standards of model performance¹¹, we can evaluate the the risk that homophily in friendship retention will lead researchers using the CF model to falsely reject the null (the standard inferential approach used in applications of the model).¹²

In this section, we explain the Monte Carlo simulation procedure used in our analysis, which is adapted from code used in Fowler and Christakis (2008b) and Fowler et al. (2011). The R code used to generate our results will be posted on the *Social Networks* website along with the electronic version of our article.

The simulation proceeds as follows:

1. A normally distributed trait Y_{t_0} is randomly generated at time t_0 for a population of $n=1000$ actors where $Y_{t_0} \sim N(50, 10)$.¹³
2. Differences in Y are computed for all dyads of actors i and j in the same manner as CF:

$$d_{i,j} = -|Y_{i,t_0} - Y_{j,t_0}|$$

arguing that such effects are generically unidentified in observational data for a person i when some latent trait “ X_i directly influences $Y_{i,t}$ for all t .” In such cases, even controlling for $Y_{i,t-1}$ and $Y_{j,t-1}$ is not sufficient to identify the causal effect of a network tie $A_{i,j}$ on $Y_{i,t}$.

¹¹By contrast, it is rare to test a model with a non-zero null hypothesis using frequentist statistics and it is not entirely clear how such a model should be evaluated.

¹²These Type I errors could happen in a variety of ways. First, many researchers test hypotheses for which we have weak priors and a null hypothesis of zero may be a valid initial assumption. In such cases, it is important to know whether CF’s model may generate spurious findings. Moreover, even if we have strong priors that social influence is taking place, we can never be sure that the expected effect is present in the data we are analyzing. It is important to know whether the CF approach could generate a spurious positive effect in those cases.

¹³Fowler and Christakis (2008b) draw their trait values from the empirical distribution of BMI in the Framingham data. Since we do not have access to those data, we use the initial distribution from the simulation in Fowler et al. (2011) instead.

The difference term is negatively valued so that dyads with similar traits have high values.

3. Ties $A_{i,j}$ are created as a function of $d_{i,j}$ using a probit model based on a latent variable $A_{i,j}^*$. These ties are directed (i.e., $A_{i,j}$ does not imply $A_{j,i}$).

$$A_{i,j}(t_0) = \begin{cases} 1 & \text{if } A_{i,j}^* > \epsilon_{i,j} \sim N(0,1) \\ 0 & \text{if } A_{i,j}^* \leq \epsilon_{i,j} \sim N(0,1) \end{cases}$$

where

$$A_{i,j}^* = \eta_0 + \eta_1 d_{i,j}$$

η_0 represents the baseline propensity to form ties and η_1 represents the coefficient for homophily (positive values indicate higher levels of homophily).

4. All actors receive a normally distributed, independent shock u_i to their trait Y_{t_0} where $u_i \sim N(0,5)$.¹⁴
5. In this step, CF assume that ego traits Y_{t_1} are updated as a function of their previous trait value Y_{t_0} and influence from their alters.¹⁵ However, since we assume there is no peer influence, ego traits at t_1 are

¹⁴Fowler and Christakis (2008b) use shock values from the empirical distribution of changes in BMI in the Framingham data. Again, since we do not have those data, we use the shock distribution from the simulation in Fowler et al. (2011) instead.

¹⁵In CF's simulation, all egos' values of Y are updated as a weighted average of their own current value of $Y_{t_0} + u_i$ and the average value of $Y_{t_0} + u_i$ for their alters:

$$Y_{i,t_1} = (1 - b_1)(Y_{i,t_0} + u_i) + b_1 \left(\frac{\sum_j A_{ij}(Y_{j,t_0} + u_j)}{\sum_j A_{ij}} \right)$$

where b_1 is represents the relative influence of alters on egos. Since we set peer influence to 0, our results are not sensitive to this assumption.

simply the sum of their previous value Y_{i,t_0} and their shock u_i :

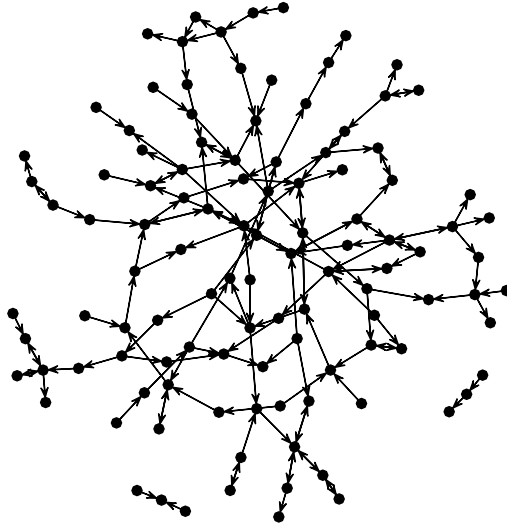
$$Y_{i,t_1} = Y_{i,t_0} + u_i$$

6. All actors update their friendship ties $A_{i,j}(t_1)$ as in steps 2–3.
7. Following CF, we estimate a linear regression model using generalized estimating equations with an independent correlation structure for all dyads who are friends at t_0 and t_1 :

$$Y_{i,t_1} = \beta_0 + Y_{i,t_0}\beta_1 + Y_{j,t_0}\beta_2 + Y_{j,t_1}\beta_3 + \epsilon \text{ where } A_{i,j}(t_0, t_1) = 1 \quad (2)$$

