Sizing Up The World: Early Greek and Indian Approximations

by
Vincent H. Malmström
Professor Emeritus of Geography
Dartmouth College
Hanover, New Hampshire, USA

History tells us that as early as 1750 B.C. the Babylonians had devised a calendar whose new year began with the Vernal Equinox – the point in the sun's annual 'migration' from tropic to tropic when it crosses the Equator on its way from the southern hemisphere into the northern hemisphere. Although we currently mark that point in our present Gregorian calendar on either March 20th or 21st, at the time that the Babylonians began recording it, the event took place on April 4th instead, a day which witnessed not only the transit of the sun at noon but also the transit of Arcturus, the sixth brightest star in the heavens, twelve hours later at midnight. As the Babylonian astronomic lore was subsequently passed on to neighboring peoples, both to the Phoenicians and Greeks in the West and the Indus and Hindu civilizations to the East, these associations between solar and stellar phenomena were, of course, a critical part of the 'package'. By the time of Homer (8th century B.C.), the Greeks were already recording the equinox on March 29th and by the time of Alexander the Great, it was falling on March 25th instead. Small wonder then, that Hipparchus, a Greek astronomer and mathematician of the 2nd century B.C., had already concluded that this 'westward shift' of the stars -- a process that he termed 'precession' -- was due to the slow 'wobble' of the Earth on its axis.

The eastern beneficiaries of Babylonian knowledge were no doubt just as aware of this gradual westward shift of the stars through time, but they seem to have been content with continuing to employ the original starting date of the year that the Babylonians had established. Thus, during the time that the Julian calendar was in use by the Europeans (i.e., up until 1582), the Hindus used the equivalent of April 4th as the starting date of their new year, whereas in the present Gregorian calendar, they currently celebrate it on April 14th-15th instead.

Although there is some question as to when the Babylonian calendar first reached India, the most likely time of its diffusion into the sub-continent was during the heyday of the Achaemenian Empire, probably around the middle of the 6th century B.C. Cyrus the Great had founded his first capital at Pasargadae, in the mountains of southern Iran, in 559 B.C. and had defeated the Medes and destroyed their capital at Ecbatana (modern Hamadan) in 550 B.C. Eleven years later he conquered Babylon, which he made his winter capital, but during a military campaign in northern Persia in 531 he met his death. In 522 B.C. his son Darius relocated the main capital of the empire to the foothills below Pasargadae where he laid out the magnificent city of Persepolis ("City of the Persians"), orienting not only the city itself to the rising sun on the summer solstice over the mountain now known as Kuh-e-Khatun but all of whose structures he similarly aligned to the solstices. (See Figure 1.) However, just as interesting was the fact that he adopted the Babylonian date of the Vernal Equinox as the beginning of the Iranian New Year, which became known as the 'Nowruz' and was celebrated as the 'holiest day of the year' among the Zoroasterians.

