Technological Leadership and Endogenous Growth

Susanto Basu,* James Feyrer,† & David Weil‡

First Draft: July 1998
This Draft: February 8, 2002

Abstract

This paper investigates the interaction between technological leadership and spillovers through a theoretical model where technological followers benefit from spillovers from a technological leader. Simulations of the model show that technological leadership is persistent and history dependent. A more patient nation may choose to remain behind an impatient technological leader in order to benefit from spillovers. The result is lower steady state growth than would occur if the more patient nation took leadership. The probability of a persistent, impatient leader is higher the more easily technology flows between nations.

Introduction

A typical feature of endogenous growth models is that the rate of growth is a function of the discount rate.[1] Countries with lower discount rates are more patient and invest more in developing new technologies. The higher investment rate is rewarded with more rapid technological progress and higher long run growth rates. If taken at face value, models of this type predict that the income gap between two countries with different discount rates will grow over time.

*Department of Economics, University of Michigan
†Department of Economics, Dartmouth College. E-mail : james.feyrer@dartmouth.edu
‡Department of Economics, Brown University. E-mail : david_weil@brown.edu

[1] We need to add some citations. Aghion and Howitt (1992), Romer (1986), Grossman and Helpman (1991)

However, as noted by Gerschenkron (1962), Nelson and Phelps (1966) and others[2], technological growth need not be solely through the creation of new technologies as implied by the endogenous growth literature. Due to the non-rivalry of ideas, the presence of technological spillovers allows a less technologically advanced nation to grow in the absence of any domestically created technological advances.

It seems reasonable to think that the rate of technological spillovers will be a function of the distance from the technological frontier. Countries far from the frontier will have a large backlog of technologies to adopt, and will naturally choose to adopt those technologies that have a high rate of return.[3] As they near the frontier, however, the number of technologies available for adoption will diminish and the difference between adaptation and creation of new technologies will blur. In the steady state, any country investing less in new technologies than the leader will lag behind. Indeed, for any given level of investment in technology, there will be a unique position below the technological frontier.

If the technological leader is also the most patient country in the world, it seems clear that this is a stable arrangement. The rest of the countries in the world will be impatient relative to the leader and will therefore want to invest less on new technologies than the leader, even in autarky. As followers, these countries will grow more rapidly than they would in autarky, while investing less on developing new technologies.

What happens in the case when the technological leader is not the most patient nation is less clear. Suppose that a relatively impatient nation is put in the position of technological leader by historical accident and that a more patient nation is in the following role. Are there any conditions under which this can be a stable relationship?

The more patient nation is faced with a choice: invest less in technology, grow more slowly than if alone in the world, and take advantage of technological spillovers; or invest more in technology, grow at the autarky rate, and provide spillovers to the follower. This tradeoff turns out to be consequential. If a less patient nation is the technological leader in equilibrium, the world as a whole grows less rapidly. This

[2]Several papers in the endogenous growth literature have explicit models of spillovers, including Grossman and Helpman (1991), Basu and Weil (1998) and Howitt (2000)

[3]Potential growth rates seem to be higher for countries away from the technological frontier. Evidence for this is the lack of examples of wealthy nations (presumably near the frontier) with sustained growth rates of greater than about 4%. On the other hand, there are several examples of backward nations sustaining growth rates of nearly 10% for decades.

paper will examine the nature of this tradeoff and see when the more patient nation chooses to follow and when they choose to lead.

The paper will describe the leadership choice in stages. Section one introduces the model and describes the nature of balanced growth paths for exogenous levels of investment in technological development. Section 2 endogenizes the choice of technology investment and determines the optimal long run level of investment for a leader and a follower. Given these optimal levels of investment, Section 3 describes the balanced growth paths that are candidates to be equilibria in the long run. Section 4 will then evaluate which of the possible balanced growth paths is a Nash equilibrium.

1 The Model

The model is based on the basic model from Lucas (1988), simplified by using log utility and neglecting population growth and capital depreciation. Endogenous growth is produced through constant returns to accumulable factors and the steady state growth rate is a decreasing function of the discount rate.[4] Preferences over per-capita consumption streams in country i are given by

$$U_i = \int_0^\infty e^{\theta_i t} \ln[c_i(t)] dt \tag{1}$$

Output is determined by a constant returns to scale Cobb-Douglas production function two factors of production, physical capital and human capital augmented labor. Labor can be allocated to production or to the accumulation of human capital. Output per capita in country i is [5]

[4] This model was chosen for simplicity. Models with more explicit microfoundations for innovation such as Aghion and Howitt (1992) and Grossman and Helpman (1991) have a similar relationship between the discount rate and the long run level of growth. This is the feature of endogenous growth models that we wish to exploit.

[5] Throughout the paper, we are analyzing the solution to the social planner's problem. The results of the paper continue to hold in a decentralized model with individual agents. In particular, the results hold if we introduce a composite human capital measure that allows spillovers between individuals and the aggregate economy. The production function has the form

$$y_{i,j}(t) = Bk_i(t)^\alpha [u_{i,j}(t) h_{i,j}(t)^\psi h_i(t)^{1-\psi}]^{1-\alpha}$$

where individual j's proportion of labor used in production is $u_{i,j}(t) \in [0,1]$ and individual j's level of physical capital is $k_{i,j}(t)$. The effective level of human capital for individual j in country i is a constant returns to scale combination of the individual's level of human capital, $h_{i,j}(t)$ and

$$y_i(t) = Bk_i(t)^\alpha [u_i(t)h_i(t)]^{1-\alpha} \tag{2}$$

To further simplify the analysis, the marginal product of capital is set by an exogenous interest rate. This allows us to express output without physical capital. Removing capital does not qualitatively affect the results.

$$y_i(t) = Au_i(t)h_i(t), \quad A = B\left(\frac{\alpha B}{r}\right)^{\frac{1}{1-\alpha}} \tag{3}$$

The proportion of output not paid to physical capital is consumed each period. We assume that capital flows across borders to equalize the marginal product of capital but that international borrowing for consumption smoothing is not possible. We further assume that the discount rate is higher than the world interest rate, $\theta_i < r, \ \ \forall i$. Under this condition, nations will want to borrow, so that the liquidity constraint will be binding. The proportion of labor not used for producing output, $(1 - u_i)$, is used to produce human capital.