To illustrate the results, Figure 1 provides a sample network at the end of one simulation (here $n=120$ since networks of the size used in our simulations are too dense to parse visually).

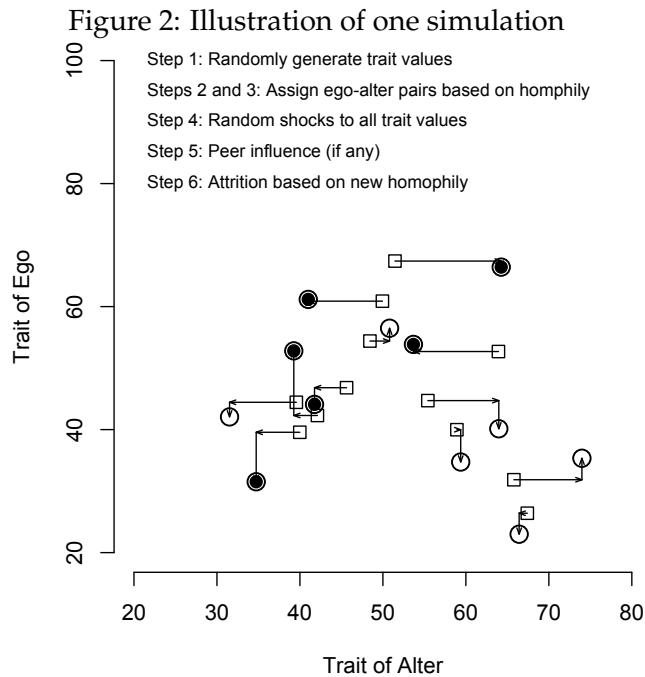
Figure 1: Sample network



Sample network: $\eta_0=1$ in step 6; $\eta_1=0.05$ in steps 3 and 6; s.d.(u)=5; $\eta_0=-2.5$; $n=120$ (isolates not displayed).

The simulation process is illustrated in Figure 2. Open squares rep-

resent values of Y for ego-alter pairs after initial friendships have been formed. There is some correlation due to homophily. The actors then both experience shocks to their values of the trait, which are represented by arrows. The new values of the trait are indicated by the circles at the end of each path. The GEE should estimate the effect of the alter's shock on the ego controlling for the alter's previous Y value. However, at the friendship retention stage, some of the pairs cease to be friends. Only those pairs indicated by the solid circles remain in the data; those that are open circles have ceased to be friends.



This procedure above modifies CF's original approach in two key ways. The most important modification is step 6, which repeats the friendship model from step 3, allowing for friendships to end based on homophily after a shock to Y . This step is crucial in longitudinal network data as discussed above. While the shock u to Y_0 is assumed to be randomly dis-

tributed, some people may cease to be friends as a result of (or for reasons that are correlated with) the shock that they received, inducing a correlation in u for those dyads whose friendships persist. Our simulation is intended to test whether this correlation could appear to be a causal influence effect even if we control for lagged values of the trait.

The way we model friendship formation and dissolution also differs from CF’s approach. We introduce a latent variable probit model where a tie exists if a deterministic component ($A_{i,j}^* = \eta_0 + \eta_1 d_{i,j}$) is greater than a stochastic error term ($\epsilon_{i,j} \sim N(0, 1)$). By contrast, CF generate a probability of a tie that is a weighted average of Y_0 and a random component and then conduct a random draw with that probability to determine whether a tie exists. This process combines two sources of random noise. The first is meant to model factors other than homophily in friendship choice, while the second models the inherently stochastic component of friendship formation. However, this partition is not readily interpretable—any unobservable influence on the outcome variable in a statistical model can reasonably be included in the stochastic component if it is not also systematically related to the independent variables.