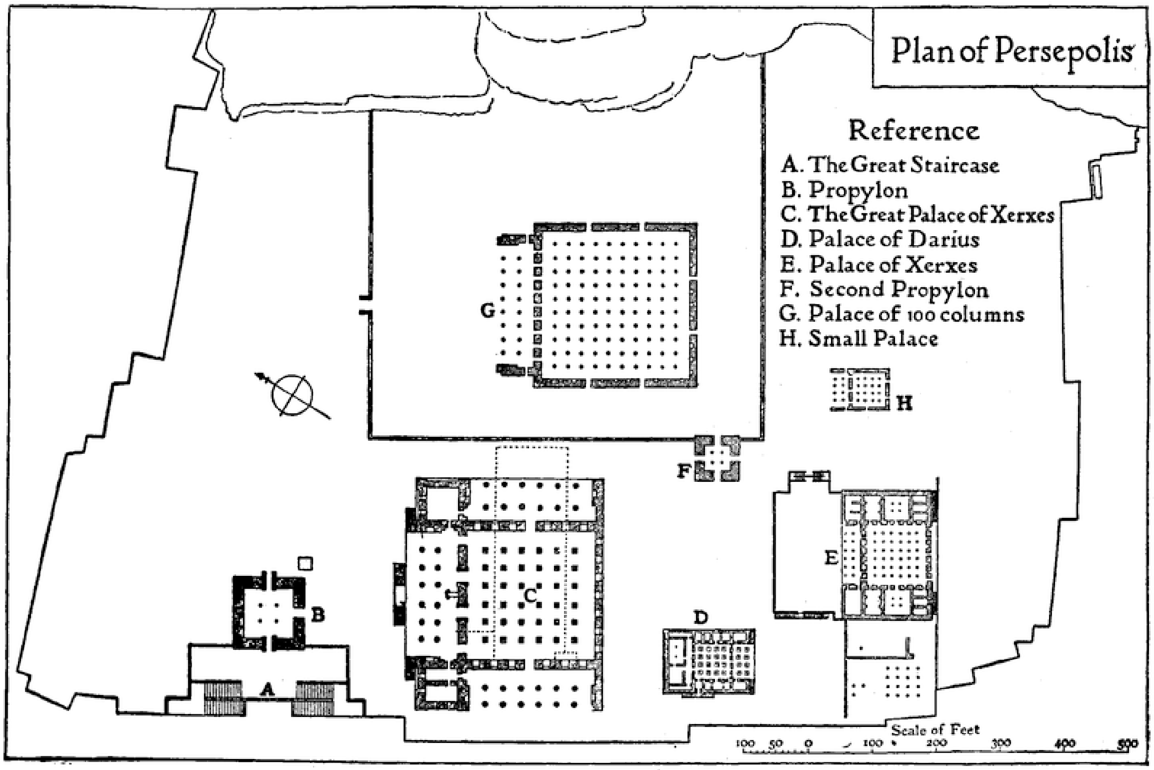


Figure 1. Plan of Persepolis. Note the north arrow near the middle of the left-hand side of the diagram. All the structures within the site will be seen to be oriented to an azimuth of ca. 60º, which marks the summer solstice sunrise over the mountain peak known as Kuh-e Khatun (elevation 3510 m, or 11,513 ft.), the most commanding topographic feature within view. Located at 31º27' N. latitude and 50º18' E. longitude, it is visible for 143 km, or 88.9 miles. The opposite azimuth (240º), i.e., the southwest façade of the buildings, defines the position of the winter solstice sunset.

On the other hand, in the west, Babylonian knowledge had already reached the Greeks as much as three or four centuries earlier, but in both areas, the arrival of Babylonian calendrics was no doubt accompanied by a familiarity with Babylonian mathematics and the so-called 'Pythagorean theorem' as well.

Among the Greeks, one of the earliest beneficiaries of this knowledge was a mathematician and geographer by the name of Eratosthenes. Born in the city of Cyrene in present-day Libya, most likely in the year 276 B.C., Eratosthenes had been educated in both the flourishing Greek colony of Alexandria, at the western edge of the Nile Delta in Egypt, but also in Athens. Combining his exposure to the teachings of Pythagoras with an account he had gleaned from a traveler to the upper reaches of the Nile, he developed an hypothesis that he believed would allow him to calculate the size of the Earth. Whereas Pythagoras had provided the tool – an understanding of trigonometry – the traveler had supplied a tantalizing clue, namely that at noon on the summer solstice (June 22), the light of the sun reached the bottom of a deep well near the present town of Aswan. To Eratosthenes, this meant that the sun was vertically overhead, i.e. 90º above Aswan on that day, so if he were to measure the altitude of the sun at Alexandria at noon on the same day, he surmised that the difference in their heights, expressed as a fraction of a full circle, would enable him to determine the distance which separated the two towns. Then, by multiplying this fraction until it yielded a full circle of 360º, he would obtain the circumference of the Earth.

The contemporary measure of distance in the Hellenic world was the stadion, of which there were at least three different lengths that are recorded: the longest was the Olympic stadion that measured 192.27 meters, while in Attica, the region surrounding Athens, a length of 185 meters was used, and in Egypt, where it was called the Royal Egyptian stadion, it measured only 157.5 meters in length. Since the stadion was also divided into six hundred 'feet', we find that a mainland Greek foot would have been equivalent to 31or 32 cm, while an Egyptian foot measured only 26 cm – which may have represented a biological reality.