At any given time, the country with the highest level of human capital is the technological leader. Let L be the subscript for the nation with the highest level of human capital. The leader's accumulation of human capital is unaffected by the actions of follower countries. The form of the human capital accumulation equation for the leader is based on the single country model of Lucas (1988)

$$\dot{h}_L(t) = [1 - u_L(t)]\,\phi h_L(t), \quad h_L(t) \geq h_i(t) \quad \forall i \tag{4}$$

At any given time, nations with a lower level of human capital than the leader are followers. In addition to domestically produced human capital, followers benefit from spillovers from the human capital of the leader. For a follower, the size of the spillover is a function of the level of human capital relative to the technological leader. The larger the difference between the two, the larger the return to investment in human capital. In other words, the farther a follower is behind the leader, the larger the spillover.

$$\dot{h}_i(t) = [1 - u_i(t)]\left\{\phi h_i(t) + \beta\left[h_L(t) - h_i(t)\right]\right\}, \tag{5}$$

the average level of human capital within country i, $h_i(t)$. As with physical capital in the Romer (1986) model, there is a spillover from human capital so that social returns to human capital are larger than individual returns to human capital.

$$h_i(t) < h_L(t)$$

Human capital accumulation for a follower can be seen as a combination of domestic idea production (the ϕh_i term) and spillovers (the $\beta[h_L - h_i]$ term). Human capital will not increase without some investment, but the return to investment is much larger when the technological leader is far ahead.[6]

Equation (4) can be seen as a special case of (5) since the β term will disappear when country i has the highest level of human capital, $h_i = h_L$. When a country makes the transition from technological follower to technological leader, their human capital accumulation equation is continuous.

1.1 Steady State Growth

Long run balanced growth paths will have the feature that investment in human capital is constant. In order to simplify the analysis, we will first look at the case where investment in human capital is exogenous and fixed. In the next section we will examine the case where u_i is endogenous.

The relationship between any two countries can be described as a system of two differential equations with two endogenous variables, h_1 and h_2. The system is described by equations (4) and (5) where the country with a higher level of human capital is denoted by the subscript L. Dropping the time indexes for simplicity, we can express (4) and (5) and as growth rates.

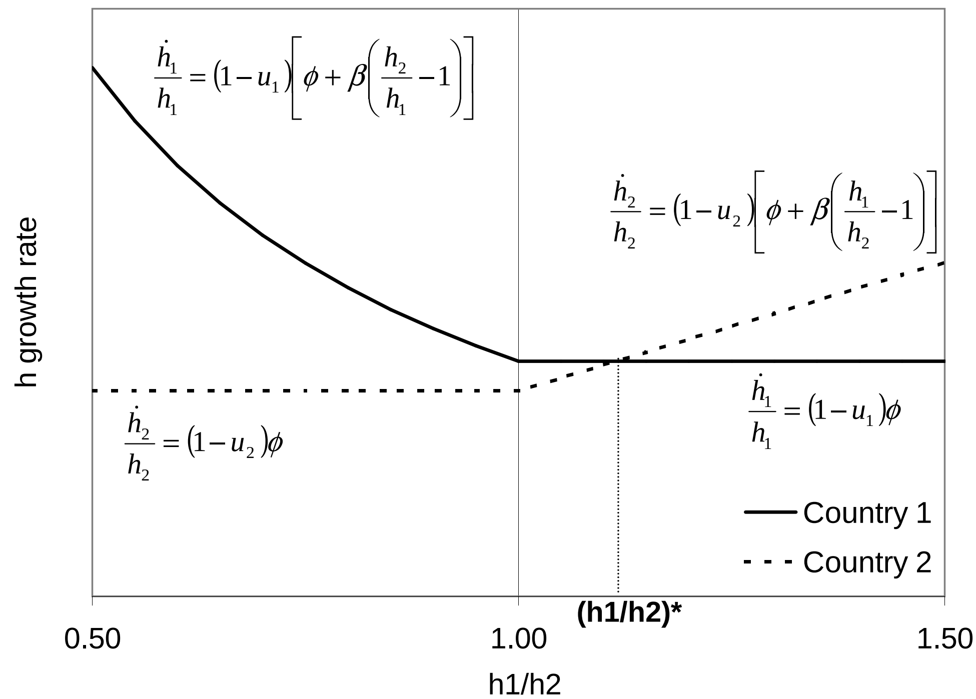
$$\frac{\dot{h}_L}{h_L} = (1 - u_L)\phi, \qquad h_L \geq h_i \quad \forall i \tag{6}$$

$$\frac{\dot{h}_i}{h_i} = (1 - u_i)\left[\phi + \beta\left(\frac{h_L}{h_i} - 1\right)\right] \qquad h_i \leq h_L \quad \forall i \tag{7}$$

Figure 1 shows the determination of the steady state. The center of the x-axis is the point where $h_1 = h_2$. To the left of this point, $h_2 > h_1$ and country one benefits from spillovers. To the right, $h_1 > h_2$ and country two benefits from spillovers. For the country benefiting from spillovers, the follower, the size of the spillover is a function of their relative backwardness. The follower's $\dot{h}/h$ function increases away from $(h_1/h_2) = 1$. For the country with higher human capital, the growth rate is not a function of the relative levels of human capital and is therefore constant. This

[6]Pack and Westphal (1986) [Good place to cite literature on R&D effort producing both knowledge and adaptation]

Figure 1: Steady State between Two Countries



is represented in Figure 1 by the horizontal segments. The steady state is the point where h_1 and h_2 grow at the same rate. In Figure 1, this is the crossing point of the two lines, $(h_1/h_2)^*$.

Whether the steady state is to the left or the right of the $(h_1/h_2) = 1$ is determined by the relative levels of investment in human capital. Suppose that the level of human capital is the same in both countries, $(h_1/h_2) = 1$. Neither country benefits from spillovers so the country with higher investment in human capital will have higher growth. For Figure 1, country one has higher investment in human capital accumulation $(1 - u_1) > (1 - u_2)$.

The leader will always be the nation with highest human capital investment. Equating (6) and (7) gives the stable relative levels of human capital for a follower. This will be referred to as the steady state level of backwardness.

$$\left(\frac{h_L}{h_i}\right)^* = 1 + \frac{(u_i - u_L)}{(1 - u_i)}\frac{\phi}{\beta} \tag{8}$$

2 Endogenous Steady States

In this section we extend the analysis of the previous section by endogenizing the level of investment in human capital. We begin by examining balanced growth paths where the level of investment in human capital accumulation is constant.[7] Through dynamic optimization we can determine the long run levels of human capital accumulation that result from optimizing behavior.

Along a balanced growth path a country must either lead or follow. For each of these long run outcomes, they must satisfy a set of first order conditions at all times. For these first order conditions we can calculate the balanced growth level of investment in human capital accumulation.

