Science 14
Lab 3 - DC Circuits

Theory

All DC circuit analysis (the determining of currents, voltages and resistances throughout a circuit) can be done with the use of three rules. These rules are given below.

1. Ohm's law. This law states that the current in a circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance in the circuit. Mathematically, this can be expressed as

$$I = \frac{V}{R} \quad . \qquad (1)$$

Ohm's law can be applied to an entire circuit or to individual parts of the circuit.

2. Kirchoff's node rule. This rule states that the algebraic sum of all currents at a node (junction point) is zero. Currents coming into a node are considered negative and currents leaving a node are considered positive. For the situation in figure 1, we have

$$-I_1 + I_2 + I_3 = 0 \qquad \text{or} \qquad I_1 = I_2 + I_3$$

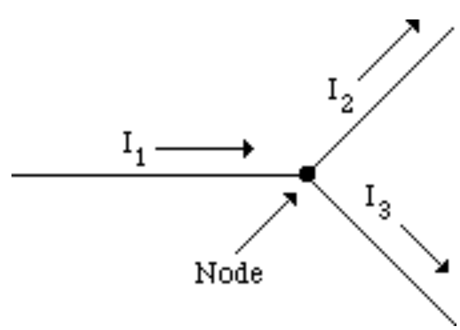


Figure 1

This is a statement of the law of conservation of charge. Since no charge may be stored at a node and since charge cannot be created or destroyed at the node, the total current entering a node must equal the total current out of the node.

3. Kirchoff's loop rule. This rule states that the algebraic sum of all the changes in potential (voltages) around a loop must equal zero. A potential difference is considered negative if the potential is getting smaller in the direction of the current flow. For the situation in figure 2, we have

$$+V_1 - V_2 - V_3 = 0 \qquad \text{or} \qquad V_1 = V_2 + V_3$$

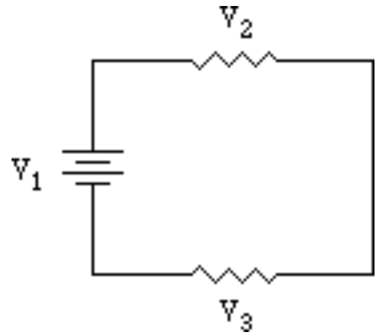


Figure 2

This is a statement of the law of conservation of energy. Since potential differences correspond to energy changes and since energy cannot be created or destroyed in ordinary electrical interactions, the energy dissipated by the current as it passes through the circuit ($V_2 + V_3$) must equal the energy given to it by the power supply (V_1).

To illustrate the application of these rules, some common electrical circuits are analyzed below.

The Simple Series Circuit

Consider the circuit shown in figure 3. It consists of a single loop. There is only one path through which a current can flow. Such a circuit is called a series circuit. Let V_1 , V_2 and V_3 be respectively the potential differences across the resistances R_1, R_2 and R_3; and let I_1, I_2 and I_3 be respectively the currents flowing through those same resistances. Let V_T be the electromotive force supplied by the power supply and I_T and R_T be respectively the total current flowing in the circuit and the total resistance in the circuit.

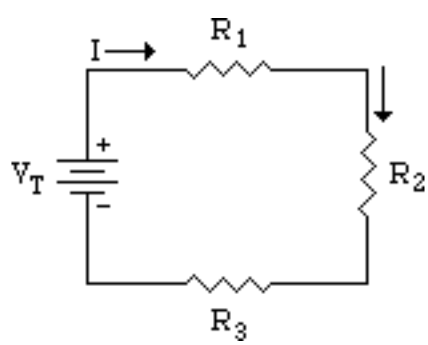


Figure 3

Writing Kirchoff's loop rule for this circuit yields

$$V_T = V_1 + V_2 + V_3 \quad . \quad (2)$$

Using Ohm's law, equation (2) can be rewritten as

$$I_T R_T = I_1 R_1 + I_2 R_2 + I_3 R_3 \quad . \quad (3)$$

But Kirchoff's node rule tells us that since charge can not pile up in any part of the circuit and since there is only one path for the current to follow, the current in any one part of the circuit must be equal to that in any other part of the circuit. In other words, $I_T = I_1 = I_2 = I_3$. Therefore, equation (3) can be simplified to

$$I_T R_T = I_T(R_1 + R_2 + R_3) \quad . \quad (4)$$

Equation (4) tells us that the total resistance is equal to the sum of the individual resistances.