For anyone about to undertake a calculation of global dimensions, one can only hope that he had very accurate local data with which to begin. In this regard, it appears that Eratosthenes was not only well prepared but also felicitously favored as well. Although his primary problem was to convert a degree of latitude into an understandable measure of overland distance, in this instance by converting it into one of the versions of the stadion, short of employing a team of surveyors to go out and run lines through unforgiving expanses of trackless and blistering desert this was not really possible. Instead, he felt that he could utilize his knowledge of trigonometry, first to fix the coordinates of at least two key locations, and then to measure the angle that a straight line between them would make with the meridian, i.e. a north-south line, to determine their distance apart. Not surprisingly, he probably chose Alexandria as the first of his 'survey points', and Cairo as his second, for he lived in the former and could relatively easily visit the latter. As long as his measurements remained coupled to degrees of latitude and longitude, they could be expressed in any units he wished, whether real or imaginary. As it turned out, though he may have been attempting to approximate 'stadia', because he could not get an accurate measure of the number of such units that equated to a degree of latitude, he would probably be obliged to arbitrarily adopt a figure of his own.

Let us suppose that he began by measuring the latitude of the waterfront library in the heart of Alexandria. Although the present grid of latitude and longitude was non-existent at the time, i.e. about 200 B.C., by careful measurement he could have rather easily established that it was located just a hair beyond 31º north of the Equator. This he could have done by measuring the vertical angle of the sun at noon, either on the spring equinox (March 20) or the autumnal equinox (September 22). Let us say he did it in March, knowing that, if in September he did the same thing in downtown Cairo, he could find the difference in latitude between the two cities. Imagine his surprise when his reading at the autumnal equinox in Cairo yielded a latitude of 30º, meaning that the city of Cairo lay almost exactly one degree to the south of Alexandria! This fortuitous revelation meant that the choice of any even number would serve well as his module of one degree of latitude, but why he specifically chose the number 700, no one really knows.

In any case, the first additional measurement he needed to address was the fact that Cairo not only lies south of Alexandria; it also lies east of it, but by how much? Obviously, he knew that the sun reached noon at an earlier time there than it did in Alexandria, so if he could find out by how many minutes this was, he would be able to establish the longitude of his second 'survey point' relative to the first.

How Eratosthenes actually measured the time difference between noon in Cairo and then in Alexandria is totally a matter of conjecture – was it by a water clock or a sand glass? In any case, the difference turned out to be only five minutes, but that told Eratosthenes exactly what he needed to know. Cairo is exactly 1.25º east of Alexandria, and, by employing trigonometry once again, he knew that at latitude 30º the east-west length of a degree had already diminished by an amount equivalent to the cosine of 30º, as compared to what it measured at the Equator, namely .866. Thus, by multiplying the 1.25º first by his module of 700, he obtained a distance of 875, which he then scaled down by .866 to a rounded value of 758. Thus, he had, in effect, obtained both the length of the adjacent side of his triangle (700), and the opposite side of his triangle (758) and he was now ready to use the tangent function to determine the size of the angle that a straight line between Alexandria and Cairo would produce. This proved to be a little under 48º (specifically 47.26º), so at this juncture, he could either divide the *adjacent* side of the triangle by the *cosine* of the angle, or divide the *opposite* side of the triangle by the *sine* of the angle, and that would give him the length of the hypotenuse, or the third side of the triangle. The latter was, of course, the distance between the two cities, which proved to be 1032 of his 'imaginary' units.

Had Eratosthenes gone a further step and converted these values into stadia using the Royal Egyptian measure of 157.5 meters, he would have obtained the following distances: one degree of latitude = 110.25 km; Cairo's distance east of Alexandria = 119.4 km; and the straight-line distance between the two cities = 162.5 km, all of which are very good approximations of their actual lengths and/or distances