2.1 The Technological Leader

This section will examine the balanced growth level of investment in human capital accumulation for a country which is the leader on the balanced path. The current value Hamiltonian the country with the highest level of human capital (dropping the time indexes for simplicity) is

$$H_L = \ln(Au_L h_L) + m_L \left[(1-u_L)\phi h_L\right], \quad h_L(t) \geq h_i(t) \ \ \forall i \tag{9}$$

An optimal path must satisfy the following first order conditions,

$$\frac{\partial H_L}{\partial u_L} = \frac{1}{u_L} - m_L \phi h_L = 0 \tag{10}$$

$$\dot{m}_L = -\frac{\partial H_L}{\partial h_L} + m_L \theta_L = -\frac{1}{h_L} - m_L \phi(1-u_L) + m_L \theta_L \tag{11}$$

We are interested in balanced growth paths where the long run investment in human capital is constant. The leader's balanced path can be solved without any knowledge of the follower's behavior. The two first order conditions, (10) and (11), can be combined to solve for the growth rate of the costate variable m_L,

$$\frac{\dot{m}_L}{m_L} = -\phi + \theta_L \tag{12}$$

For u_L to remain constant, equation (10) implies that m_L and h_L must have

[7]This is similar to the constant long run saving rate in the Cass-Ramsey model. In both cases, constancy of the choice variable is a consequence of transversality conditions.

growth rates of equal magnitude and opposite sign. With (12) this allows us to solve for the steady state growth rate of human capital and output.

$$\frac{\dot{h}_L}{h_L} = -\frac{\dot{m}_L}{m_L} = \phi - \theta_L \tag{13}$$

Plugging this result in to the leader's human capital accumulation equation (6) we can determine the steady state value of u_L^* which satisfies the first order conditions,

$$\frac{\dot{h}_L}{h_L} = (1 - u_L)\phi = \phi - \theta_L$$
$$u_L^* = \frac{\theta_L}{\phi} \tag{14}$$

For a country which is the leader on a balanced growth path, optimization implies that the level of human capital accumulation is described by (14). In our simplified dynamic system (with no capital) the leading nation is always on the balanced growth path without transitional dynamics.

2.2 Technological Followers

This section will examine the balanced growth level of investment in human capital accumulation for a country which is a follower on the balanced path. The current value Hamiltonian for an individual in the follower nation is,

$$H_i = \ln(Au_ih_i) + m_i \left\{(1 - u_i)\left[\phi h_i + \beta\left(h_L - h_i\right)\right]\right\}, \tag{15}$$
$$h_i(t) < h_L(t) \ \forall i$$

An optimal path must satisfy the following first order conditions,

$$\frac{\partial H_i}{\partial u_i} = \frac{1}{u_i} - m_i\left[\phi h_i + \beta\left(h_L - h_i\right)\right] = 0 \tag{16}$$

$$\dot{m}_i = -\frac{\partial H_i}{\partial h_i} + m_i\theta_i = -\frac{1}{h_i} - m_i(\phi - \beta)(1 - u_i) + m_i\theta_i \tag{17}$$

Again, we are interested in balanced growth paths where the long run investment in human capital is constant. From the preceding section we saw that a technological leader chooses a constant growth rate of human capital which is independent of the actions of a follower nation. From the point of view of a technological follower, this growth rate is exogenous in a long run steady state. Let g_L be the growth rate chosen by the leader.

As before, we are considering long run steady states where the level of u_i is constant. In Section 1 we showed that there is a stable steady state between two nations choosing constant levels of investment in human capital accumulation. In this steady state, the ratio h_L/h_i is constant.

Using these two features of the steady state we can solve for the level of u_i^* which satisfies the first order conditions for a follower. The two first order conditions, (16) and (17), can be combined to solve for the growth rate of the costate variable m_i,

$$\frac{\dot{m}_i}{m_i} = -u_i \beta \frac{h_L}{h_i} - (\phi - \beta) + \theta_i \tag{18}$$

If u_i and the ratio h_L/h_i are to remain constant, (16) implies that m_i and h_i must have growth rates of equal magnitude and opposite sign. The constancy of h_L/h_i allows us to equate the growth rate of h_i with the growth rate of h_L determined in the leader's problem by equation (13).

$$\frac{\dot{m}_i}{m_i} = -\frac{\dot{h}_i}{h_i} = -\frac{\dot{h}_L}{h_L} = -g_L \tag{19}$$

Equating (18) and (19),

$$\beta \frac{h_L}{h_i} = \frac{g_L + \theta_i}{u_i} + \frac{\beta - \phi}{u_i} \tag{20}$$

Using the follower's human capital accumulation equation (7) we can determine the optimum u_i^* where the follower grows at the same rate as the leader along a balanced growth path.

$$\phi + \beta \frac{h_L}{h_i} - \beta - u_i \phi - u_i \beta \frac{h_L}{h_i} + u_i \beta = g_L \tag{21}$$

Using (20) to substitute out h_L/h_i terms,

$$\phi + \frac{g_L + \theta_i}{u_i} + \frac{\beta - \phi}{u_i} - \beta - u_i \phi - u_i \left(\frac{g_L + \theta_i}{u_i} + \frac{\beta - \phi}{u_i} \right) + u_i \beta = g_L \tag{22}$$

Rearranging and multiplying through by u_i to get a quadratic equation in u_i

$$[\beta - \phi]\, u_i^2 + [-2(\beta - \phi) - 2g_L - \theta_i]\, u_i + [\beta - \phi + g_L + \theta_i] = 0 \tag{23}$$

Solving for u_i,[8]

$$u_i^* = \frac{2(\beta - \phi) + 2g_L + \theta_i - \sqrt{4g_L(\beta - \phi) + [2g_L + \theta_i]^2}}{2(\beta - \phi)} \tag{24}$$

Substituting in $g_L = \phi(1 - u_L^*)$ from (6) we can express the steady state investment in human capital for a follower in terms of the steady state level investment in human capital for the leader.

$$(1 - u_i^*) = \frac{-2\phi(1 - u_L^*) - \theta_i - \sqrt{4\phi(\beta - \phi)(1 - u_L^*) + [2(1 - u_L^*) + \theta_i]^2}}{2(\beta - \phi)} \tag{25}$$

Equation (25) describes the optimal choice of investment in human capital accumulation along the balanced growth path for a follower as a function of the leader's investment in human capital. Assume that human capital in country i relative to the leader is at the stable level defined by equation (8), $h1/h2 = (h1/h2)^*$, and that $u_i(t) = u_i^*\ \forall t$. As long as the lead country grows at the rate g_L, country i will satisfy the first order conditions at all times. This is therefore the only optimal long run choice if country i is a follower.