The result of this double-randomness is that CF’s simulations do not generate sufficiently high correlations in the outcome variable among friends even when ties are formed on the basis of “complete homophily” (Fowler and Christakis 2008*b*, 1405). When we replicated CF’s simulations with homophily set to its maximum value, the mean correlation between ego and alter on the outcome variable was 0.12 (2.5-97.5 percentile range: 0.05, 0.18). In practice, social network datasets often display higher levels of correlation among friends. For instance, Halliday and Kwak (2009) find a BMI correlation of 0.19 among adolescent peers in the Add Health data after accounting for school fixed effects. The correlation in reported vote choice between respondents to the 2000 American National Election Study and their friends ranged from 0.43 to 0.57 (results available upon request). Even accounting for the effects of projection (i.e., respondents falsely perceiving that their friends agree with them), these results suggest that simulations should consider higher levels of homophily. While some of these correla-

tions may be due to contagion, we cannot assume such an effect. As such, we designed our simulations to cover a wide range of homophily on the trait of interest.

5 Monte Carlo results

Following standard procedure in Monte Carlo evaluations of statistical models, we set the true contagion effect (i.e., the parameter b_1) to 0 and estimate mean bias and coverage levels for the CF model. In our simulations, we set η_0 to -2.5 at the friendship formation stage in order to generate realistic numbers of friendships at t_0 .¹⁶ We then vary η_1 at both stages to generate realistic levels of homophily in both friendship formation and retention and also vary η_0 at the friendship retention stage to consider different friendship attrition rates.¹⁷

- Homophily in friendship formation: We vary the homophily parameter η_1 in step 3 to cover five possible levels: 0.0, 0.0125, 0.025, 0.0375, and 0.05.
- Homophily in friendship retention: We vary the homophily parameter η_1 in step 6 to cover three levels: 0.0, 0.025, and 0.05.
- Levels of friendship retention: We vary η_0 in step 6, considering values of 0.0, 0.5, and 1.0 to represent realistic variation in attrition rates between t_0 and t_1 . We also include the value of 1.85 to cover the maximum FHS friendship retention rate of 0.96 (see footnote 7 above).¹⁸

¹⁶CF's Framingham subjects typically only name one friend due to the nature of the instrument used. However, this structure is unusual and we do not mimic it here. (See Thomas and Blitzstein N.d. for a related discussion of how binary networks with censored outdegree information may generate inflated social influence effects.)

¹⁷In practice, it is possible that homophily in friendship formation and retention could generate feedback effects in a more elaborate multi-stage simulation (as, for instance, new, more similar acquaintances displace old, less similar friends), but we do not attempt to model the complexities of those potential dynamics here, especially given our lack of substantive knowledge about how such a process would operate.

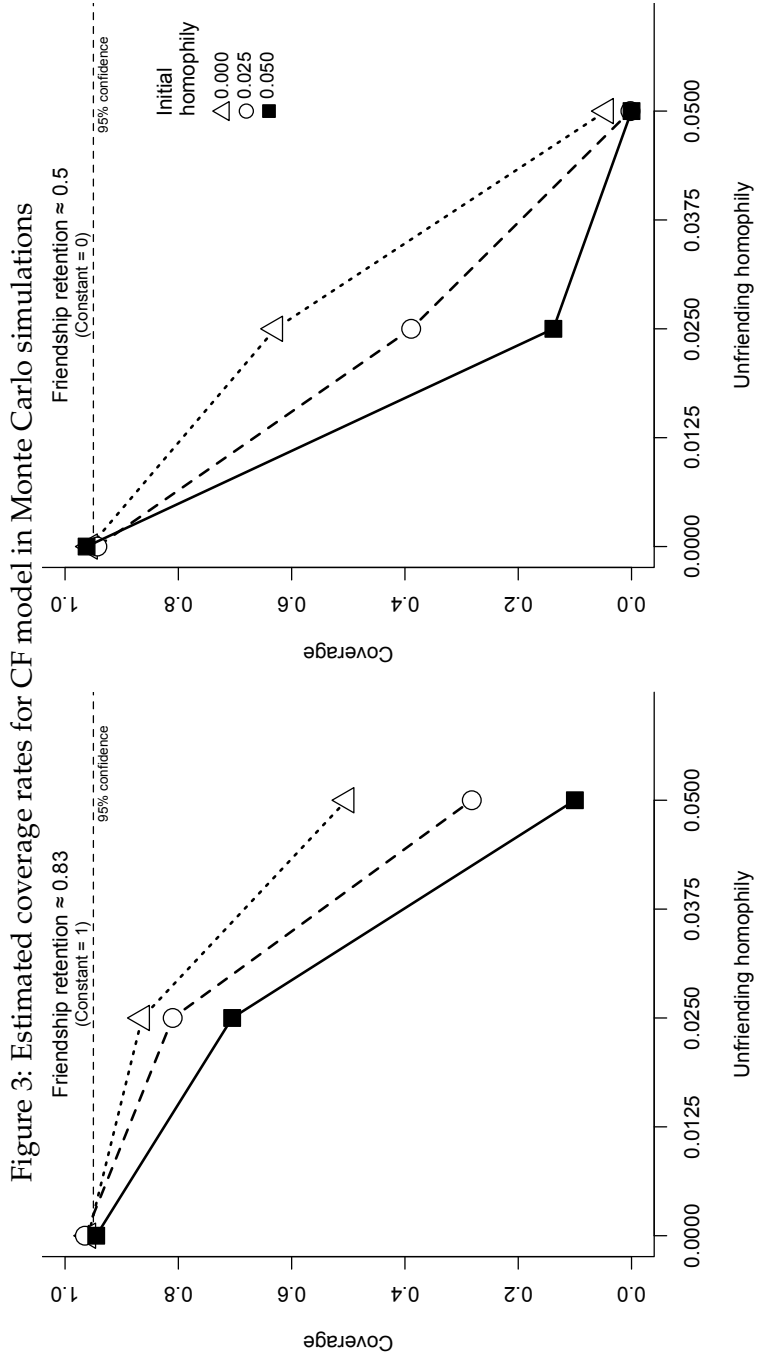
¹⁸Additional simulations in which we also vary the standard deviation of the shock u to Y_0 generate similar results. They are thus omitted but available on request.