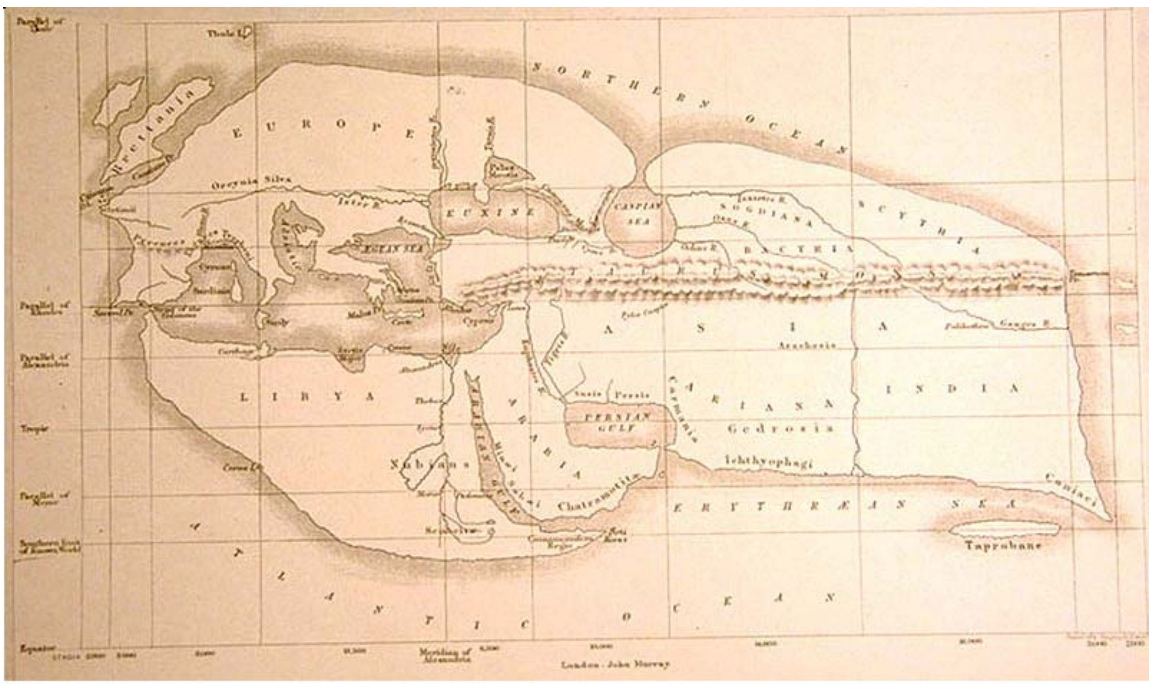
With reference to his principal endeavor – determining the size of the Earth – his choice of a module of 700 for 1º of latitude was also a very felicitous one. When he made his measurement of the noon sun angle at Alexandria on the summer solstice, he obtained a value of 7.2º from the vertical, yielding a total of 5040 stadia between that city and Syene, present-day Aswan. Inasmuch as 7.2º represented 1/50 of a full circle of 360º, by multiplying 5040 by 50 he obtained a total of 252,000 stadia for the size of the Earth. This, in turn, would have equated to a circumference of 39,690 km, compared to its Metric value of 40,000 km, or, expressed in the English system, about 24,650 miles versus its actual circumference of 24,900 miles – again very accurate results.

By way of a footnote, it is nevertheless interesting to observe that had he employed a module of 600 instead, and chosen to use the Attica stadion with its length of 185 meters, he would have obtained the following values: one degree of latitude = 111 km; Cairo's distance east of Alexandria = 120 km; and the straight-line distance between the two cities = 163.5 km, all of which are equally good or better approximations of their true lengths and/or distances than those he had already settled on.

Regarding his primary project, the results he would have obtained had he employed the latter options would have been the following: 4320 stadia for the distance between Alexandria and Aswan, and 216,000 stadia for the size of the Earth, equating to 39,960 kilometers or 24,815 statute miles. It is ironic, therefore, that he would have been even more accurate in his computations had he made these choices, but probably no one else would ever have been the wiser, unless they had made the same observations that we have just made here and now.

An even greater irony resulted from the actions of his successor as the Head Librarian at Alexandria some 400 years later. By now the Hellenic era had passed and the Roman era was in full flower. One Claudius Ptolemaeus, often referred to simply as Ptolemy, took it upon himself to recalculate the size of the Earth, most likely by using the Olympic stadion as his measure. In any event, his revised circumference was about a quarter too small, but for the Europeans, who lacked any knowledge of Eratosthenes' work, it was eagerly adopted as fact when they first discovered it in the 1400's. This may not have been the first time that a later 'revision' of an earlier finding proved to be less accurate than the original, but it certainly was not the last.