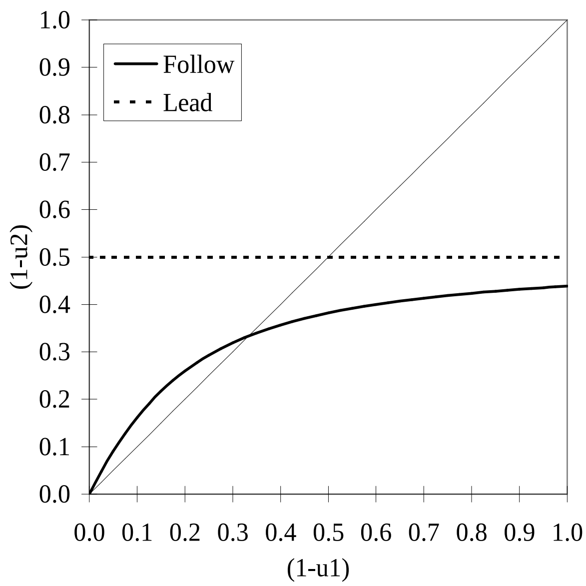
3 Optimal Long Run Equilibria

On a balanced growth path, each of the countries in our model must either lead or follow. If they lead, they must satisfy the steady state optimality condition for a leader, equation (14), and if they follow, they must satisfy the steady state optimality condition for a follower, equation (25). Figure 2 shows both these possibilities for country two in our model as a function of country one's investment in human capital accumulation, u_1. The straight line, labelled "Lead," is invariant to the level of u_1 because if country two leads, their behavior is independent of country one. The curved line, labelled "Follow," shows the optimal long run response of country two

[8] only the positive root results in $u_i^* \in [0, 1]$

to different levels of u_1. These two lines represent the only possible choices of human capital accumulation for country two along an optimal balanced growth path.

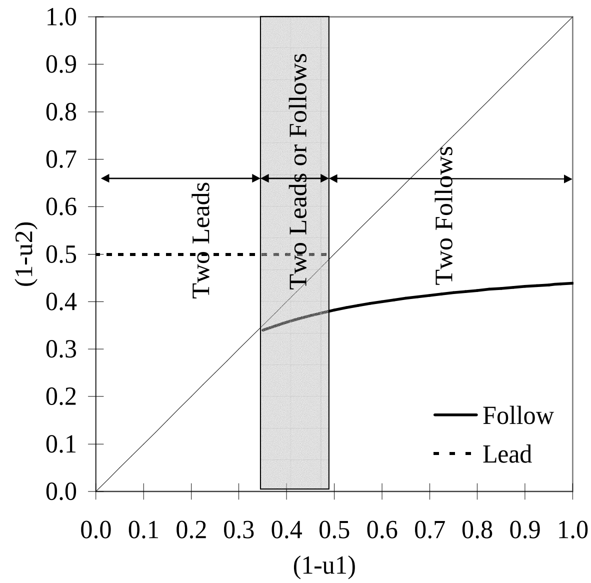
Figure 2: Optimal Responses for Country Two



A closer look at Figure 2 will show that portions of each line will not result in a steady state as described in Section 1. Recall from Section 1 that the country with the highest level of investment in human capital is always the leader in the steady state. To the right of the 45^o line, the Lead line indicates that country two's optimal level of investment in human capital as a leader is lower than country one's investment. Country two cannot lead if they invest less than country one in human capital accumulation. Similarly, a portion of the Follow line lies above the 45^o line. If country two's optimal level of human capital accumulation as a follower is larger than country one's level, two cannot be a follower. Redrawing the graph to include only the portions of each line which represent potential steady states results in Figure 3.

If country one has low enough investment in human capital accumulation, country two has no choice but to lead in the long run. This is represented in Figure 3 by the region to the left of the grey rectangle. If country one has sufficiently high human capital investment country two must follow on the balanced path (the area to the right of the grey rectangle). In between the extremes, both leading and following are possible long run equilibria. This is represented in Figure 3 by the

Figure 3: Optimal Responses for Country Two



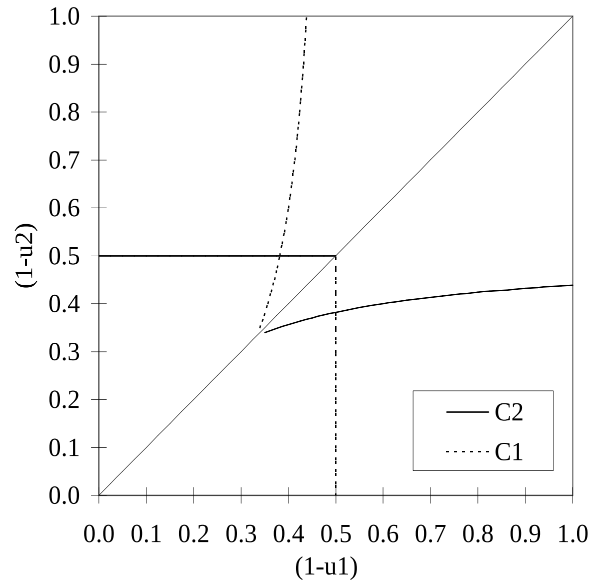
shaded area between the two intersections with the 45^0 line.

3.1 Potential Equilibria

Any long run steady state must be a result of optimizing behavior on the part of *both* countries. Figure 3 shows the responses of country two to a fixed level of investment in human capital for country one. These responses satisfy the first order conditions for optimality set out earlier in the paper. We can also draw the corresponding graph showing country one's optimal responses to a fixed level of investment in human capital in country two. By combining the two graphs we can illustrate potential steady states where both countries satisfy their optimality conditions.

Figure 4 graphs the response functions for both countries. The straight lines represent paths where the country leads, the curved line represents paths where the country follows. The two points where the curves cross in Figure 4 represent possible equilibrium points; one with country one leading (lower right) and the other with country two leading (upper left). For each of these equilibrium points, one of the countries is investing the fixed level of $(1 - u_L)$ described by (14) which is optimal if they lead on the balanced growth path. The other country responds by investing the fixed level of $(1 - u_i)$ described by (25) which is optimal for a follower given the

Figure 4: Symmetric Response Functions - Two Possible Equilibria



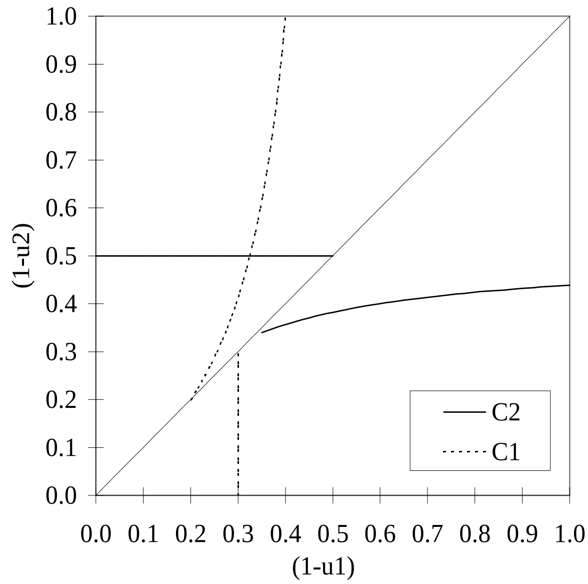
leader's behavior.

