

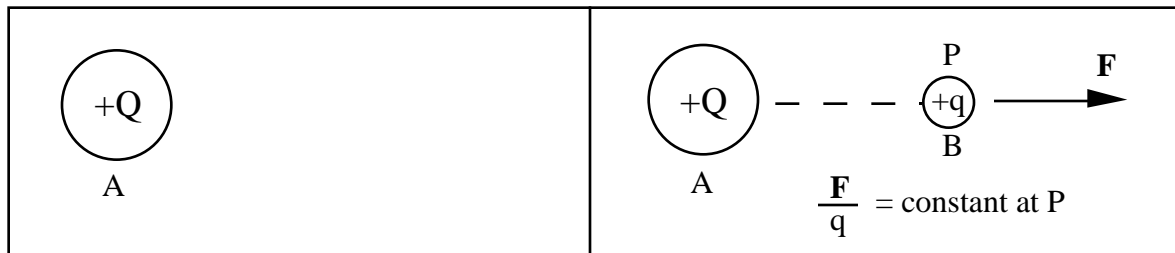
Science 14

Lab 1 - Potential Plotting

Theory

Consider an isolated conductor, A, carrying a positive charge Q , as shown in figure (1a). If body B, with positive charge q_0 ($Q \gg q_0$) is moved to a position near A, it will experience a repulsive force of electrical nature directed along a straight line connecting the centers of the two bodies as shown in figure (1b). For a given Q , the magnitude of the force on B will depend on the distance separating the centers of the two bodies and on the magnitude of q_0 . At a given position, however, the ratio of the force on B to the charge q_0 will be a constant.

These experimental facts can be explained by saying that the charge on A modifies the state of affairs in the space around it in such a way that any charged body entering that space will experience a force of electrical nature. This modified state of affairs is called an electric field. (Actually, the charge on B also creates an electric field; but in this case we can neglect its effects. Why?) An electric field can not be seen. Its presence can only be detected by its effects on charged particles. Because its effects are directional, the electric field is a vector quantity.



(a)

(b)

Figure 1

For the concept of the electric field to be useful, we need some method for describing it, both qualitatively and quantitatively. In what follows, two such methods will be described. The first will be dynamical in nature and will illustrate the vector nature of electric fields; the second will involve energy considerations and will illustrate a scalar side to electric fields. Finally, the two ways of viewing an electric field will be tied together to provide a convenient experimental method for mapping (i.e. determining the shapes of) electric fields.

One way of describing an electric field is to take a positive test charge q and systematically place it at various positions in the field. (Again, q must be much much smaller than the charge creating the field. Why?) At each position, measure the magnitude and direction of the force F experienced by the test charge. The magnitude of the electric field at that point would then be the ratio of F to q and the direction of the field would be the same as the direction of F . (Why do we use the ratio of F to q rather than simply the value of F to describe the magnitude of the electric field?)

We could then represent the field graphically by drawing an arrow at each point. The length of the arrow would be proportional to the magnitude of the electric field at that point and the head of the arrow would indicate its direction. The electric fields of an isolated positive charge and of an isolated negative charge represented in this way are shown in figures (2a) and (2b).

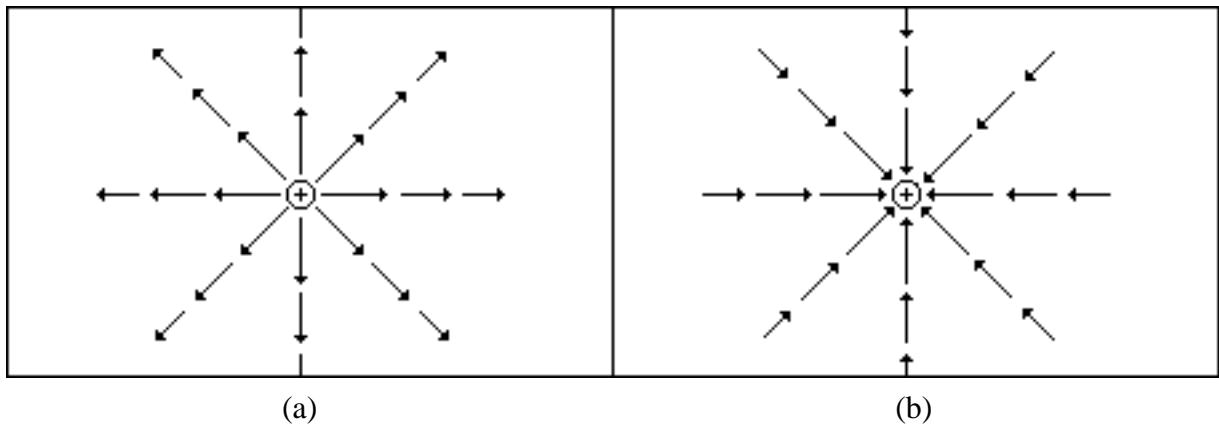
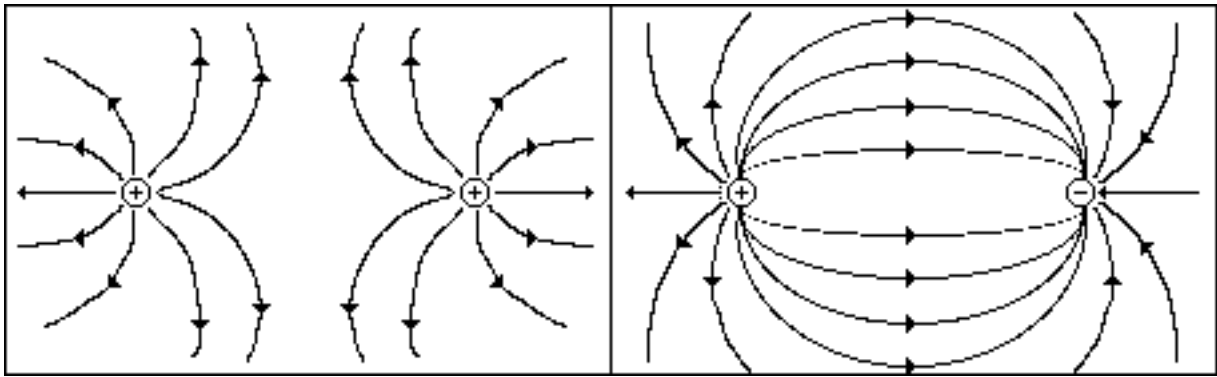