These values correspond to realistic levels of homophily and friendship retention. For instance, O'Malley and Christakis (N.d.) use FHS data to test whether they observe homophily in friendship formation and dissolution using BMI and related measures. Their hierarchical logit models find no effect for homophily at the formation stage, which would correspond to a η_1 value of 0.0 in step 3, but find a statistically significant and substantively large coefficient on BMI for friendship retention. While we cannot directly compare coefficient values given differences in data and estimation techniques, the value of 0.05 for η_1 in step 6 appears to be an appropriate comparison.

Complete results from the Monte Carlo simulations, which were performed 1,000 times for each unique combination of model parameters, are presented in Table 1 at the end of the document. The first result of note is that our simulations cover realistic ranges of both ego-alter trait correlation and friendship attrition. As we discuss above, observed ego-alter trait correlations can be quite high. In the simulations, these correlations range from approximately 0.0 to 0.6 at t_0 and 0.0 to 0.4 at t_1 . Similarly, the current simulation yields friendship retention rates of approximately 50% in the high attrition case (retention constant $\eta_0 = 0$), 69% in the moderate attrition case ($\eta_0 = 0.5$), 84% in the low attrition case ($\eta_0 = 1.00$), and 97% in the very low attrition case ($\eta_0 = 1.85$).

Figures 3 and 4 plot how well the GEE estimate of equation 2 performed for varying values of the homophily coefficients when the constant $\eta_0 = 1$ and $\eta_0 = 0$ at the friendship retention stage (those for $\eta_0 = 0.5$ and $\eta_0 = 1.85$ are not plotted but are available in Table 1). These values approximately correspond to attrition rates in FHS (18%) and Add Health (50%), respectively. For visual clarity, the figures include bias and coverage levels for η_1 at the friendship formation stage of 0, 0.025, and 0.05. (When η_1 equals 0.0125 and 0.0375 at the friendship formation stage, the results fall between the three lines in Figures 3 and 4. They are thus omitted from the figures but described in Table 1.)

First, Figure 3 presents the probability that the estimated 95% confidence interval covers the true contagion parameter of 0. When homophily

