FEEDBACK CONTROLLER DESIGN FOR SENSITIVITY-BASED DAMAGE LOCALIZATION

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A method is developed for locating structural damage using only measured natural frequency changes induced by damage. The damage localization method exploits multiple sensitivity enhancing controllers, each of which provides an independent set of modal frequency information that is used to identify damage variables based on a least-squares technique. The method provides significant improvement in damage localization and ability to tolerate measurement noise on natural frequency shifts due to damage over similar localization methods that use only open-loop modal data. A first-order sensitivity matrix relates natural frequency changes in both the open-loop and closed-loop systems to damage variables and is evaluated from an analytic model. Single-input and multi-input control laws are designed to enhance the change in natural frequencies due to damage. Multi-input control laws are designed using a minimum-gain eigenstructure assignment method in order to maintain a well-conditioned sensitivity matrix and generate independent modal data, while also minimizing the number of actuators required. This study found that the resolution of measured natural frequency changes due to damage can be significantly improved by careful selection of damage-sensitive closed-loop poles targeted by the eigenstructure assignment method. As a result, measured closed-loop natural frequency changes due to damage exhibit better

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signal-to-noise ratios than open-loop frequency changes. The method is demonstrated nu-
merically using a cantilevered beam to show how multiple sensitivity enhancing controllers
can locate damage and assess damage extent in the presence of measurement noise on natural
frequencies.

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1. INTRODUCTION

Increasing performance demands on load-carrying structures, along with high expected
reliability, has necessitated the development of elaborate NDE (Nondestructive Evaluation)
methods. Among them, vibration-based damage detection methods have received attention
due to their simplicity and autonomous monitoring capability.

Natural frequency (eigenvalue) and mode shape (eigenvector) changes have been the most
frequently studied vibration based damage metrics [1]. Inverse or model updating ap-
proaches to damage identification use optimization methods to update mass and stiffness
matrices of an analytic model using measured modal data. Using the structure’s eigenvalue
equation for damage localization requires both eigenvalue and eigenvector measurements to
reconstruct stiffness and mass parameters. This means that perturbation or damage result-
ing in changes in system parameters cannot be determined explicitly without knowing the
perturbation in both eigenvalues and eigenvectors simultaneously. However, mode shape
measurement requires multiple sensors to approximate the infinite number of degrees of free-
dom (DOFs) in real distributed parameter structures. When mode shape measurements are
incomplete, mode expansion or model reduction inevitably deteriorates the solution of the
eigenvalue equation through inverse approaches [2], [3]. Moreover, measurement error and
noise are critical threats to the credibility of measured mode shape information [4]. On
the other hand, modal frequencies (eigenvalues) give more reliable evidence of damage in a
structure, compared to the mode shape (eigenvector), in that frequencies are easier to measure and are less sensitive to measurement errors [5]. Hence, damage detection methods that require only measured modal frequencies are generally preferred to those that require both frequencies and mode shapes.

The model updating method can be cast into a form in which only frequencies are required. Sensitivity-based model updating methods use the first order approximation between system parameter perturbations and modal frequency changes to determine damage parameters using a least-squares approach. Usually, the sensitivity matrix is obtained using an analytic model by finding derivatives of the eigenvalues with respect to stiffness perturbations for each potential damage location. The updating process typically iterates the solution until the damage variables satisfactorily converge to the true system perturbations. One problem with this method is that the number of unknowns (system parameters to be updated) is usually much larger than the number of measurable modal frequencies. In other words, the number of identifiable damage locations is limited to the number of measurable modes. It is quite difficult to accurately measure more than a few modal frequencies above the lowest one. As a result, recent research has focused on methods of increasing the modal frequency data available for sensitivity-based model updating.