Had Columbus not believed it he certainly would not have assumed he could set off across the Atlantic with Cathay as his goal. Although he managed to reach the Bahamas on his first voyage, until his dying day he was certain that his landfall was just off the coast of India, so he called the people he encountered "Indians". It was not until 1513 when Balboa climbed a mountain as he traversed the Isthmus of Panama and first beheld the Pacific Ocean that Europeans finally realized that they were separated from Asia not by one ocean but by two oceans and a vast intervening continent!



A copy of what is purported to be Eratosthenes' map of the world. Obviously, his knowledge of place-names far outstripped his knowledge of their whereabouts. It is interesting, however, because it is clear that even though he was aware of the sizable eastward bend of the Nile, he considered Alexandria and Syene to have lain on exactly the same meridian.

Fortunately, the knowledge of mathematics and astronomy initially generated by the Sumerians and Babylonians not only diffused westward via the Phoenicians to the Greeks, but also eastward to the Persians, the Indus Valley civilizations, and Hindu India. The fact that Hindu astronomers adopted the original Babylonian date of the Vernal Equinox as the beginning of their calendar strongly suggests that Mesopotamian (or

Achaemenian) seafarers had been responsible for introducing this notion into India well before the dawn of the present era.

In any case, throughout history the safest anchorages for anyone arriving by sea along the west coast of India would have been in protected inlets such as those where the ports of Kozhikode (known to the western world as Calicut) and Cochin are located. (Indeed, Vasco da Gama, the first European to reach India, also chose to land at Calicut.) Although spices such as pepper and cardamom may have prompted the first visits of foreign traders to these shores, those arriving from the Persian Gulf region undoubtedly brought with them 'riches' of their own – the ancient cultural heritage of the Mesopotamian cultures. It therefore comes as no surprise that on their arrival in Calicut, they defined its meridian in the same manner as both the Babylonians and Greeks had done in their own homelands, namely by the midnight transit of Arcturus, the sixth brightest star in the heavens, that took place almost exactly twelve hours after the noon transit of the sun on the Vernal Equinox. However they may have first recorded the Calicut meridian, it currently equates to 75º43' East Longitude.

Interestingly, in the late 5th century, India spawned one of the greatest scholars and mathematicians of all time – a member of the Brahmin caste known as Aryabhata (born in 476, died in 550). Aryabhata not only had the advantage of having learned trigonometry – whether from the heirs of the late-arriving Greeks or from the much earlier arriving Mesopotamians or Achaemenians, we cannot know for certain -- but he also had the advantage of his local geography, for he resided in a region where he could personally observe the sun at the northernmost point in its annual 'migration' between the tropics. Indeed, thanks to its geographic location, India was the first 'tropical' country in the world to be reached by a civilization privy to such knowledge. Thus, it was during Aryabhata's lifetime, and most probably due to his initiative, that the first astronomical meridian in Hindu India was established. It ran through the city now known as Ujjain, which is located just shy of the Tropic of Cancer at 23º10' North latitude. (Actually, for observers of the sun who were obliged to measure its altitude by the use of a gnomon, i.e., any vertical pillar or post, the height they obtained was that of the upper limb of the sun, rather than its center, which modern astronomers employ. This automatically added 16' to their measurement, which meant that Ujjain's observatory yielded the precise latitude of the sun at its northernmost extreme – 23º26' N.) Thus, while it is manifestly clear why Ujjain's specific latitudinal position had been chosen, one might question why Aryabhata elected to establish his meridian at this particular longitude. However, once we realize that it too, lay at precisely 75º 43' East Longitude, we can only conclude that he must have known of the Mesopotamians having defined this meridian when they first arrived at the port of Calicut, and thus rule out either its being a fortuitous coincidence or Aryabhata's own independent creation.

On the other hand, the southwest coast of India is about the least auspicious region in the country in which to practice astronomy, for there the monsoon comes early and stays late, producing several months of cloudy weather during which the skies are frequently obscured. (See Figure 3, which shows a climate graph of Cochin, one of the ports in Kerala where foreign sailors early took refuge in a safe anchorage.)