For different parameter values, there may be only one crossing point and only one possible equilibrium. Figure 5 illustrates this possibility. In Figure 4 both countries share the same discount rate. For Figure 5 country one's discount rate was increased. The increased discount rate results in lower optimal investment, shifting country one's response functions to the left. The equilibrium point where one remains in the lead is eliminated.

For any given set of parameters β and ϕ, the existence of one or two potential equilibria will be a function of the discount rates of the two countries. We are interested in identifying the set of discount rates pairs where there are two potential long run equilibria, as illustrated by the crossing points in Figure 4. These are the conditions under which it is possible for a less patient country to lead in the long run.

Suppose that country two has a lower discount rate than country one. For there to be two crossing points, the optimal level of investment by country two as a follower must be lower than country one's autarky investment at some point to the right of the 45^o line. Since the optimal response of two is an increasing function of one's investment in human capital, it is the case that two's lowest possible investment level is found where their response curve crosses the 45^o. This investment level can be

Figure 5: Only One Possible Equilibrium



solved by setting he follower's optimal response, (25) equal to the leaders invesment level, $(1 - u_L)$. Solving and simplifying we find that the minimum sustainable investment for country two is

$$\underline{(1 - u_i)} = \frac{\phi - \theta_i}{\beta + \phi} \tag{26}$$

The minimum possible level of investment is decreasing in the discount rate (less patient nations are willing to invest less), and decreasing in the spillover parameter β (larger spillovers induce less investment).[9]

It is impossible for two to be a follower if the leader is investing less than two's minimum in human capital accumulation. The condition where two can potentially follow can therefore be stated as

$$(1 - u_F^*) > \underline{(1 - u_i)} \tag{27}$$

This condition simply states that the leader must invest more than the follower's minimum investment to retain leadership. Plugging in (14) and (26) we can describe the conditions needed for two potential equilibria in terms of the discount rates of

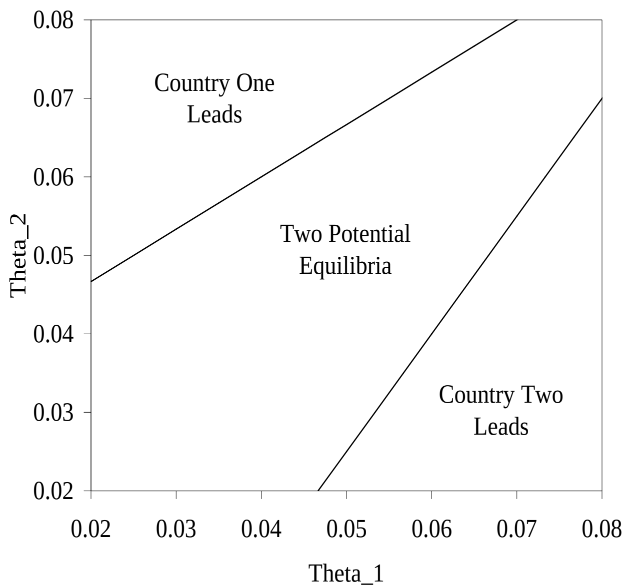
[9]Indeed, as $\beta \to 0$ the minimum level of investment by country two approaches the autarky level.

the two countries and the exogenous parameters.

$$\frac{\theta_L}{\phi} < \frac{\beta + \theta_i}{\beta + \phi} \tag{28}$$

Again, it is worth noting the effect of changes to the spillover parameter. As long as $\phi > \theta_i$ increasing β will increase the allowable discount rate of the leader.[10] In other words, as spillovers increase, you can sustain a larger gap between the leader and follower's discount rates. As $\beta \to \infty$, any gap between the discount rates is sustainable as long as the leader has positive growth.[11] As $\beta \to 0$, no gap between the two discount rates is sustainable and the more patient nation will always lead. Figures 6 and 7 illustrate the properties of Equation 28.

Figure 6: Potential Equilibria as a Function of Discount Rates

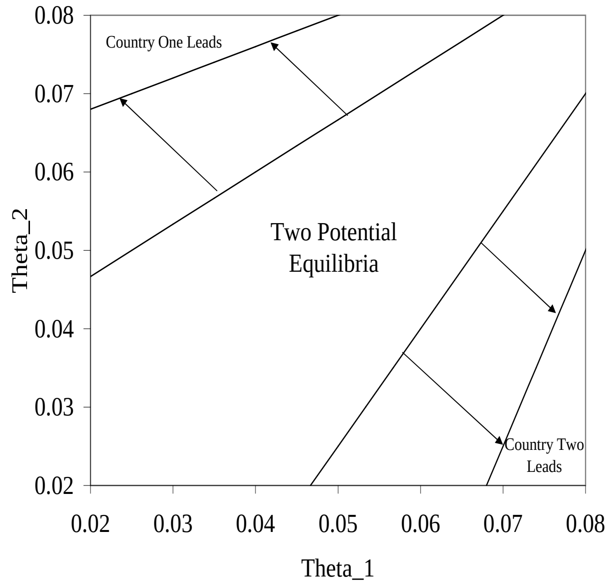


Near the central region of Figure 6 the gap between the two discount rates is small and either country can potentially lead in the long run. In the upper left and lower right corners of the graph, the gap between the discount rates is larger and the country with the lower discount rate always will lead in the long run. Figure 7 shows that as the magnitude of spillovers, represented by the parameter β, increases the size of region where there are two potential equilibria.

[10]From (25) we can see that requiring $\phi > \theta_i$ is equivalent to requiring that country two would have positive growth in autarky

[11]Again, see (25)

Figure 7: The Effect of Increasing Beta



In the case where there are two potential equilibria (i.e., two crossing points exist), these equilibria represent the only long run choices of investment in human capital where both countries satisfy their respective first order conditions along balanced growth paths. This does not imply, however, that both configurations are optimal for both countries. It may be the case that one country will wish to deviate from one of the long run paths in order to move to the other long run path. The next section will describe the circumstances under which this is the case. In doing so, we will be able to establish that one of the two potential balanced growth paths is a Nash equilibrium.