Recently, Cha and Gu [6] studied a mass addition technique for structural parameter updating. They show that adding known masses to a multi-spring-mass system and measuring its new eigendata can correct the mass matrix of the perturbed model. Subsequently, the stiffness matrix can also be updated by equating the eigenvalue equation to a new, mass-added system. However, this method needs both natural frequency and mode shape measurements. Messina et al. [7] propose a sensitivity-based method which is combined with statistical correlation to find multiple damage locations in truss and three-beam test structures. After inserting the first-order sensitivity matrix into a correlation equation, the damage scaling coefficients are sought, which avoids taking a pseudo-inverse of the sensitiv-
ity matrix. A numerical simulation study is presented in [7] to validate the method. In the study, more than 10 measured modal frequencies are used for detecting multiple damage locations. The effectiveness of a second order approximation is also investigated. Nalitolela et al. [8] introduce the possibility of using a mass or stiffness added model to extract additional natural frequencies. After perturbing mass or stiffness at specific coordinates of the beam, sensitivity analysis is performed. In order to measure significant natural frequency changes, substantial perturbation of the structure is essential. Physically adding mass or stiffness to structures makes the method difficult to implement in practice. Lew and Juang [9] incorporate the concept of a virtual passive controller to overcome this limitation. They used output and dynamic feedback controllers to generate additional closed-loop modal frequencies in order to identify multiple damage locations in a cantilevered beam. Here, no physical mass or stiffness attachments to the structures are required. In the study, a damage variable vector is defined as a percentage of stiffness loss for each potential damage location. Although the stability of the closed-loop system is guaranteed by the nature of the energy dissipative passive controller, neither the sensitivity of frequency shifts for each passive controller nor the effect of measurement noise is considered. In general, frequency shifts due to small damage in structures are insignificant [5], [10]. Hence, without sensitivity enhancement, it is difficult to measure modal frequency changes accurately in the presence of measurement noise. Recently, Palacz and Krawczuk [11] investigate the effects of measurement error on damage detection using vibration parameters. The study shows that very small errors in measured natural frequencies can ruin the localization of damage in cantilevered beam.

Ray and Tian [12] recently proposed a new approach to improve sensitivity of closed-loop natural frequency toward stiffness and mass damage. They employ a damage-sensitive pole placement technique. In that study, feedback control laws for sensitivity enhancement are investigated through numerical simulation. A control law targeting stiffness damage
detection is designed by reducing frequencies of the first three modes using a single point force actuator. Sensitivity to the thickness reduction at the root of the beam increases by a factor of approximately 40 (first mode) to a factor of five (third mode). Ray, Koh, and Tian [13] extends these results to consider sensitivity enhancing of fatigue crack damage and it includes an experimental validation of the SEC concept for a cantilevered beam under bending.

Both [12] and [13] use sensitivity enhancing control law designs for single-input systems to increase the linear sensitivity of natural frequency variations to damage. For a completely controllable system, the control gains are uniquely determined in the single-input case. In the multi-input case, however, an infinite number of gain sets can satisfy closed-loop pole placement. Hence, additional control law design objectives can be achieved in the multi-input case. Juang et al. [14] propose an eigenstructure assignment technique seeking a minimum norm gain solution for an output feedback multi-input system. In that study, a null-space technique along with singular value decomposition is adopted to expand admissible eigenvector space. They suggest the open-loop eigenvector as a good candidate for desired closed-loop eigenvector set, which eventually leads to minimum control gains and thus minimum control effort.

This study develops a damage localization method to locate damage using damage-sensitive feedback controllers. The sensitivity-based damage localization method of [7] is combined with the concept of damage-sensitive control laws of [12] and [13] to produce a damage localization method that relies on fewer measured modal frequencies than a comparable open-loop system only and is robust to measurement noise. The key contributions of the paper are 1) demonstration of the use of the SEC concept to provide sufficient and independent equations to solve for unknown parameters or damage locations, and 2) development of a localization method that lessens the dependence on the analytic model of the structure. Multiple actuators along a structure can contribute additional sets of measured closed-loop
modal frequencies in order to maintain a sensitivity matrix relating damage variables and measured frequencies that is well-conditioned and thus whose pseudo-inverse is defined. All possible actuator locations are investigated to increase the orthogonality of the equations. An eigenstructure assignment technique is also presented to determine the minimum norm gains for multi-input closed-loop systems. In essence, damage-sensitive control laws improve the resolution of measured natural frequency changes. A simulation comparison is made between the proposed method with and without the use of sensitivity enhancing control laws using a cantilevered beam.