Equations (2) and (4), although specifically describing the circuit in figure 3, can easily be generalized for a series circuit containing n elements. Therefore, application of our three rules leads to these general relationships for series circuits:

$$I_T = I_1 + I_2 + \ldots + I_n \quad . \quad (5a)$$

$$V_T = V_1 + V_2 + \ldots + V_n \quad . \quad (5b)$$

$$R_T = R_1 + R_2 + \ldots + R_n \quad . \quad (5c)$$

The Simple Parallel Circuit

Consider the circuit shown in figure 4. It consists of three loops. There is more than one path for the current to follow in going from the power supply through the circuit and back to the power supply. Such a circuit is called a parallel circuit. Let V_1, V_2 and V_3 be respectively the potential differences across the resistances R_1, R_2 and R_3; and let I_1, I_2 and I_3 be respectively the currents flowing through those same resistances. Let V_T be the electromotive force supplied by the power supply and I_T and R_T be respectively the total current flowing in the circuit and the total resistance in the circuit.

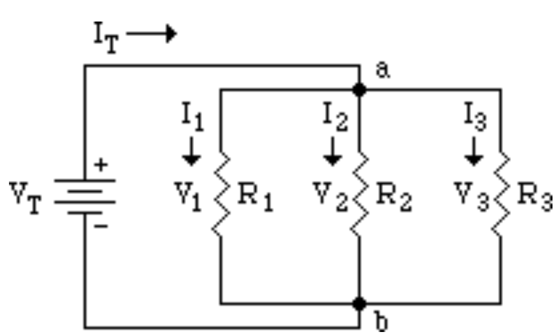


Figure 4

Writing Kirchoff's node rule for point a yields

$$I_T = I_1 + I_2 + I_3 \quad . \qquad (6)$$

Using Ohm's law, equation (6) can be rewritten as

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \quad . \qquad (7)$$

But if the concept of potential is to have any meaning, a single point can have only one value of potential. Therefore, the potential at points a and b must be single valued and no matter which path is taken the potential difference between a and b must be the same. This means $V_T = V_1 = V_2 = V_3$ and equation (7) becomes

$$\frac{V_T}{R} = V_T\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad . \qquad (8)$$

Equation (8) tells us that the reciprocal of the total resistance of a parallel circuit is equal to the sum of the reciprocals of the individual resistances.

Equations (6) and (8), although specifically describing the circuit in figure 4, can be easily generalized for a parallel circuit of n elements. Therefore, application of our three rules leads to these general relationships for parallel circuits:

$$I_T = I_1 = I_2 = \cdots = I_n \qquad (9a)$$

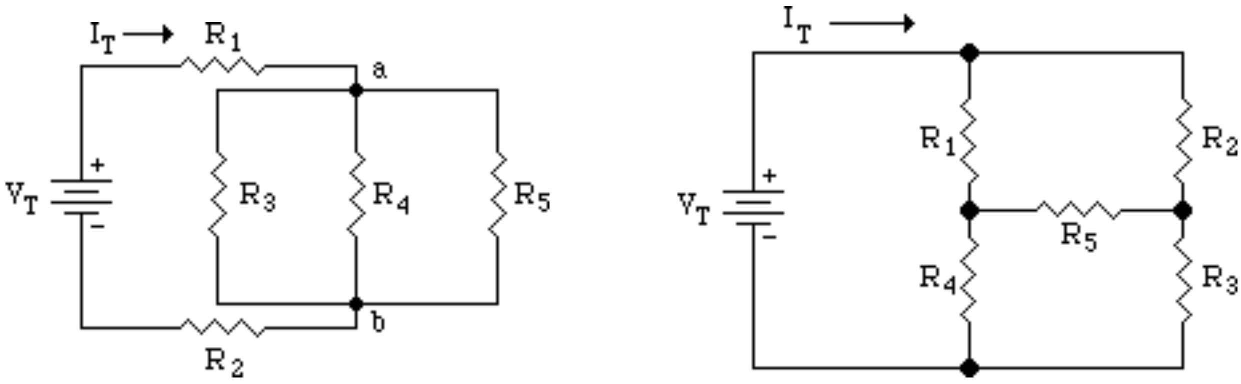
$$V_T = V_1 = V_2 = \cdots = V_n \qquad (9b)$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \qquad (9c)$$

Complex Circuits

Consider the circuits shown in figure 5. Neither circuit can be classified as a simple series circuit or a simple parallel circuit. Circuits such as these fall into one of two categories: (1) circuits which can be broken down into a combination of simple series and simple parallel circuits and (2) circuits that can not. Figure 5a is an example of the first category and figure 5b is an example of the second.