At this point, we should note that the insights gained from analyzing the potential long run equilibria continue to hold when we add transitional dynamics to the analysis. Some of the potential equilibria described in the Figures 6 and 7 will turn out to be unsustainable as Nash Equilibria, but the basic shape of the regions remains intact. Long run paths where less patient nations are technological leaders continue to be possible and the likelihood of this outcome increases with the rate of technological spillovers.

4 Nash Equilibria

In this section we will examine the two potential equilibria described in the previous section and determine the conditions under which these points are Nash equilibria. Our discussion will encompass variation in three variables, the discount rates, θ_1 and θ_2, and the initial relative levels of human capital, $h_1(0)/h_2(0)$.

In the long run, for any given set of parameters there are only two possible configurations; country one leading and country two leading. For each of these configurations, the long run investment in human capital accumulation and the long run human capital ratio will be constant as determined in the previous section. Given a set of discount rates and the initial human capital ratio, any optimal paths of human capital accumulation must satisfy the differential equation system given by the first order conditions, (10), (11), (16), and (17) at all points in time. The problem is to find initial values for the control variables, $u_1(0)$ and $u_2(0)$, which put each country on one of the two potential balanced growth paths.

There are two possible sets of initial conditions which lead to optimal balanced growth paths given θ_1, θ_2 and $h_1(0)/h_2(0)$:

$$\left[u_1^L(0), u_2^F(0)\right]$$

$$\lim_{t\to\infty} u_1^L(t) = u_1^{L*}, \quad \lim_{t\to\infty} u_2^F(t) = u_2^{F*}, \quad \lim_{t\to\infty} \frac{h_1(t)}{h_2(t)} = \left(\frac{h_1}{h_2}\right)^* > 1 \tag{29}$$

$$\left[u_1^F(0), u_2^L(0)\right]$$

$$\lim_{t\to\infty} u_1^F(t) = u_1^{F*}, \quad \lim_{t\to\infty} u_2^L(t) = u_2^{L*}, \quad \lim_{t\to\infty} \frac{h_1(t)}{h_2(t)} = \left(\frac{h_1}{h_2}\right)^* < 1 \tag{30}$$

where the superscripts L and F indicate the relative position of the country in the steady state. If the two countries start with initial $\left[u_1^L(0), u_2^F(0)\right]$ their optimal paths will lead to country one leading and country two following along a balanced growth path. As t gets large, each country's investment in human capital accumulation will approach the balanced growth levels described by equations (14) and (25). Similarly, the human capital ratio approaches the steady state level of backwardness described by (8).

For (29) country one leads along the balanced growth path; for (30) country two leads along the balanced growth path. There is no reason to assume that both these paths exist. As we showed in the last section, there are conditions under which only one long run configuration is supportable in the long run.

Let

$$[u_1^L(0..\infty), u_2^F(0..\infty)] \tag{31}$$

$$[u_1^F(0..\infty), u_2^L(0..\infty)] \tag{32}$$

be the entire paths of $u_1(t)$ and $u_2(t)$ implied by (29) and (30). The paths (31) and (32) are the only possible Nash equilibria given a set of initial parameters because they are the only long run paths which satisfy the first order conditions for an optimum. Each of these paths is a Nash equilibrium if and only if the path for each country is the optimal strategy taking the other country's path as given.

To evaluate whether (31) is a Nash equilibrium, suppose that country one takes the path for country two in (31), $u_2^F(0..\infty)$ as given. Country one may choose $u_1^L(0..\infty)$ and lead, or they may pick a starting value $u_1^{F'}(0)$ and path $u_1^{F'}(0..\infty)$ which satisfies the first order conditions and leads them to a balanced growth path following country two. If $u_1^{F'}(0..\infty)$ is preferred to $u_1^L(0..\infty)$ then (31) is not a Nash equilibrium. Similarly we check to see if $u_2^F(0..\infty)$ is preferable to $u_2^{L'}(0..\infty)$, a leading path for country two given that country one chooses $u_1^L(0..\infty)$.

At this point it is important to note that we are solving for open-loop Nash equilibria. That is, for given initial conditions we pick a time path of saving rates for country one and then solve for the optimal time path of saving rates for country two. If country one's time path of saving rates is also optimal given country two's reaction then we conclude that we have found a Nash equilibrium.

Clearly, these Nash equilibria are not necessarily subgame-perfect. Of course, it would be desirable to solve for subgame-perfect equilibria, since these equilibria specify the behavior of the countries both on and off the equilibrium path. By contrast, we solve only for the behavior on the equilibrium path. For example, at $t = 10$ the equilibrium will specify values of u_1 and u_2. But these values are optimal only if the equilibrium actions have been followed for $t < 10$. If one country has deviated (or if there has been an unexpected shock to the human capital of one country), then the countries will not know what to do at subsequent points in time. However, there is no general theory of subgame-perfect Nash equilibrium for dynamic games (as opposed to repeated games). In fact, there are only a few special examples of dynamic games where it has proved possible to solve for subgame-perfect equilibria, and our model is not in this set. Thus, we consider non-subgame-perfect Nash equilibria, while remaining cognizant of the limitations of this solution concept.

We have noted one reason why we may have "too many" equilibria-we do not discard some of them by applying subgame perfection. However, we may also have "too few" equilibria, since we do not investigate Nash equilibria where actions are conditioned upon the history of play (as in trigger strategies). Thus, for example, we do not consider equilibria with strategies of the form "Save more, or we will punish you." We do not consider this omission to be a serious one. Since our model is based on countries with a large number of small agents, we do not think it likely that such agents would coordinate on sophisticated punishment strategies.

4.1 Simulations

The comparisons described in the previous section can be accomplished through numerical simulations. Given some set of conditions θ_1, θ_2, and $h_1(0)/h_2(0)$, the starting values in (29) and (30) are straightforward to find using the limiting conditions on u_1 and u_2. If $\lim_{t\to\infty} u_1(t) = -\infty$, the starting value $u_1(0)$ is too low. Similarly, if $\lim_{t\to\infty} u_1(t) = \infty$, $u_1(0)$ is too high. Finding $[u_1(0), u_2(0)]$ pairs which satisfy the limit conditions is sufficient to find the entire optimal paths of (31) and (32).