2. BACKGROUND THEORY

2.1. SENSITIVITY ENHANCING CONTROL

A damage-sensitive feedback controller drives the poles of the closed-loop system to the locations in the complex plane where modal frequency shifts are more sensitive to damage [12],[13]. The concept of sensitivity enhancement can be easily demonstrated by numerical simulation of a controllable structural model. For example, the FE cantilevered beam model considered here consists of 8 elements and 9 nodes with a point force actuator at node 2 as shown in Figure 1. Single-input pole placement (SIPP) is employed to move the first four modes to frequencies slightly lower than open-loop frequencies in complex plane. Damage-sensitive feedback control increases the frequency shifts under 10% reduction of Young’s modulus in the first element (closest to the root) of the beam. The percentage of frequency shifts in the first four modes of the closed-loop system are shown in Figure 2 as function of closed-loop frequency. As the closed-loop frequency of each mode decreases, the frequency shifts increase, illustrating sensitivity enhancement.
2.2. SENSITIVITY-BASED DAMAGE LOCALIZATION

Damage localization using eigenvalue sensitivity analysis requires two consecutive processes: 1) computing the eigenvalue sensitivity matrix from an analytic model, and 2) estimating the unknown parameters or damage variables using natural frequencies measured from a real structure. The sensitivity matrix is typically computed from a finite element (FE) model of the structure. With this sensitivity matrix and measured natural frequencies, damage variables, i.e., possible locations of damage occurrence and their extent, should be identified. Here, the damage variable is defined as the fraction of thickness reduction for each finite element. For example, a damage vector \( \{v_d\} = \{1, 1, 0.95, 1, 1, 1, 1\} \) means that the third element has a 5% thickness reduction while the other seven elements are undamaged. The damage vector \( \{v_h\} \), whose elements are all equal to unity, represents the nominal or healthy system.

The analytic model of a distributed parameter structure such as a cantilevered beam can be conveniently expressed by a second order equation as

\[
\begin{align*}
M\ddot{x} + C\dot{x} + Kx &= F, \\
\end{align*}
\]

where \( M, C, K \) and \( F \) are mass, damping, stiffness and input force matrices, respectively.

For an equivalent control model, a state-space equation is defined as

\[
\dot{X} = AX + BF
\]

where,

\[
A = \begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix},
B = \begin{bmatrix}
0 \\
M^{-1}
\end{bmatrix}
\]

\[
X = [x \quad \dot{x}]^T.
\]

Denoting \( A_d \) as a damaged system matrix (by the perturbation of damage variables), the solution of the characteristic equation yields the \( i \)th natural frequency of damaged system.
Perturbations of the damage variable \( \{ \delta v \} \) and the system’s natural frequency changes \( \{ \delta \omega \} \) are defined as

\[
\{ \delta v \} = \{ v_h \} - \{ v_d \} \\
\{ \delta \omega \} = \{ \omega_h \} - \{ \omega_d \},
\]

where \( \omega_h \) and \( \omega_d \) represent the natural frequencies of the healthy and damaged structures, respectively.

The nonlinear relation between \( \{ \delta \omega \} \) and \( \{ \delta v \} \) can be linearized by the first order multi-variable approximation.

\[
\{ \delta \omega \} = S \{ \delta v \}
\]

where

\[
S = \begin{bmatrix}
\frac{\partial \omega^1}{\partial v_1} & \frac{\partial \omega^1}{\partial v_2} & \cdots & \frac{\partial \omega^1}{\partial v_r} \\
\frac{\partial \omega^2}{\partial v_1} & \frac{\partial \omega^2}{\partial v_2} & \cdots & \frac{\partial \omega^2}{\partial v_r} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \omega^p}{\partial v_1} & \frac{\partial \omega^p}{\partial v_2} & \cdots & \frac{\partial \omega^p}{\partial v_r}
\end{bmatrix}
\]

The sensitivity matrix, \( S \) is defined as derivatives of natural frequencies \( \omega^j \) up to \( p \) modes with respect to \( r \) damage variables \( v_j \). Hence, the size of the \( S \) matrix is \( p \times r \).

Once the sensitivity matrix is formed, the actual damage variables can be estimated from the measured natural frequency shifts \( \{ \delta \omega \}_m \) and the pseudo-inverse of sensitivity matrix.

\[
\{ v_d \} = \{ v_h \} - S^+ \{ \delta \omega \}_m
\]

Although an iterative variable updating could improve the converged solution of equation (10) as in [8], its success also depends heavily on the accuracy of the analytic model. In
this study, the damage variables are found directly from equation (10) with no iteration in order to minimize dependence of the damage localization method on analytic model.

The accuracy of estimated damage variables \( \{v_d\} \), depends on the condition number of sensitivity matrix, \( S \). When the number of measurable natural frequencies is less than the number of unknown damage variables \( p < r \), additional sets of closed-loop natural frequencies can be included to identify the damage variables. However, added closed-loop natural frequency vectors should be mutually independent. In other words, the condition number, or the ratio of the largest to smallest singular value of the sensitivity matrix should be as small as possible. Otherwise, the sensitivity matrix can become ill-conditioned and computing the pseudo-inverse, \( S^+ \) might not be possible. Hence, the condition number of the sensitivity matrix is a critical measure of the adequacy of additional closed-loop information.