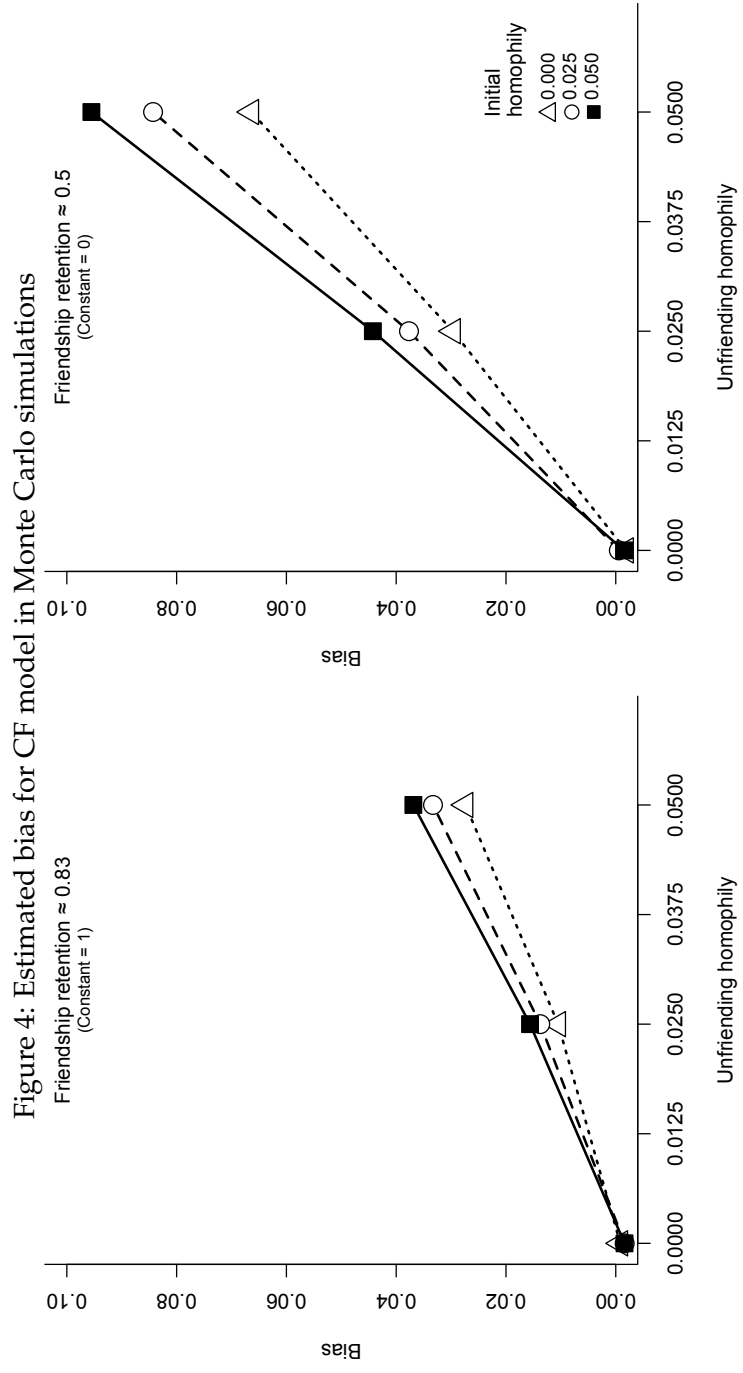


in friendship retention is 0, the confidence intervals for the CF models accurately bracket the true value approximately 95 percent of the time. However, as homophily in friendship retention increases, coverage rates decline dramatically. For instance, when initial homophily is 0 and friendship retention is high (the open triangles in the left panel of Figure 3), coverage falls to approximately 50 percent as homophily in friendship retention increases. In the most extreme cases, the confidence interval almost never includes the true value of the influence coefficient. Even when friendship attrition is very low (approximately 4%), Table 1 indicates that the CF model still has some coverage problems (decreasing as low as 81 percent) when homophily in friendship retention is high (see lines 46 to 60 of Table 1). In additional simulations, we find that coverage problems worsen further as sample size increases, thereby increasing the likelihood that the CF model will falsely reject the null hypothesis that the influence effect is 0 (in practice, CF's data frequently include far more than 1000 observations).

As expected, coverage degrades because the model displays an upward bias, as is evident in Figure 4, which presents the mean value of the estimated peer effect (which has been set to 0 in the simulations). When unfriending is not affected by homophily, the estimator is unbiased. But as homophily in friendship retention increases, a correlation emerges in changes in the trait of interest between egos and alters, which the model interprets as evidence of social influence. As a result, estimated bias levels increase substantially—up to 0.10 in the worst case. Again, even in the left panel where friendship attrition is not large, the bias is substantial if friendship attrition is closely related to homophily.

Is the level of bias described above meaningful? As a point of comparison, we note that estimated peer effect coefficients for continuous variables in the literature are often in the range described in Table 1 and Figure 4. For example, the coefficient on alter's current BMI is 0.053 in the FHS data (SE=0.018) and 0.033 in Add Health data (SE=0.014) (Fowler and Christakis 2008b).¹⁹ Of course, our simulations cannot prove that the results in

¹⁹It would be worthwhile to repeat this exercise with a binary variable as in the CF studies of smoking or depression.



any given study are spurious, but they do suggest that the risk of falsely rejecting the null hypothesis is high for the CF model when homophily in friendship retention is present.

6 Conclusion

In this paper, we have argued that the “unfriending” problem complicates efforts to estimate causal influence effects in longitudinal social network data. While previous studies have demonstrated how homophily in friendship formation can confound estimates of peer effects, we are the first to demonstrate that homophily in friendship *retention* can create a similar problem by inducing an association between random shocks to an outcome variable for dyads that remain friends after the shock. We provide evidence for this hypothesis using an adaptation of Christakis and Fowler’s Monte Carlo simulation. Our simulations show that, when friendship attrition is present, the CF model suffers from serious bias and coverage problems as homophily in friendship retention increases. This association is large enough that, under certain parameter values, it could account for most or even all of the observed associations in published estimates.