To see if (31) is a Nash equilibrium we first take the strategy for country two, $u_2^F(0..\infty)$, as given. The optimal strategy where country one leads, $u_1^L(0..\infty)$ is already known. Plugging $u_2^F(0..\infty)$ into the equations of motion we look for a starting value $u_1^{L'}(0)$ for country one which results in country one following country two on a balanced path and which satisfies the first order conditions for a maximum for country one. Call this strategy $u_1^{F'}(0..\infty)$. If

$$U_1(u_1^{F'}(0..\infty)) > U_1(u_1^L(0..\infty)) \tag{33}$$

we can eliminate (31) as a Nash equilibrium. Similarly, we can take the strategy for country one, $u_1^L(0..\infty)$, as given and check to see if country two would deviate. If

$$U_2(u_2^{L'}(0..\infty)) > U_2(u_2^F(0..\infty)) \tag{34}$$

we can eliminate (31) as a Nash equilibrium. If (31) passes both tests country one as the leader is a Nash equilibrium. We can similarly test the case where country two leads, (32).

4.2 Nash Equilibria for Fixed θ_1

In Section 3, we were able to identity the conditions under which there are two potential candidates to be Nash equilibria.[12] Through numerical simulations we will identify *which* of those potential equilibria is sustainable as a Nash equilibrium. This will turn out to depend critically on the initial levels of human capital.

We begin with the result that a Nash equilibrium always exists when the more patient nation is the leader at time $t = 0$. A less patient nation never chooses to pass a more patient leader. Our investigation therefore focuses on the case where a more patient nation is initially behind a less patient leader.

In the following discussion, we will assume that country one is always the leader at time $t = 0$ and that country two has a lower discount rate than country one. One central result from simulations is that the leader at time $t = 0$, country one, always has higher utility by acquiescing to the decision of the follower. That is, if country two, starting behind, chooses to stay behind, country one has higher utility by continuing to lead. Similarly, if country two chooses to become the leader, country one has higher utility from following.

Country two is therefore the decision maker. One possibility is that country two's utility is always maximized by overtaking the leader, regardless of initial levels of human capital. If this is the case, the only Nash equilibrium is with country two leading.[13]

First we will consider the case where the discount rate for country one is fixed. We would like to find conditions under which country two's optimal choice is always to become the leader regardless of initial human capital levels. These conditions will define the boundary between where there are two potential Nash Equilibria and where the only Nash equilibrium is for country two to lead. Suppose that country one is the current leader and that country two is at the steady state value

[12] If there is only one candidate, there is no need for simulations. The more patient country leading in the long run is a Nash Equilibrium

[13] If country two starts ahead, they stay ahead due to a lower discount rate. If country two starts behind, they overtake country one. Country two leading is therefore the only long run Nash equilibrium regardless of initial conditions.

for backwardness,

$$\frac{h_2(0)}{h_1(0)} = \left(\frac{h_2}{h_1}\right)^* \tag{35}$$

This starting position is chosen because any path from a lower starting position will approach this point even if country two is to remain a follower. Therefore, if country two chooses to overtake from an initial position at the steady state level of backwardness, they will also choose to overtake starting at any level below the steady state. At human capital ratios above the steady state level of backwardness, simulations show that overtaking becomes more attractive. Therefore, if country two chooses to overtake from the steady state level of backwardness, they will also choose to overtake from any point above this level. Therefore, if country two chooses to overtake country one when the starting position is the steady state level of backwardness, they will choose to overtake country one for any starting position.

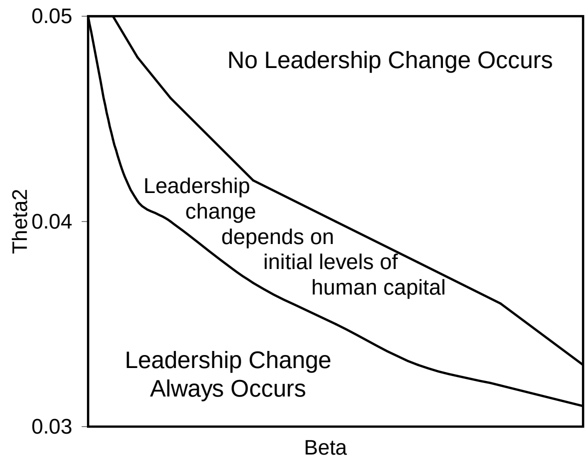
We are examining the candidate Nash equilibrium (29) where country one remains the leader along the balanced path. Since there are no transitional dynamics for a leader and country one starts in the lead, country one will always choose u_1^{L*} described by equation (14). Country two takes this behavior as given. Country two has two options, remaining at the steady state level of backwardness and following, or passing country one and becoming the leader.

Suppose that we hold θ_1 constant and look for the critical θ_2, θ_2^L where country one remaining the leader ceases to be a Nash equilibrium for any starting ratio of human capital. A country with a discount rate equal to or lower than θ_2^L will always overtake country one along the balanced growth path. This value identifies the boundary where two following is a potential Nash Equilibria. The critical θ_2^L will change with the spillover parameter, β.

There is also a second critical value of interest. When $\theta_2 > \theta_2^L$, country two will remain behind country two if they start at the steady state level of backwardness. Suppose that country two's discount rate is ϵ higher than the critical level $\theta_2 = \theta_2^L + \epsilon$. If country two's initial position is the steady state level of backwardness, they will choose to follow. However, as previously noted, overtaking becomes more attractive at higher starting level of human capital. Therefore, we will identify a second critical value, $\theta_2^F > \theta_2^L$ where country two will choose to follow if they start with any human capital level below the leader.

Figure 8 shows the value of the critical values θ_2^L and θ_2^F for rising β. As β gets larger the critical values fall. In other words, as spillovers get larger, the gap in discount rates where country two will choose to lag behind a less patient leader gets larger. They are willing to give up larger amounts of future growth in exchange for spillovers.

Figure 8: Nash Equilibria, $h_2(0)/h_1(0) < (h_2/h_1)^*$,$\theta_1 = .05$

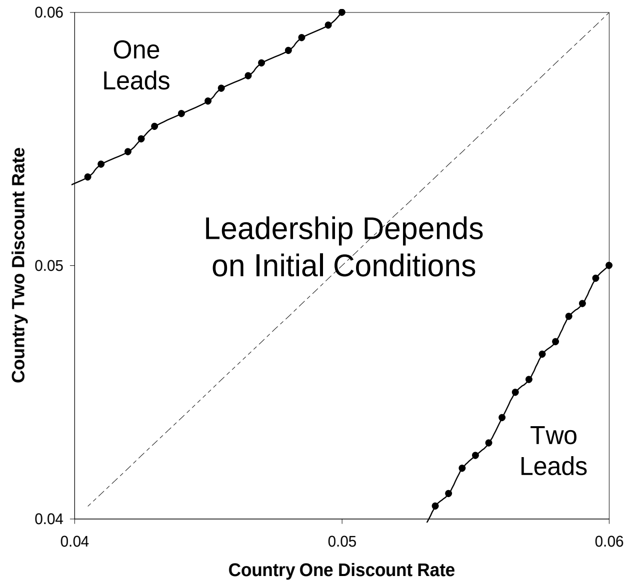


For the lower left hand corner, country two is willing to overtake country one regardless of starting position. Therefore, country two leading is the only Nash equilibrium. For the upper right corner, country two will stay behind country one as long as they start behind. The relatively small zone between the two critical values identifies the parameter values for which the precise initial conditions matter. If country two is initially close to country one in this region, they will overtake, otherwise, they will remain behind. This region identifies the only parameter values where initial conditions matter.