Given a controllable system model \((A, B)\), the \( i \)th natural frequency of the closed-loop system formed by applying control law \( F = -GX \) to the actual system is obtained by the solutions to

\[
\text{det}[(A - BG) - (\omega_c)_i^2 I] = 0.
\]

As mentioned in references [12] and [13], by choosing the gain matrix \( G \) properly, closed-loop poles can be arbitrary placed in the complex plane. Conversely, assigning the desired closed-loop pole locations provides a state feedback control gain. Thus, state feedback closed-loop systems will not only provide additional information for equation (8), but can also enhance the sensitivity of the natural frequency change by systematic selection of closed-loop poles. As a result, a new sensitivity matrix having \( p \) modes of closed-loop natural frequencies from \( q \) closed-loop systems is calculated as
Here, $\omega_{pq}^p$ means $p$th natural frequency of the $q$th closed-loop system. As in equation (10), damage variables can be identified using the closed-loop sensitivity matrix, $S_c$, which is computed from the analytic model, and measured closed-loop natural frequency shifts, $\{\delta\omega_c\}_m$.

$$\{v_d\} = \{v_h\} - S_c^+\{\delta\omega_c\}_m$$  \hspace{1cm} (13)

Note that the number of rows in $S_c$ ($pq \times r$) increases by a factor of $q$, compared to the open-loop sensitivity matrix $S$ ($p \times r$). Although, theoretically, the number of damage variables $\{v\}$ can be increased up to the number of rows ($pq$) of $S_c$, the unique solution of equation (13) is not guaranteed unless the sensitivity matrix is well-conditioned. Hence, the sets of closed-loop systems should be chosen such that the condition number of sensitivity matrix is minimized. In this study, each closed-loop system has a different actuator location, which is shown to improve the independence of the rows of $S_c$ and thus minimizes the condition number.
2.3. MULTI-INPUT CONTROLLER DESIGN

Previous studies [12, 13] on damage-sensitive feedback control investigate only single-input closed-loop systems. In this section, the multi-input case is investigated from the viewpoint of damage-sensitive controller design for damage localization.

First, a general approach to the determination of feedback gain matrices from [15] is summarized. The eigenvalue equation for a closed-loop system can be written as

\[(A + BG)\Psi_k = \Psi_k \lambda_k\]  

(14)

where, \(\Psi_k\) represents an \(k\)th eigenvector corresponding to the assigned or desired closed-loop eigenvalue \(\lambda_k\). Alternatively, equation (14) can be partitioned in a homogeneous form as

\[
\begin{bmatrix}
A - \lambda_k I & B \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\Psi_k \\
G \Psi_k
\end{bmatrix} = 0.
\]

(15)

Equation (15) should have a nontrivial solution to satisfy the sufficient condition for the existence of assigned eigenvalues and their corresponding eigenvectors.

Defining

\[
\Gamma_k = [A - \lambda_k I \mid B],
\]

(16)

Singular-Value Decomposition (SVD) can be used to find a set of orthogonal basis vectors spanning the null space of equation (16):

\[
\Gamma_k = U_k \Sigma_k V_k^* = U_k \begin{bmatrix}
\sigma_k & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
V_{\sigma k}^* \\
V_{0 k}^*
\end{bmatrix}.
\]

(17)

Thus, equation (17) gives

\[
\Gamma_k V_{0 k} = 0, \quad \Gamma_k \phi_k = 0
\]

(18)

where

\[
\phi_k = \begin{bmatrix}
\Psi_k \\
\tilde{\Psi}_k
\end{bmatrix}.
\]

(19)
From equation (15) and (19), the gain matrix $G$ is embedded in $\tilde{\Psi}_k$. Thus $G$ can be obtained by taking the inverse of matrix $\Psi_k$, the assigned eigenvectors, which are selected by the designer in advance:

$$G = \tilde{\Psi}_k \Psi_k^+$$

(20)

Since an infinite number of eigenvector and eigenvalue combination can be assigned in equation (14), usually additional constraints are imposed, such as minimizing the norm of the gain matrix or improving robustness toward system uncertainties. In this study, a gain-minimization technique from the study of Juang et al. [14] is applied to design the multi-input controller. Eventually, these multi-input closed-loop systems are considered to be additional candidates for building the sensitivity matrix of equation (12).