These results suggest that caution is required in interpreting findings of peer effects that rely on maintained ties over time. Though this paper focuses on friendship network data, the logic of the “unfriending” problem applies to estimates of peer effects in any longitudinal network data in which homophily plays a significant role in tie dissolution. For instance, Grannis (2010) discusses how inferences about network structure change if one allows for tie dissolution over time in the Ph.D. exchange network among sociology graduate programs. If there is significant homophily in tie retention in this network (for instance, by methodological or substantive focus), one might obtain misleading estimates of “influence” effects among graduate programs.

Going forward, more research is clearly needed on models of peer influence in observational data. For instance, the actor-based model of Steglich, Snijders and Pearson (2010) attempts to account for many of the concerns

described above. However, our simulation results suggest that it is essential to test the properties of such models using simulations in which the true parameter values are known.

In addition, more research is needed on how to *assess* models of peer influence. While the parsimonious approach presented here has limitations, it provides a flexible framework for more complex simulations of friendship ties and social influence. Many desirable modifications are possible, including incorporating more than two time periods; allowing peer effects to vary by friendship duration; estimating the effect of latent traits; including environmental confounders; or allowing for correlations in shocks among peers. As research proceeds on models of network formation (e.g., Christakis et al. 2010), it may soon also be possible to simulate random networks that more closely mirror important features of human networks such as clustering, mutuality, and transitivity. As our theoretical understanding of networks improves, so too will our ability to test statistical models of social influence in observational data.

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Table 1: Simulation results

| | Retention constant | | Formation homophily | | Retention homophily | | Bias | Coverage probability | Ego-alter correlation | | Friends/ person | | Friendship retention rate |
|----|--------------------|---------------|---------------------|---------------|---------------------|---------------|-------|----------------------|-----------------------|---------|-----------------|---------|---------------------------|
| | $\eta_0(t_1)$ | $\eta_0(t_0)$ | $\eta_1(t_0)$ | $\eta_1(t_1)$ | $\eta_1(t_0)$ | $\eta_1(t_1)$ | | | (t_0) | (t_1) | (t_0) | (t_1) | |
| 1 | 0.00 | 0.0000 | 0.0000 | 0.000 | -0.00 | 0.96 | -0.00 | -0.00 | -0.00 | 6.2 | 3.1 | 0.50 | |
| 2 | 0.00 | 0.0000 | 0.0000 | 0.025 | 0.03 | 0.63 | -0.00 | -0.00 | -0.00 | 6.2 | 3.1 | 0.50 | |
| 3 | 0.00 | 0.0000 | 0.0000 | 0.050 | 0.07 | 0.05 | -0.00 | -0.00 | -0.00 | 6.2 | 3.1 | 0.51 | |
| 4 | 0.00 | 0.0125 | 0.0125 | 0.000 | -0.00 | 0.96 | 0.19 | 0.15 | 0.15 | 6.4 | 3.2 | 0.50 | |
| 5 | 0.00 | 0.0125 | 0.0125 | 0.025 | 0.03 | 0.54 | 0.19 | 0.15 | 0.15 | 6.4 | 3.2 | 0.50 | |
| 6 | 0.00 | 0.0125 | 0.0125 | 0.050 | 0.08 | 0.01 | 0.19 | 0.15 | 0.15 | 6.4 | 3.2 | 0.50 | |
| 7 | 0.00 | 0.0250 | 0.0250 | 0.000 | -0.00 | 0.94 | 0.36 | 0.27 | 0.27 | 7.1 | 3.5 | 0.50 | |
| 8 | 0.00 | 0.0250 | 0.0250 | 0.025 | 0.04 | 0.39 | 0.36 | 0.27 | 0.27 | 7.1 | 3.5 | 0.50 | |
| 9 | 0.00 | 0.0250 | 0.0250 | 0.050 | 0.08 | 0.00 | 0.36 | 0.27 | 0.27 | 7.1 | 3.6 | 0.50 | |
| 10 | 0.00 | 0.0375 | 0.0375 | 0.000 | -0.00 | 0.94 | 0.49 | 0.36 | 0.36 | 8.1 | 4.1 | 0.50 | |
| 11 | 0.00 | 0.0375 | 0.0375 | 0.025 | 0.04 | 0.26 | 0.49 | 0.36 | 0.36 | 8.1 | 4.1 | 0.50 | |
| 12 | 0.00 | 0.0375 | 0.0375 | 0.050 | 0.09 | 0.00 | 0.49 | 0.36 | 0.36 | 8.1 | 4.1 | 0.50 | |
| 13 | 0.00 | 0.0500 | 0.0500 | 0.000 | -0.00 | 0.96 | 0.59 | 0.42 | 0.42 | 9.6 | 4.8 | 0.50 | |
| 14 | 0.00 | 0.0500 | 0.0500 | 0.025 | 0.04 | 0.14 | 0.59 | 0.42 | 0.42 | 9.6 | 4.8 | 0.50 | |
| 15 | 0.00 | 0.0500 | 0.0500 | 0.050 | 0.10 | 0.00 | 0.59 | 0.42 | 0.42 | 9.6 | 4.8 | 0.50 | |
| 16 | 0.50 | 0.0000 | 0.0000 | 0.000 | -0.00 | 0.94 | -0.00 | -0.00 | -0.00 | 6.2 | 4.3 | 0.69 | |
| 17 | 0.50 | 0.0000 | 0.0000 | 0.025 | 0.02 | 0.77 | -0.00 | -0.00 | -0.00 | 6.2 | 4.3 | 0.69 | |
| 18 | 0.50 | 0.0000 | 0.0000 | 0.050 | 0.05 | 0.16 | -0.00 | -0.00 | -0.00 | 6.2 | 4.2 | 0.68 | |
| 19 | 0.50 | 0.0125 | 0.0125 | 0.000 | -0.00 | 0.94 | 0.19 | 0.15 | 0.15 | 6.4 | 4.4 | 0.69 | |
| 20 | 0.50 | 0.0125 | 0.0125 | 0.025 | 0.02 | 0.69 | 0.19 | 0.15 | 0.15 | 6.4 | 4.4 | 0.69 | |