4.3 Nash Equilibria for General θ_1, θ_2

Figure 9 shows the possible configurations of Nash Equilibria given the discount rates for country one and country two. If the gap between the two discount rates is high enough, the only Nash equilibrium is for the more patient nation to lead. This is represented by the upper left and lower right regions. When the two discount rates differ by less, there will be two possible equilibria. Only one of these equilibria is a Nash Equilibrium. Which one is dependent on initial conditions.

Figure 9: Nash Equilibria as a Function of Discount Rates

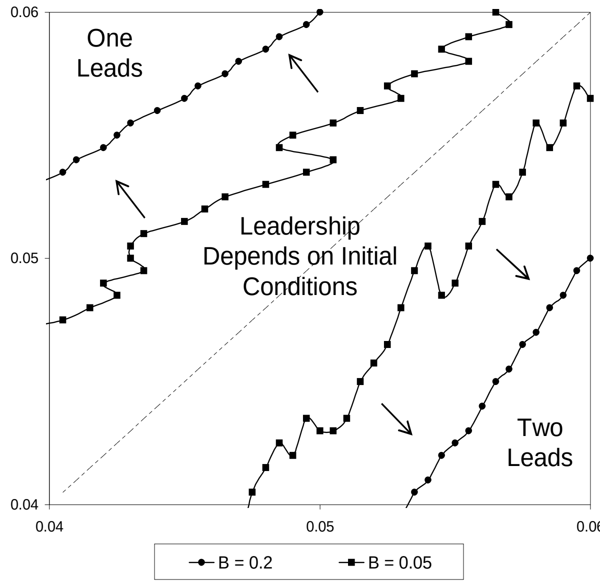


The larger the size of the spillover, (larger β) the larger the area with multiple possible equilibria. A less patient follower faces a choice: grow more slowly and receive spillovers, or grow more quickly without spillovers. Obviously, this tradeoff depends crucially on the size of the spillover (see Figure 8). For large spillovers, a much less patient nation may choose to follow in order to benefit from the spillover. On the other hand, if the benefits from spillovers are small, a small differential in discount rates may result in the more patient nation overtaking.

Figure 10 shows the effect of increasing β on the size of the multiple equilibrium zone. As β increases, the area with two potential equilibria increases and it is more likely that a more patient nation will remain a technological follower. If we consider the case where $\beta \to 0$, the model reverts to a standard endogenous growth model where the more patient nation always becomes the leader in the long run. This case would be represented in Figure 10 by a shrinking of the multiple equilibrium area to a line. If both countries had the same discount rate and $\beta = 0$, the only Nash equilibrium is the initial leader remaining the leader forever.

It should be noted that Figures 9 and 10 are very similar to Figures 6 and 7. The simulations served to show that for a portion of the region of Figure 6 labelled "Two Potential Equilibria", some of the the points are unsustainable as Nash Equilibria regardless of initial conditions. However, the shapes of the regions, and the impact of

Figure 10: The Effect of Increasing Beta



increasing the spillover parameter remain unchanged after the simulations exercise.

5 Conclusion

Our model shows that it is possible to have a stable equilibrium where a less patient nation is the technological leader. This appears to be the current situation in the world. As judged by savings rates, the United States appears to have one of the highest discount rates among the industrialized nations yet invests a higher per capita share of GDP on research and development than any other nation. This situation is more likely to occur in a world with rapid transfers of technology from the technological leader to followers.

If we assume that the speed of transfers is increasing over time, we should see that technological leadership is more persistent over time. With a low transfer coefficient, a relatively small shock to discount rates will be sufficient to induce a change in technological leadership. With a large transfer coefficient, leadership changes require a much larger differential in discount rates.

This may be the case. The US has been the world technological leader for at least eighty years and arguably for well over a century. There appear to be few signs that US technological leadership is diminishing. This is a relatively long period of time when compared to other technological leaders over the previous three centuries.

References

Aghion, P. and P. Howitt, "A Model of Growth Through Creative Destruction," *Econometrica*, March 1992, *60* (2).

____ **and** ____ , *Endogenous Growth Theory*, Cambridge, MA: MIT Press, 1998.

Barro, Robert J. and Xavier Sala-i-Martin, *Economic Growth*, Cambridge, MA: MIT Press, 1995.

Basu, Susanto and David Weil, "Appropriate Technology and Growth," *Quarterly Journal Of Economics*, November 1998, *113* (4), 1025–54.

Brezis, Paul Krugman, and Tsiddon, "Leapfrogging in International Competition: A Theory of Cycles in National Technological Leadership," *American Economics Review*, December 1993.

Gerschenkron, Alexander, *Economic Backwardness in Historical Perspective*, Cambridge, MA: The Belknap Press, 1962.

Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy*, MIT press, 1991.

____ **and** ____ , "Endogenous Innovation in the Theory of Growth," *Journal of Economic Perspectives*, Winter 1994, *8* (1).

Howitt, Peter, "Endogenous growth and cross country income differences," *AER*, September 2000, *90* (4), 829–46.

Lucas, Robert E., "On the Mechanics Of Economic Development," *Journal of Monetary Economics*, 1988, *22*, 3–42.

Mankiw, N. Gregory, David Romer, and David Weil, "A Contribution to the Empirics of Economic Growth," *Quarterly Journal Of Economics*, 1992, *107*, 407–437.

Nelson, Richard and Edmund Phelps, "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economics Review*, 1966, *56*, 69–75.

Pack, Howard and Larre E. Westphal, "Industrial Strategy and Technological Change: Theory Versus Reality," *JDE*, June 1986, *22* (1), 87–128.

Romer, Paul, "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 1986, *94*, 1002–1038.

——, "The Origins of Endogenous Growth," *Journal of Economic Perspectives*, Winter 1994, *8* (1).