Circuits in the first category can be analyzed using the concept of equivalent circuits. This concept states that any group of resistors in a simple series or simple parallel arrangement can be replaced by a single resistor which would leave unaltered the potential difference between the terminals of the group and the current in the rest of the circuit. The value of the single resistor can be determined using equations 5(a-c) and 9(a-c). The circuit with the single resistor is equivalent in every respect to the original circuit. Circuit analysis then becomes a matter of reducing each



(a) (b)

Figure 5

series of parallel resistance subgroup in a combination circuit to its equivalent resistance until what is left is a simple series or simple parallel cirucit.

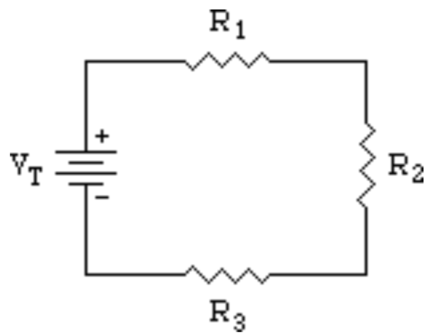


Figure 6

Thus to analyze the circuit in figure 5a, we replace the circuit elements between points a and b by a single resistor R_e ($R_e = 1/R_1 + 1/R_2 + 1/R_3$) as shown in figure 6. The circuit in figure 6 is equivalent to the circuit in figure 5a and can be easily analyzed using equations 5(a-c).

Circuits in the second category cannot be reduced to equivalent circuits and must be analyzed by applying Kirchoff's rules to each loop separately and then solving the resulting simultaneous equations. For example, if we divide the circuit shown in figure 5b into three loops as shown in figure 7 and assume the currents i_1, i_2 and i_3 to flow clockwise in each loop, application of Kirchoff's rules yields the following:

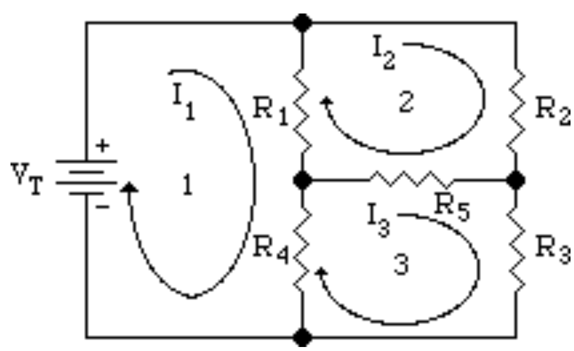


Figure 7

$$\text{Loop 1:} \quad V_T = I_1(R_1 + R_2) - I_2R_1 - I_3R_4$$

$$\text{Loop 2:} \quad 0 = I_2(R_1 + R_2 + R_5) - I_1R_1 - I_3R_5$$

$$\text{Loop 3:} \quad 0 = I_3(R_5 + R_3 + R_4) - I_2R_5 - I_1R_4$$

With these equations I_1, I_2, and I_3 can be determined. Note that I_1, I_2, and I_3 are defined in figure 7 so that, for example, I_2 flows through R_2, I_1 flows through the battery, and ($I_1 - I_2$) flows through R_1. If, upon solving the equations we find that $I_2 > 0$, this means current flows downward through R_2. If $I_1 - I_2 > 0$, current flows downward through R_1, etc. So both the magnitude and the sense of the current flowing through each element of the circuit is determined.

References

The following sections in Halliday and Resnick's Fundamentals of Physics (2nd edition) are pertinent to this lab. They should be read before coming to lab.

1. Chapter 28
2. Chapter 29 sections 29-1 to 29-6

Experimental Purpose

The purpose of this lab is use the three rules given in the introduction to make theoretical predictions about the values of currents and voltages in three actual circuits constructed in the lab and then to check these predictions by actually measuring the currents and voltages.

Procedure

Before beginning, a few words on the equipment to be used and it's effects on the measurements is in order.

The Carbon Resistor

A common type of resistor used in electrical circuits is made from a carbon composition in the form of a small solid cylinder with a wire lead attached to each end. The nominal resistance value is specified by a color code that is shown in figure 8.