Figure 2

The electric fields created by single isolated charges are very simple. When two or more charges are placed close together, however, their electric fields at any point add vectorially and depending on the number of charges, their relative magnitudes and their respective positions, the resulting electric fields can be very complicated. These more complicated fields can be mapped in the same way, but when representing them graphically it is more convenient to use lines of force. A line of force is an imaginary line drawn in such a way that its direction (the direction of the tangent at that point) is the same as the direction of the field at that point. The strength of the field is indicated by the density of the lines of force. A high density indicates a strong field and vice versa. The fields of a positive and a negative charge of equal magnitude and of two equal positive charges represented in this way are shown in figures (3a) and (3b).



(a)

Figure 3

(b)

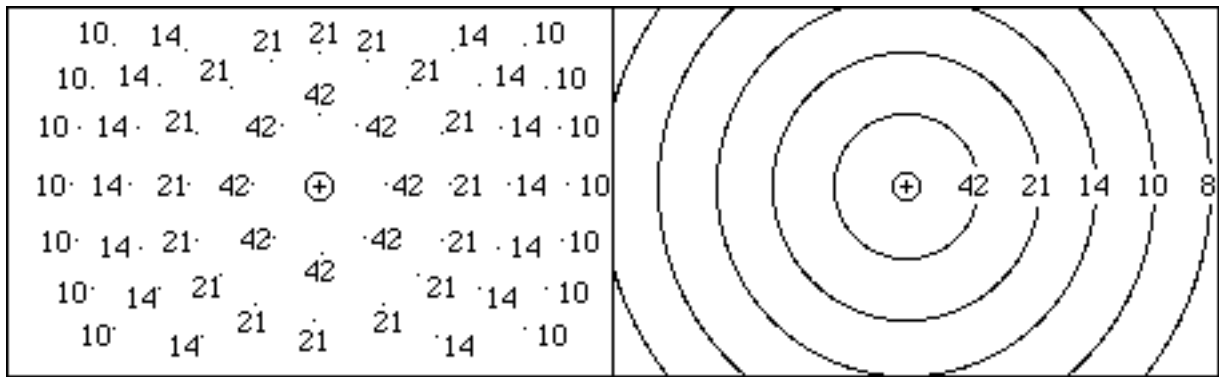
The second way of describing an electric field is similar in technique to the first method. Consider again our isolated positive charge shown in figure (1a). To bring body B in from infinity to point P, we must do work against the repulsive force of the field. In other words, we must supply energy to body B. It can be shown experimentally that for a given Q the amount of work W depends only on the distance between the center of A and point P and on the magnitude of q_0 . It is independent of the path we chose to get from infinity to P.

As before, however, it is not the value of W we are primarily concerned with, but rather the ratio of W to q_0 . (Why?) At any given position, this ratio is a constant whose value gives the amount of energy which must be supplied to a unit positive charge to bring it from infinity to position P. If we assume q_0 to have zero energy at infinity, it also gives the total energy per unit charge at point P. Since the energy supplied to body B is not lost but is stored, the ratio is called the body's electric potential energy (or electric potential for short). Mathematically, this is expressed as

$$V = \frac{W}{q_0} \quad (1)$$

Since W and q_0 are scalar quantities, V is also a scalar quantity.