3. SIMULATIONS AND RESULTS

For numerical simulations, an aluminum cantilevered beam is modelled using 64 uniform finite elements. The dimensions of the beam are $200\,mm(W) \times 9.5\,mm(H) \times 650\,mm(L)$. This represents the truth model of the structure, which is used to generate damage cases and to which state feedback control laws are applied. The state-feedback control laws for generating closed-loop systems are developed from an 8-element cantilevered beam model(Figure 1) that has the same dimension of the 64-element model. Hence, the location for each node of 8-element model coincides with a corresponding node of the 64-element model.

The open-loop poles of the first three modes are $-0.67\pm115.77i$, $-26.3\pm725.08i$, and $-206.6\pm2022.1i$, respectively. For sensitivity enhancement, the damped natural frequencies of the first three modes are lowered from open-loop poles such as $(A)$ as shown in Table 1. For comparison, another set of closed-loop poles, case $(B)$ is considered whose real parts of
eigenvalues are increased respectively. The reason for comparing these two cases is to show that the locations of closed-loop poles in complex plane explicitly govern the amount of sensitivity enhancement, as illustrated in section 2.1, and consequently, also affects damage localization.

The actuator for the state-feedback closed-loop system is a vertical force input applied at a nodal point. Table 1 provides target closed-loop pole locations for five single-input control laws, with actuator locations at nodes 2, 3, 4, 6, and 7, and for five two-input control laws. Double digits for actuator location indicate multi-input cases; for example, actuator location 23 represents a multi-input closed-loop system having dual actuators at node 2 and 3. Stiffness damage is assumed as the damage case and is simulated as a 3% thickness reduction in a single element. Damage variables are selected as a set of potential single-element damage locations in the truth model. Three different damage variable vectors are considered as shown in Table 2. A damage vector having 11 damage variables can identify 11 potential damaged elements from total 64 elements, while one with 16 damage variables can identify 16 possible single-element damage locations.

Figure 3(a) presents the percent change in open-loop natural frequencies for the first four modes due to thickness reduction in a single beam element of the 64-element truth model. It shows that the maximum open-loop frequency shifts are less than or equal to 0.3%, as damage location moves from the root to the tip of beam. Figure 3(b) and (c) illustrate the percent natural frequency shifts for closed-loop systems (A) and (B), having respectively, both single-input actuators at nodes 2 and 6. In contrast to the open-loop case, the closed-loop system with damage-sensitive controller (A) exhibits a huge enhancement in frequency shifts for the first three controlled modes. Note that damage-insensitive controller (B) does not show much difference from the open-loop case in frequency changes. It is also clear from Figure 3 that under sensitivity enhancing control, the actuator location (node 2 and 6) can significantly change the pattern of frequency shifts due to damage along the
beam. This contrast is considered to improve the independence of each closed-loop system’s
corresponding contribution to the sensitivity matrix of equation (12).

Table 3 presents several combinations of closed-loop systems which have \( p \) modes, \( q \) actu-
tors, and \( r \) damage variables along with the resulting sensitivity matrix condition numbers.
In this table, a quantitative measure of the performance of localization is provided by two
statistical indices, \( \alpha \) and \( \beta \); that indicate accuracy of localization and severity of damage,
respectively.

\[
\alpha = \bar{s}, \quad s_k = \sqrt{\frac{\sum_{i=1}^{r} (v_i - \bar{v})^2}{r - 2}}
\]

\[
\beta = \bar{\gamma}, \quad \gamma_k = \frac{v_k}{v_t}
\]

where, \( \alpha \) represents the mean of standard deviations \( (s_k) \) of total \( r - 1 \) damage variables
which are identified as healthy ones; i.e., \( i \neq k \) (where \( k \) is the correct location of damage).
\( \beta \) indicates the mean of ratios \( (\gamma_k) \) between the true value of a damage variable \( (v_t = 0.97) \)
and the identified value of the \( k \)th damage variable \( (v_k) \). Hence, the smaller \( \alpha \) and \( \beta \), the
better damage localization and the more accurate assessment of its severity.