$\eta_0 = -2.75$ at t_0 ; true contagion effect $b_1=0$; $u \sim N(0, 5)$; $n=1000$; 1000 simulations per row (continued on next page)

Table 1: Simulation results (continued)

| | Retention constant $\eta_0(t_1)$ | Formation homophily $\eta_1(t_0)$ | Retention homophily $\eta_1(t_1)$ | Bias | Coverage probability | Ego-alter correlation (t_0) | Ego-alter correlation (t_1) | Friends/ person (t_0) | Friends/ person (t_1) | Friendship retention rate |
|----|----------------------------------|-----------------------------------|-----------------------------------|-------|----------------------|-------------------------------|-------------------------------|-------------------------|-------------------------|---------------------------|
| 21 | 0.50 | 0.0125 | 0.050 | 0.05 | 0.10 | 0.19 | 0.15 | 6.4 | 4.4 | 0.68 |
| 22 | 0.50 | 0.0250 | 0.000 | -0.00 | 0.95 | 0.36 | 0.27 | 7.1 | 4.9 | 0.69 |
| 23 | 0.50 | 0.0250 | 0.025 | 0.02 | 0.63 | 0.36 | 0.27 | 7.1 | 4.9 | 0.69 |
| 24 | 0.50 | 0.0250 | 0.050 | 0.06 | 0.03 | 0.36 | 0.27 | 7.1 | 4.8 | 0.68 |
| 25 | 0.50 | 0.0375 | 0.000 | -0.00 | 0.95 | 0.49 | 0.36 | 8.1 | 5.6 | 0.69 |
| 26 | 0.50 | 0.0375 | 0.025 | 0.03 | 0.50 | 0.49 | 0.36 | 8.1 | 5.6 | 0.69 |
| 27 | 0.50 | 0.0375 | 0.050 | 0.06 | 0.01 | 0.49 | 0.36 | 8.1 | 5.6 | 0.68 |
| 28 | 0.50 | 0.0500 | 0.000 | -0.00 | 0.95 | 0.59 | 0.42 | 9.6 | 6.6 | 0.69 |
| 29 | 0.50 | 0.0500 | 0.025 | 0.03 | 0.40 | 0.59 | 0.42 | 9.6 | 6.6 | 0.69 |
| 30 | 0.50 | 0.0500 | 0.050 | 0.06 | 0.00 | 0.59 | 0.42 | 9.6 | 6.6 | 0.68 |
| 31 | 1.00 | 0.0000 | 0.000 | -0.00 | 0.96 | -0.00 | -0.00 | 6.2 | 5.2 | 0.84 |
| 32 | 1.00 | 0.0000 | 0.025 | 0.01 | 0.86 | -0.00 | -0.00 | 6.2 | 5.2 | 0.83 |
| 33 | 1.00 | 0.0000 | 0.050 | 0.03 | 0.50 | -0.00 | -0.00 | 6.2 | 5.1 | 0.82 |
| 34 | 1.00 | 0.0125 | 0.000 | -0.00 | 0.95 | 0.19 | 0.15 | 6.4 | 5.4 | 0.84 |
| 35 | 1.00 | 0.0125 | 0.025 | 0.01 | 0.87 | 0.19 | 0.15 | 6.4 | 5.4 | 0.84 |
| 36 | 1.00 | 0.0125 | 0.050 | 0.03 | 0.37 | 0.19 | 0.15 | 6.4 | 5.3 | 0.82 |
| 37 | 1.00 | 0.0250 | 0.000 | -0.00 | 0.96 | 0.36 | 0.27 | 7.1 | 6.0 | 0.84 |
| 38 | 1.00 | 0.0250 | 0.025 | 0.01 | 0.81 | 0.36 | 0.27 | 7.1 | 5.9 | 0.84 |
| 39 | 1.00 | 0.0250 | 0.050 | 0.03 | 0.28 | 0.36 | 0.27 | 7.1 | 5.9 | 0.83 |
| 40 | 1.00 | 0.0375 | 0.000 | -0.00 | 0.94 | 0.49 | 0.36 | 8.1 | 6.9 | 0.84 |
| 41 | 1.00 | 0.0375 | 0.025 | 0.01 | 0.78 | 0.49 | 0.36 | 8.1 | 6.8 | 0.84 |