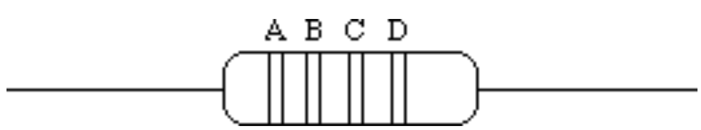


Figure 8

The first three bands give the resistance in ohms in the form $R = AB \times 10^{C}$, where A,B, and C are integers between 0 and 9. The first band is A, the second B and the third C. The color code for the integer is

0 - black	5 - green
1 - brown	6 - blue
2 - red	7 - violet
3 - orange	8 - grey
4 - yellow	9 - white

The fourth band specifies the tolerance, i.e. the allowed deviation from the nominal value, according to

5 % - gold
10 % - silver
20 % - (no band)

For example, suppose band A is red, band B is violet and C is yellow. The value of the resistance would then be 27×10^{4} ohms (270K or 270,000 ohms). Obviously, this type of resistor is intended for applications where high presision is not important (a common case).

In addition to the resistance value, an important parameter is the allowable power dissipation:

$$\text{Power} = VI = I^2R = \frac{V^2}{R}$$

Since the rate of heat loss depends on the surface area of the resistor, this rating is determined by the physical size of the resistor. Most of those we shall use are rated 1W; other common sizes are .5W and 2W. If the rating in exceeded, the resistor may get too hot (changing its R value) or burn out.

The Multimeter

A "multimeter" is a variable-range voltmeter and ammeter, which will be used to measure the voltages and currents in this experiment. A perfect voltmeter would have infinite resistance (draw zero current) and a perfect ammeter would have zero resistance (zero voltage drop); but a real meter has a certain resistance and therefore affects the circuit in which it is used. The resistance is made as nearly perfect as possible, within the limitations of size, cost, ruggedness, sensitivity, etc., for a particular application.

We will use a meter of a basic type in which the pointer is attached to a movable coil of many turns of fine wire. A current through the coil causes it to rotate. The details of its operation do not concern us now; we need to know only that its performance is determined by two parameters, I_m and R_m. A current Im passing through the meter causes the pointer to deflect full scale; R_m is the resistance of the basic meter. If Im is very small, the meter is highly sensative. Clearly, the voltage required for a full scale pointer deflection is $V_m = I_m R_m$. In addition, the manufacturer specifies the accuracy, as a percentage of the full scale reading. The specifications of the Simpson model 257 are:

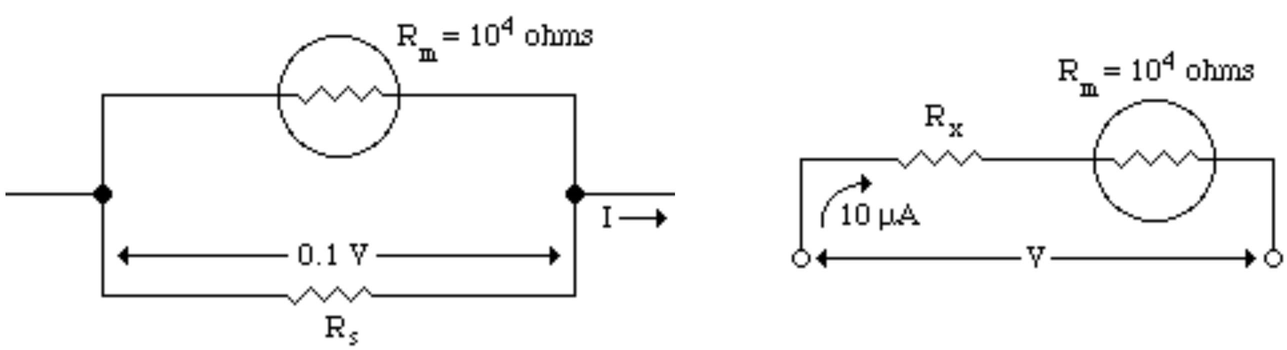
I_m	10 mA
R_m	10^4 W
V_m	100 mV
accuracy	1.5% of full scale

The basic meter can be converted into a multirange ammeter by adding different resistors in parallel with it, and into a multirange voltmeter by adding different resistors in series; these are added internally, by means of a switch on the front panel. Thus, the resistance of the meter depends on the range setting; but for the above specification, at full scale:

on any dc current range, the drop across the meter is 0.1V;

on any dc voltage range, the meter draws 10 mA

as shown in figure 9. Note that the lowest current range (10 mA) is the lowest voltage range (100 mV).