There are three important observations to be made regarding Table 3. First, the values of \( \alpha \)
and \( \beta \) decrease as the condition numbers decrease. In other words, the performance of dam-
age localization depends on the condition number of the sensitivity matrix, \( S_c \). However, \( \alpha \)
and \( \beta \) provide more objective measures for localization performance than condition numbers
since comparing the absolute values of condition numbers for each closed-loop system is less
meaningful; due to variation in the number of and placement of actuators. Secondly, as the
value of the number of damage variables \( r \) approaches \( pq \), the condition number of sensi-
tivity matrix increases and damage localization become less successful. This supports the
fact that simply satisfying the inequality, \( pq > r \) does not guarantee successful localization
[8]. Finally, in regard to sensitivity enhancement, controller \( (A) \) apparently gives better
damage localization results than controller (B), and the difference in the damage severity index, β, between two controllers is substantial. Hence, sensitivity enhancement improves the performance of damage localization.

3.1. MEASUREMENT NOISE

Figure 4 through Figure 7 illustrate the result of damage localization using the closed-loop sensitivity matrix, $S_c$. Each plot in the figure represents localization results for 11 damage variables along the beam. The vertical line of each plot denotes the true location of the damaged element. Abscissa and ordinate indicate the serial number of damage variables (Table 2) and their identified values, respectively.

First, the effect of measurement noise is examined. With the same noise level, the sensitivity enhanced closed-loop systems (A) are compared to the case of closed-loop systems without sensitivity enhancement (B). Figure 4 and Figure 5 indicate localization results using controllers (A) and (B), respectively and noise-free frequency measurements. Both results are from a sensitivity matrix, $S_c$, developed for four modes ($p$), five closed-loop systems ($q$), and 11 damage variables ($r$) (i.e., row 3 of Table 3). The location of the bar whose damage severity is 0.97 indicates the identified location of damage and its extent (3% thickness reduction). Both controllers successfully identified damage from the root to the free end of the beam. However, damage identification based on noise-free frequency measurement of closed-loop system (A) is consistently accurate in both localization and determining extent of damage, while damage identification based on closed-loop system (B) is occasionally ambiguous in damage localization or inaccurate in the determination of damage extent. Figure 6 and Figure 7 show the localization results for controllers (A) and (B) with noise-contaminated natural frequency measurements. Here, the measurement noise is assumed as -0.03%, 0.045%, -0.075%, and 0.06% error from the first four measured natural
frequencies, respectively. In this study, the fixed error ratios for each mode are imposed on measurements so that localization results for each controller can be objectively compared. With measurement noise, damage-sensitive controller (A) still maintains unambiguous localization with modest degradation in determining damage magnitude, while the controller (B), which has no sensitivity enhancement, mostly fails to locate a damaged element for all but four damage locations. Hence, closed-loop systems should have sufficient independence and enhanced sensitivity for damage localization under measurement uncertainties.

In Figure 8, a comparison is made between the performance in damage localization and the robustness toward measurement noise. The localization performance index, $\alpha$, is plotted in terms of average maximum control effort of the closed-loop systems. Again, smaller $\alpha$ denotes better ability to localize damage. Maximum control effort on the x-axis is obtained from the impulse response of each closed-loop system. In general, the magnitude of the maximum control effort depends on how far the closed-loop poles are moved from the original open-loop poles. Noticeable sensitivity enhancement can be achieved if closed-loop poles are assigned further from open-loop poles. Hence, the noise-robustness improves as the effect of sensitivity enhancement surpasses the measurement noise as shown in the upper line of the figure; the index, $\alpha$, decreases which means localizing a damaged element becomes more successful. However, the sensitivity enhancement also intensifies the nonlinear relation between the damage location and the corresponding modal frequency shift. Thus, the first-order approximation becomes less accurate as the maximum control effort increases (lower line). The comparison shows that there is a trade-off between the performance of localization and robustness toward measurement noise. However, from observing the gradients of two lines, the sensitivity enhancement is dominant in diminishing the effect of measurement uncertainties.
3.2. MULTI-INPUT CONTROLLER

In this section, multi-input closed-loop controllers are developed to complement the single-input closed-loop systems so that the required number of actuators can be minimized. As is well known, the number of identifiable damage variables is limited by the number of measurable modal frequencies and the number of closed-loop systems. For example, a sensitivity matrix having five single-input actuators along with the first three modal frequency measurements cannot identify 16 damage variables \(5 \cdot 3 < 16\) as shown in Figure 9.