The diminished impact of the monsoon as one moves north through India can readily be seen from Figure 4, where a climate graph of Indore, the nearest major weather station to Ujjain, is presented.

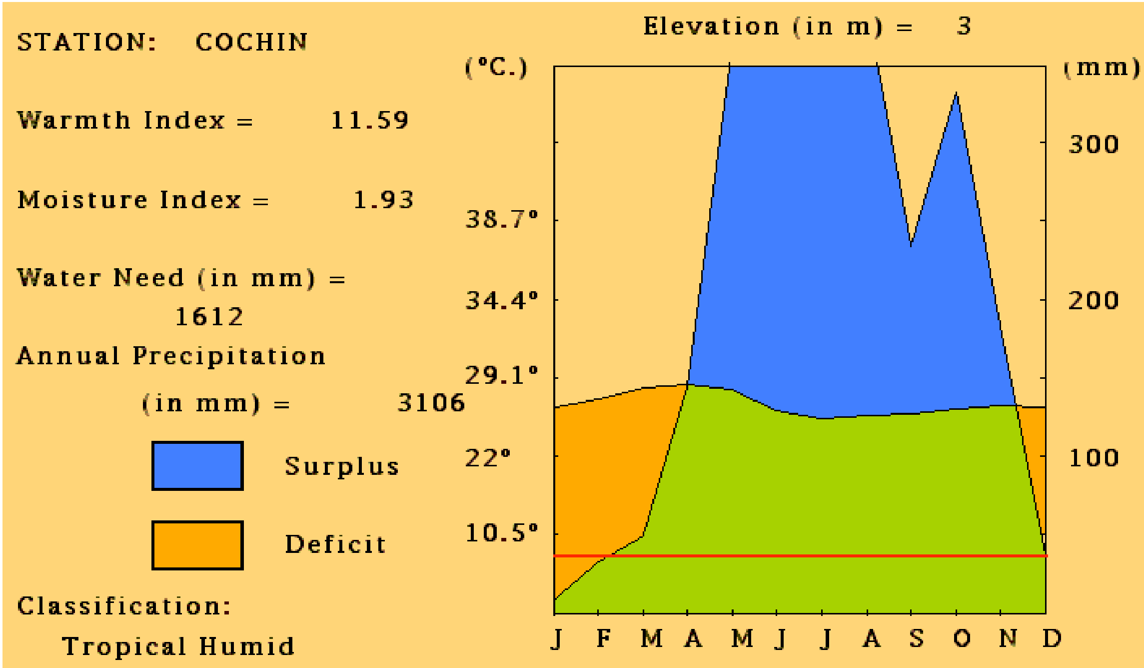


Figure 3. Water budget for Cochin, showing the limitations imposed on astronomical study in the southwest of India due to the monsoon. The heavy cloud cover markedly depresses the summer temperature as well as obscuring the sky between mid-April and mid-November.

However, as one moves north through India, one finds that the monsoon not only comes later and stays a shorter time but also that its intensity is considerably reduced. (The only exception – and it is a major one is when it encounters a topographic barrier such as the Himalayas. For example, the rainfall at Cherrapunji, a site in their southern foothills, often exceeds 10000 mm (400 inches) a year, over 2/3 of which falls within a three month time span.)