a) ammeter (full scale)

b) voltmeter (full scale)

Figure 9

Actually, because the wiring of the Simpson model 257 is slightly different from figure 9a, the voltage drop on the mA ranges is about 0.24V instead of 0.1V . The current drawn on the voltage ranges is actually 10 mA; this is conveniently expressed as 100,000 W/V, meaning that the voltmeter resistance is 10^5W times the full scale voltage for any setting. Thus, on the 10-V scale, the meter resistance is 1 MW . The Simpson 257 also has resistance ranges, on which the meter actually measures the current (through an external circuit that contains no voltage sources) due to an internal 1.5-V battery. The battery must be calibrated before each measurement: with the test leads shorted, set the pointer to full-scale deflection by using the "W ZERO" knob.

1. Use the Simpson multimeter to set the dc power supply to exactly 10V. (When using the multimeter, it is always wise to start with the least sensative scale, and then turn the selector to obtain an on-scale reading.) The power supply should remain at this setting for the remainder of the lab. The internal resistance of the power supply is negligible in this experiment. (How could you check this?)
2. Measure the actual resistance of each of your resistors. How could you check their linearity?
3. Assemble the circuit shown in figure 10.

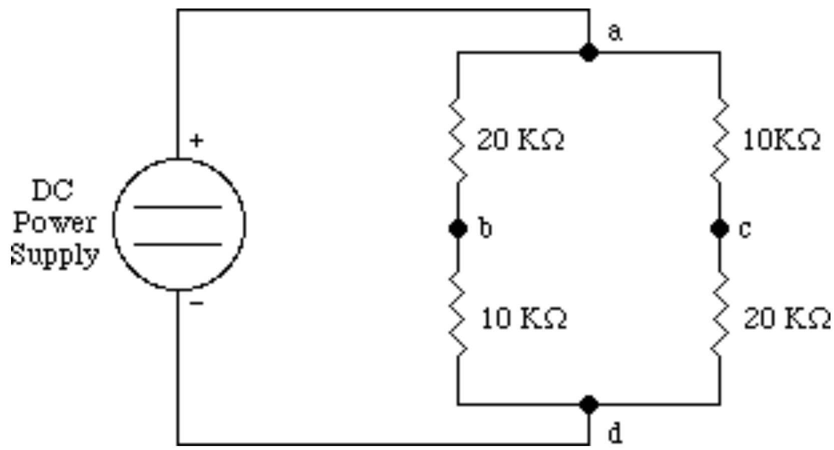


Figure 10

Consider point d to be at ground (zero potential, $V_d = 0$). This will be true for all other circuits constructed in this lab. This point should be connected to the negative terminal of the power supply but need not be otherwise grounded.

a. Compute the total resistance of the circuit R_T using equations (5c) and (9c). With the powr supply disconnected from the circuit, measure R_T.

b. Compute theoretical values for V_{ad}, V_{bd} and V_{cd}. Connect the power supply to the circuit and measure V_{ad}, V_{bd} and V_{cd}.

c. Compute a theoretical value for I_1. Measure I_1. Current is measured by setting the Simpson meter to amps and connecting it as part of the circuit, in this case by connecting the + terminal of the meter to the + of the power supply and the common terminal to point a.

d. Compute the ratio (using experimental values) V_{ad}/I_1. What does the ratio V_{ad}/I_1 represent? What value do you expect it to have? Did it have that value?

For all measurements made in this part and all other parts of the lab, compute the percent deviation between the theoretical values and the experimental values, and compare to the uncertainty in the measurements.