In order to overcome this problem without increasing the number of actuators, multi-input controllers are developed by coupling single-input actuators, in order to generate additional independent sets of closed-loop systems. In other words, with five single-input actuators, up to 15 different one or two input control laws (five single input and 10 two input) can be developed. Hence, the number of identifiable damage variables can be increased or the minimum required number of actuators can be reduced by incorporating multi-input closed-loop systems. Figure 10 illustrates the result of damage localization with ten mixed closed-loop systems, having both single-input and multi-input controllers identified in Table 2. It is apparent from the Figure 10 that the concept of including multi-input controllers can increase the number of identifiable damage variables. The inconsistent result in the first identified damage variable is due to the relatively large frequency shifts for damage at the root of the cantilevered beam. In other words, the first-order linear approximation suffers from nonlinearity, as mentioned in section 3.1. As may be seen from Table 3, the localization performance indices, \(\alpha\) and \(\beta\) decrease noticeably as compared with single input case with the same number of damage variables, when the closed-loop systems with multi-input actuators are included in the sensitivity matrix.

Note that not all of the 15 possible combinations of multi-input closed-loop systems are
linearly independent. Here, the pairs of actuators are carefully selected to enhance the independence of closed-loop systems and the condition number of the sensitivity matrix. The development of a systematic method to select an optimal combination of actuators is an issue to be addressed in future research.

4. CONCLUSIONS

An improved structural damage localization method is presented whose effectiveness is verified by using only measured closed-loop natural frequency changes before and after the damage occurs. Damage-sensitive, or sensitivity enhancing state-feedback, controllers are developed to generate multiple independent closed-loop systems. These additional closed-loop modal frequencies provide the eigenvalue sensitivity matrix to approximate damage variables in a least-square sense. The proposed method is proven to be highly tolerant to measurement noise. Also, the concept of minimum-gain eigenstructure assignment is applied to design multi-input controllers, which improves the condition of the sensitivity matrix with a minimum number of actuators.
REFERENCES

Multifunctional Smart Structures.


<table>
<thead>
<tr>
<th>Actuator locations</th>
<th>Closed-loop poles (A)</th>
<th>Closed-loop poles (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$-0.7 \pm 60i$</td>
<td>$-1.0 \pm 115.77i$</td>
</tr>
<tr>
<td></td>
<td>$-27.0 \pm 555i$</td>
<td>$-36.0 \pm 725.08i$</td>
</tr>
<tr>
<td></td>
<td>$-206.0 \pm 1800i$</td>
<td>$-255 \pm 2022i$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.7 \pm 65i$</td>
<td>$-1.2 \pm 115.77i$</td>
</tr>
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<td>$-27.0 \pm 565i$</td>
<td>$-46.0 \pm 725.08i$</td>
</tr>
<tr>
<td></td>
<td>$-206.0 \pm 1750i$</td>
<td>$-285 \pm 2022i$</td>
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<tr>
<td></td>
<td>$-27.0 \pm 650i$</td>
<td>$-33.0 \pm 725.08i$</td>
</tr>
<tr>
<td></td>
<td>$-206.0 \pm 1750i$</td>
<td>$-225 \pm 2022i$</td>
</tr>
<tr>
<td>23</td>
<td>Same as 2</td>
<td>Same as 2</td>
</tr>
<tr>
<td>24</td>
<td>Same as 3</td>
<td>Same as 3</td>
</tr>
<tr>
<td>26</td>
<td>$-0.7 \pm 95i$</td>
<td>$-2.7 \pm 115.77i$</td>
</tr>
<tr>
<td></td>
<td>$-27.0 \pm 655i$</td>
<td>$-50.0 \pm 725.08i$</td>
</tr>
<tr>
<td></td>
<td>$-206.0 \pm 1700i$</td>
<td>$-225 \pm 2022i$</td>
</tr>
<tr>
<td>27</td>
<td>Same as 6</td>
<td>Same as 6</td>
</tr>
<tr>
<td>67</td>
<td>Same as 6</td>
<td>Same as 6</td>
</tr>
</tbody>
</table>
Table 2. Number of damage variables and corresponding element number of damage locations.

<table>
<thead>
<tr>
<th>Damage Variables ($r$)</th>
<th>Element Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1 7 13 19 25 31 37 43 49 55 61</td>
</tr>
<tr>
<td>13</td>
<td>1 6 11 16 21 26 31 36 41 46 51 56 61</td>
</tr>
<tr>
<td>16</td>
<td>1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65</td>
</tr>
</tbody>
</table>

Table 3. Number of measured modal frequencies($p$), actuators($q$), damage variables($r$), condition numbers(CN) for closed-loop systems $(A, B)$, and the result of damage localization ($\alpha/\beta$) with no measurement noise. 5 actuators at nodes (2,3,4,6,7), 10 actuators at nodes (2,3,4,6,7,23,24,26,27,67).