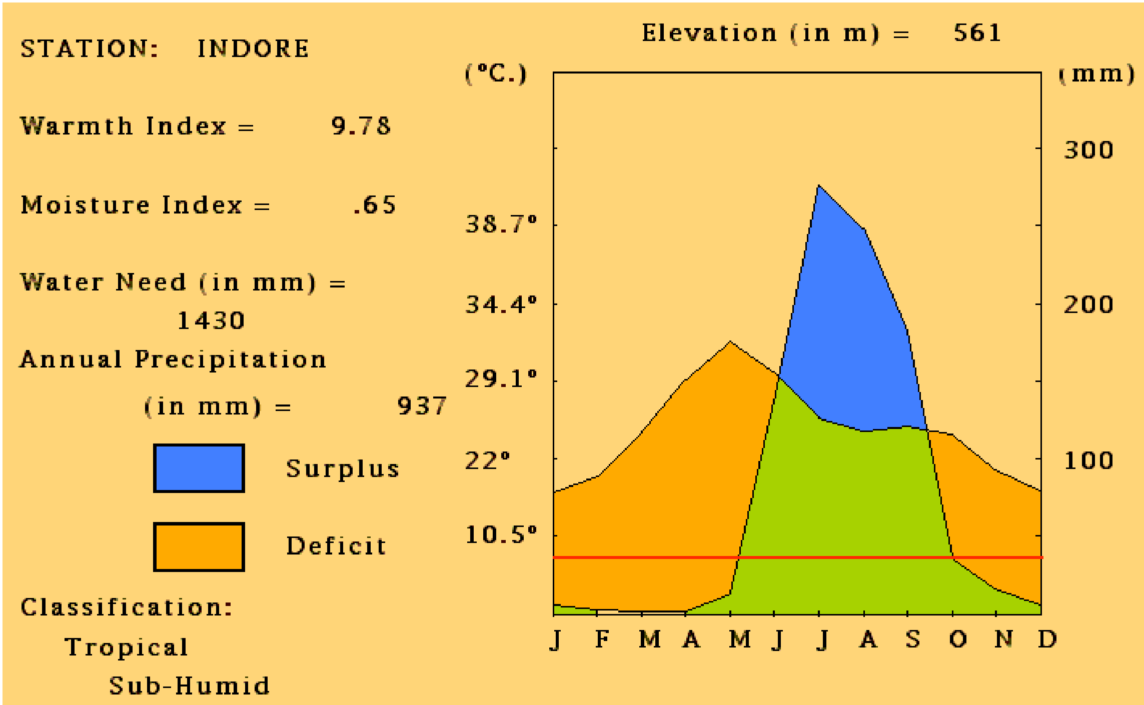


Figure 4. Water budget for the city of Indore, Madhya Pradesh state, the nearest major weather station to Ujjain. In north-central India, the length of the monsoon decreases to just over four months, thus extending the period of favorable weather conditions for viewing the sky.

In establishing the meridian of Ujjain, there can be little doubt but that Aryabhata likewise employed the transit of Arcturus (known in Hindi as Svati) to fix its location. Thus, if we use a computer program to examine the night sky over Ujjain during Aryabhata's lifetime by zeroing in on early April (the time that Hindu astronomers chose to mark the beginning of their year – no doubt because it occurred just before the 'breaking of the monsoon'), we find that in the years bracketing the year 516 A.D., Arcturus not only made its transit within 3.5º of the zenith (at altitude 86º34') but it also did so within a few minutes of midnight. (See Figure 5.) Indeed, from the literature we learn that Aryabhata's observation of the gradual westward shift of the stars not only convinced him that the Earth rotated on its axis, but also that the vernal equinox was being displaced westward at the rate of 1º for every 70 years. (Having already dismissed the likelihood of a belated Greek influence on Indian astronomy, it is only fitting that we recognize Aryabhata's observation as an independent discovery of what Hipparchus had reported some six or seven centuries earlier.) By the same token, we also know that much of the knowledge possessed by Hindu astronomers did not reach the Western world until well over a millennium later, and only then through the medium of the Arabs.

Among Aryabhata's many contributions to science were the development of the decimal system, the recognition of zero as a discrete numerical entity, rather than just a

Figure 5. The Calicut (Kozhikode)- Ujjain Meridian as defined by Aryabhata in the 5th century A.D.

'place holder' as among the Maya, and the refinement of the value of pi to 3.1416. When he made his calculation of the Earth's circumference – obviously with observations carried out along the Ujjain meridian – he obtained a value equivalent to 24,835 miles, or within 1% of its true size of 24,900 miles. (See Figure 6.)