4. Assemble the circuit shown in figure 11.

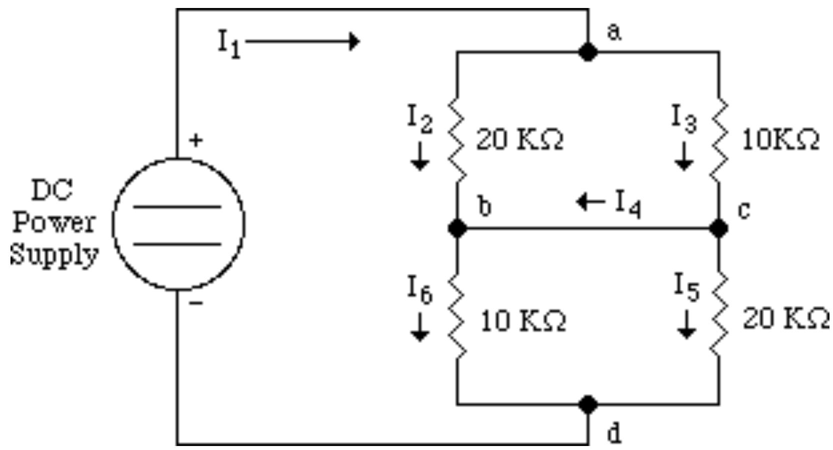


Figure 11

a. Using equations (5c) and (9c) to compute R_T. With the power supply disconnected from the circuit measure R_T. Why?

b. Compute theoretical values for V_{ad}, V_{bd} and V_{cd}. Connect the power supply to the circuit and measure V_{ad}, V_{bd} and V_{cd}.

c. Write down Kirchoff's node rules for each node in the circuit. Use Ohm's law to calculate theoretical values for all the currents, I_1 - I_6. Measure all the currents, I_1 - I_6. Plug the experimental values for the currents back into the node equations. Do the experimental values yield true statements from the node equations?

d. Compute the R_T using the ratio V_{ad}/I_1.

e. What is V_{bc}? Given that value, how can you explain a nonzero value for I_4? How could you rearrange the resistors to make I_4 zero? Suppose we placed a sensitive current meter between points b and c, replaced the 20k resistor between points c and d with an unknown resistor and replaced the 10k resistor between points a and c with a variable resistor. How could such an arrangement be used to determine the value of the unknown resistor? Explain.

5. Assemble the circuit shown in figure 12.

a. Can this resistor network be reduced by series and parallel combinations to a single resistor? Identify the three loops in this circuit and write Kirchoff's loop equation for each loop. From the three simultaneous equations (and Ohm's law) compute theoretical values for I_1 - I_6 and V_{ad}, V_{bd} and V_{cd}.

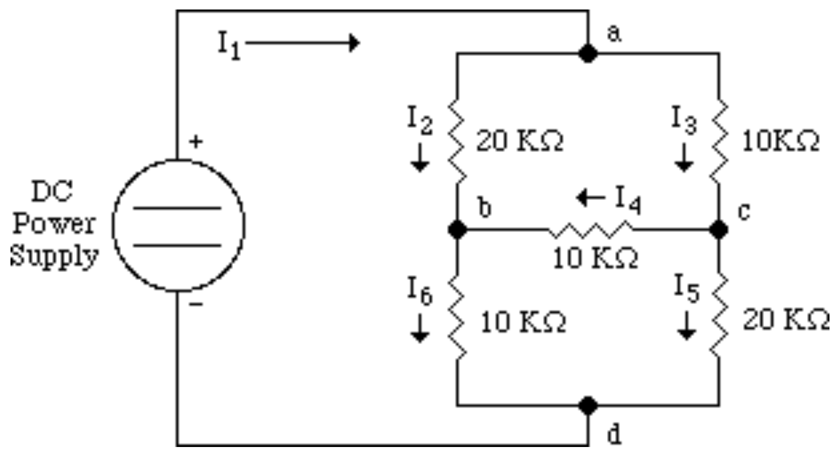


Figure 12

b. With the power supply disconnected from the circuit measure RT.

c. Measure I_1 - I_6 and V_{ad}, V_{bd} and V_{cd}.

d. Compute R_T from the ratio V_{ad}/I_1 .

e. Write Kirchoff's node rule for each node in the circuit. Plug the experimental values for I_1 - I_6 into the node equations. Do the experimental values yield true statements?

Lab Report

Include the following in your report:

1. all theoretical calculations done - beside each calculation indicate which of the three rules given in the introduction you are using;

2. all experimental values measured with computation of the % deviation between the theoretical value and the experimental values;

3. answers to all questions asked in the introduction and procedure;

4. all equations asked for (such as the node equations asked for in procedure 3c); and

5. a discussion of the accuracy of the measurements. In your discussion consider what effect the insertion of the meter into the circuit has on the quantities measured. With the meter in the circuit, would you expect the values measured to be higher or lower than the theoretical values? By what percentage higher or lower? Can the observed deviations be accounted for (in both magnitude and sense) by taking the effect of the meter into account? If not, what else could effect the measurements?