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r$</th>
<th>CN(A)</th>
<th>$\alpha/\beta(A)$</th>
<th>CN(B)</th>
<th>$\alpha/\beta(B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>11</td>
<td>268</td>
<td>0.0047/0.0019</td>
<td>2118</td>
<td>0.0074/0.0130</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>13</td>
<td>1763</td>
<td>0.0258/0.0097</td>
<td>2912</td>
<td>0.0119/0.0175</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>16</td>
<td>4092(^9)</td>
<td>0.0757/0.0342</td>
<td>4693</td>
<td>6.1395/3.6884</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>11</td>
<td>167(^4)</td>
<td>0.0012/0.0019</td>
<td>2859(^5)</td>
<td>0.0071/0.0120</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>13</td>
<td>901</td>
<td>0.0098/0.0048</td>
<td>4354</td>
<td>0.0103/0.0145</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>16</td>
<td>2613</td>
<td>0.0249/0.0219</td>
<td>7748</td>
<td>0.0106/0.0156</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>11</td>
<td>275</td>
<td>0.0038/0.0019</td>
<td>1055</td>
<td>0.0066/0.0155</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>13</td>
<td>769</td>
<td>0.0061/0.0047</td>
<td>1674</td>
<td>0.0075/0.0175</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>16</td>
<td>1478(^{10})</td>
<td>0.0150/0.0133</td>
<td>2118</td>
<td>0.0090/0.0197</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>11</td>
<td>188</td>
<td>0.0011/0.0018</td>
<td>1838</td>
<td>0.0061/0.0132</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>13</td>
<td>684</td>
<td>0.0055/0.0029</td>
<td>3064</td>
<td>0.0072 0.0154</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>16</td>
<td>1434</td>
<td>0.0072/0.0052</td>
<td>3645</td>
<td>0.0082 0.0175</td>
</tr>
</tbody>
</table>

\(^ (*)\) indicates a Figure number of damage localization result.
Figure 1. 8-element controlled cantilever beam finite element model and nodal points: 200mm(W) × 9.5mm(H) × 650mm(L).

Figure 2. Sensitivity enhancement in an 8-element cantilever beam: the percentage of natural frequency changes of the first four modes vs. closed-loop pole selections. * Denotes open-loop frequency.
Figure 3. Percentage frequency changes for damage consisting of a 3% thickness reduction in a single beam element of the 64-element model. (a): open-loop system, (b): closed-loop system (A), (c): closed-loop system (B). Single-input actuators at node 2 (—) and 6 (- - -) for closed-loop systems.
Figure 4. Localization performance of controller A (sensitivity enhancement), with four measured modal frequencies, five closed-loop systems, and 11 damage variables. Noise-free measured natural frequencies are assumed. Vertical line denotes the true location of damage.

Figure 5. Localization performance of controller B (sensitivity reduction), with four measured modal frequencies, five closed-loop systems, and 11 damage variables. Noise-free measured natural frequencies are assumed.
Figure 6. Localization performance of controller A (sensitivity enhancement), with four measured modal frequencies, five closed-loop systems, and 11 damage variables. Errors of (−0.03%, 0.045%, −0.075%, 0.06%) for the first four measured natural frequencies are assumed.

Figure 7. Localization performance of controller A (sensitivity reduction), with four measured modal frequencies, five closed-loop systems, and 11 damage variables. Errors of (−0.03%, 0.045%, −0.075%, 0.06%) for the first four measured natural frequencies are assumed.
Figure 8. The average maximum control efforts vs. the localization performance index $\alpha$ with and without measurement noise in measured frequencies. Closed-loop system (A) with four measured modal frequencies, five closed-loop systems, and 11 damage variables.
Figure 9. Localization performance of controller A (*sensitivity enhancement*), with three measured modal frequencies, five closed-loop systems, and 16 damage variables. Noise free measured natural frequencies are assumed.
Figure 10. Localization performance of controller A (*sensitivity enhancement*), with three measured modal frequencies, 10 closed-loop systems, and 16 damage variables. Noise free measured natural frequencies are assumed.