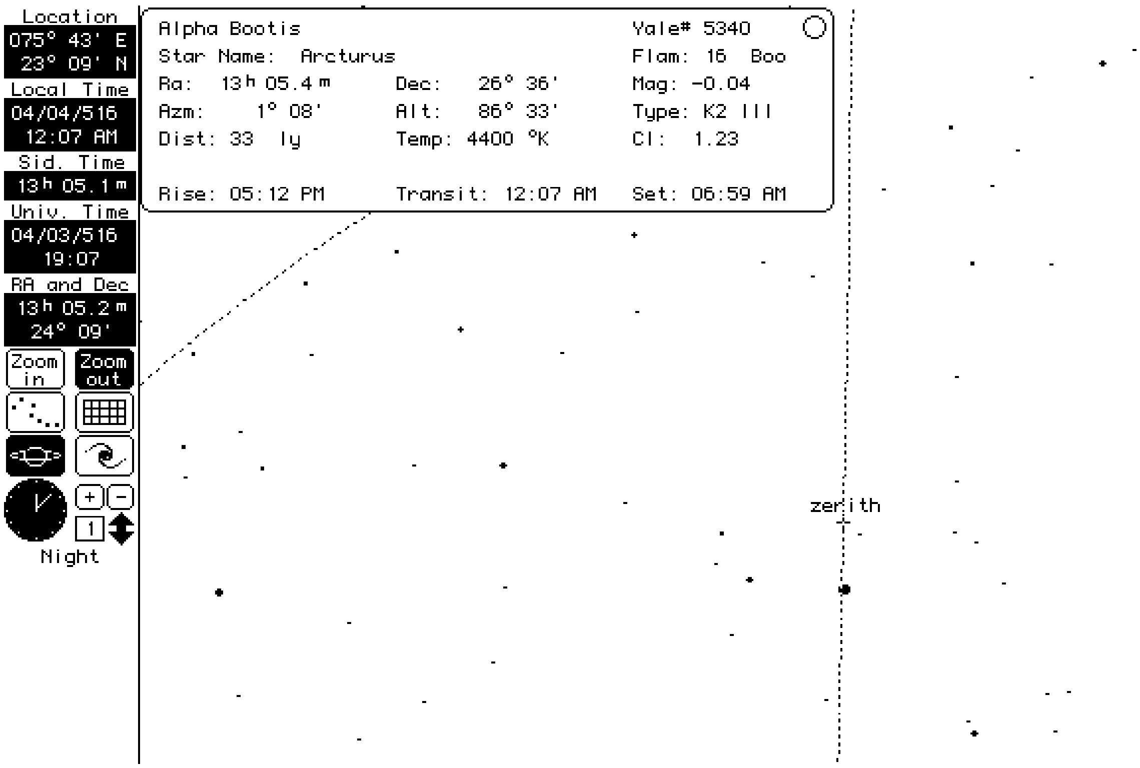


Figure 6. The transit of Arcturus over Ujjain on the evening of April 4, 516 A.D. No other asterism would have served the purpose of Aryabhata in defining the meridian more splendidly than this event. Note that the date in the diagram above is in the Julian calendar, which was the time-reckoning system in use in the Western world in the fifth century A.D. The equivalent date in the present Gregorian calendar would be April 14th, which Hindu astronomers currently use as the beginning of their New Year. (The diagram above was produced by the Voyager Program, Carina Software, San Leandro, California.)

It is also interesting to note that the Maharaja Jai Singh II, working in the early 18th century on behalf of the Moghal emperors, constructed five astronomical observatories in the Ganges plain of northern India, one of which was founded at his new capital city of Jaipur, which he located so as to precisely fix the northernmost extension of the meridian from Calicut to Ujjain. (See Figure 7.)

Figure 7. The red circles identify the five astronomical observatories founded by Maharaja Jai Singh II in the early 18th century. Of these, that at Jaipur was constructed in his new capital city and was located on what was to serve as the northernmost extension of the Calicut-Ujjain meridian. That at Varanasi was located 7.28º to the east, at a distance of 732 km from the Ujjain meridian. Along the parallel of latitude at which Varanasi is situated, the speed of the earth's rotation amounts to 1510 km an hour, which means that celestial phenomena are observed there 29 minutes ahead of their timing along the Ujjain meridian. (For example, on April 4, 516 AD, Arcturus transited at Varanasi at 18:38 Universal Time, whereas it did not transit Ujjain until 19:07 UT.) This suggests that Jai Singh may have been attempting either to relate longitude to time or to reconfirm the earth's circumference with an east-west measurement of his own in addition to the earlier north-south one made by Aryabhata. If the latter had been his intent, his calculation would have produced a result that was within one-half of one percent of the correct value – a level of precision not obtained by Europeans until well over a century and a half later!

(Revised October 2010)