$\eta_0 = -2.75$ at t_0 ; true contagion effect $b_1=0$; $u \sim N(0,5)$; $n=1000$; 1000 simulations per row (continued on next page)

Table 1: Simulation results (continued)

| | Retention constant $\eta_0(t_1)$ | Formation homophily $\eta_1(t_0)$ | Retention homophily $\eta_1(t_1)$ | Bias | Coverage probability | Ego-alter correlation (t_0) | Ego-alter correlation (t_1) | Friends/ person (t_0) | Friends/ person (t_1) | Friendship retention rate |
|----|----------------------------------|-----------------------------------|-----------------------------------|-------|----------------------|-------------------------------|-------------------------------|-------------------------|-------------------------|---------------------------|
| 42 | 1.00 | 0.0375 | 0.050 | 0.04 | 0.17 | 0.49 | 0.36 | 8.1 | 6.7 | 0.83 |
| 43 | 1.00 | 0.0500 | 0.000 | -0.00 | 0.94 | 0.59 | 0.42 | 9.6 | 8.1 | 0.84 |
| 44 | 1.00 | 0.0500 | 0.025 | 0.02 | 0.71 | 0.59 | 0.42 | 9.6 | 8.1 | 0.84 |
| 45 | 1.00 | 0.0500 | 0.050 | 0.04 | 0.10 | 0.59 | 0.42 | 9.6 | 8.0 | 0.83 |
| 46 | 1.85 | 0.0000 | 0.000 | -0.00 | 0.95 | -0.00 | -0.00 | 6.2 | 6.0 | 0.97 |
| 47 | 1.85 | 0.0000 | 0.025 | 0.00 | 0.97 | -0.00 | -0.00 | 6.2 | 6.0 | 0.96 |
| 48 | 1.85 | 0.0000 | 0.050 | 0.01 | 0.88 | -0.00 | -0.00 | 6.2 | 5.9 | 0.95 |
| 49 | 1.85 | 0.0125 | 0.000 | -0.00 | 0.94 | 0.19 | 0.15 | 6.4 | 6.2 | 0.97 |
| 50 | 1.85 | 0.0125 | 0.025 | 0.00 | 0.96 | 0.19 | 0.15 | 6.4 | 6.2 | 0.96 |
| 51 | 1.85 | 0.0125 | 0.050 | 0.01 | 0.86 | 0.19 | 0.15 | 6.4 | 6.1 | 0.95 |
| 52 | 1.85 | 0.0250 | 0.000 | -0.00 | 0.94 | 0.36 | 0.27 | 7.1 | 6.9 | 0.97 |
| 53 | 1.85 | 0.0250 | 0.025 | 0.00 | 0.94 | 0.36 | 0.27 | 7.1 | 6.8 | 0.97 |
| 54 | 1.85 | 0.0250 | 0.050 | 0.01 | 0.88 | 0.36 | 0.27 | 7.1 | 6.8 | 0.96 |
| 55 | 1.85 | 0.0375 | 0.000 | -0.00 | 0.95 | 0.49 | 0.36 | 8.1 | 7.9 | 0.97 |
| 56 | 1.85 | 0.0375 | 0.025 | 0.00 | 0.93 | 0.49 | 0.36 | 8.1 | 7.9 | 0.97 |
| 57 | 1.85 | 0.0375 | 0.050 | 0.01 | 0.84 | 0.49 | 0.36 | 8.1 | 7.8 | 0.96 |
| 58 | 1.85 | 0.0500 | 0.000 | -0.00 | 0.95 | 0.59 | 0.42 | 9.6 | 9.3 | 0.97 |
| 59 | 1.85 | 0.0500 | 0.025 | 0.00 | 0.93 | 0.59 | 0.42 | 9.6 | 9.3 | 0.97 |
| 60 | 1.85 | 0.0500 | 0.050 | 0.01 | 0.81 | 0.59 | 0.42 | 9.6 | 9.2 | 0.96 |

$\eta_0 = -2.75$ at t_0 ; true contagion effect $b_1=0$; $u \sim N(0, 5)$; $n=1000$; 1000 simulations per row