To describe our electric field using the electric potential, we use the same technique as before. We would systematically bring our positive test charge q from infinity to various positions around the charge creating the field, measuring the amount of work done in each case. At each point we could assign a number equal to the ratio of W to q. A representation of an electric field around a positive charge Q using this method is shown in figure (4a).



(a)

Figure 4

(b)

As can be seen from figure (4a), this is not a very convenient way of representing the electric field. The mass of points and numbers makes it difficult to quickly discern any regularities in the field. Close inspection of the numbers, however, does show us a way of solving this problem. If you look closely, you will notice a number of positions (Figure 4) with the same potential. If we connect all the positions with equal potentials, we get a family of lines (or surfaces if we were working in three dimensions). Since all the points in a given line (or surface) have equal potentials, the lines are called equipotentials. The equipotentials around an isolated positive charge are shown in figure (4b). The regularities of the electric field are now much easier to see. One of the regularities we note when we compare figures (4b) and (2a) is that the electric field lines are everywhere perpendicular to the equipotential surfaces. This is not peculiar to this system, but turns out to be true of any electric field. (Why?)

Although the two methods described above for mapping electric fields seem straightforward, they are experimentally difficult. To isolate small test charges, control their positions and measure either the forces they experience or the work done on them in placing them at the various positions, would require sophisticated equipment beyond the reach of most undergraduate teaching laboratories. An experimentally simple method, however, can be found if we combine the two ways of viewing the electric field. In other words, to overcome our experimental difficulties, we need to derive an expression that will relate the electric field E to the electric potential V . (Actually, we will derive an expression relating E to changes in the potential V . Why do we want to use V rather than V ?)

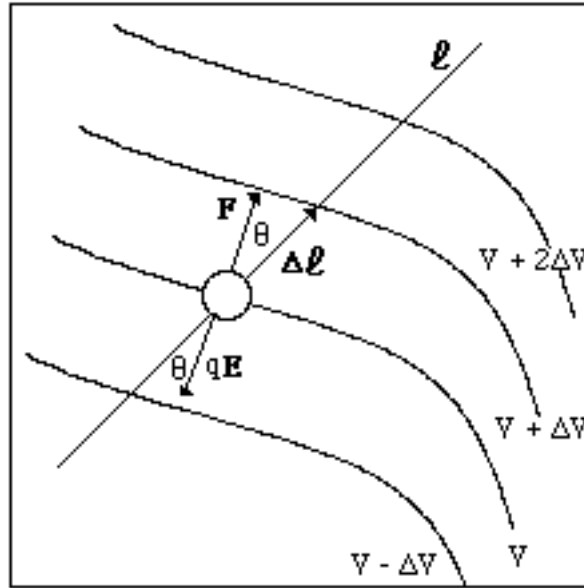


Figure 5

Consider an arbitrary electric field which has the equipotential surfaces shown in figure 5. In order to move the test charge q along the path from the equipotential with potential V to the equipotential with potential $V + DV$, we must do work on q equal to

$$W = (F \cos \theta) \Delta \ell \quad (2)$$

The amount of work done can also be computed by applying equation (1). This yields

$$\frac{W}{q} = (V + \Delta V) - V$$

or

$$W = q \Delta V \quad (3)$$

Setting the two expressions equal to each other yields

$$q \Delta V = (F \cos \theta) \Delta \ell \quad (4)$$

The force exerted by the external agent on q must be equal in magnitude to the force exerted on q by the electric field. In other words,

$$F = qE \quad (5)$$

Substituting for F in equation (4) yields

$$q \Delta V = q (E \cos \theta) \Delta l \quad (6)$$

Simplifying and rearranging terms gives

$$E \cos \theta = -\frac{\Delta V}{\Delta l} \quad (7)$$

The quantity $E \cos \theta$ is the component of the electric field in the $-\Delta l$ direction. The quantity $-E \cos \theta$ would then be the component of the electric field in the $+\Delta l$ direction. Therefore, if we let $E_1 = E \cos \theta$, the component of E in the $+\Delta l$ direction is

$$E_1 = -\frac{\Delta V}{\Delta l}$$

or written in differential form

$$E_1 = -\frac{dV}{dl} \quad (8)$$

Equation (8) tells us that if we travel in a straight line through an electric field and measure V as we go, the negative of the rate of change of V is equal to the component of E in that direction. If the potential does not change with position, then we are traveling at right angles to the electric field and are on an equipotential. If $-dV/dl$ is a maximum, then E_1 is a maximum and, in fact, is E itself.

Equation (8) shows us the way to easily experimentally map an electric field. All we need are two probes and a device that will measure the potential difference between the two probes. A simple voltmeter will measure potential differences and two metal rods can serve as probes. By connecting the probes to the terminals of the voltmeter and placing the probes in various positions in an electric field, we can monitor the potential differences between the two positions of the probes. Equipotential surfaces can be located by leaving one probe still and moving the other probe. Whenever the voltmeter registers a zero reading, the moving probe is at the same potential as the non-moving probe. By marking each such position, equipotentials can be found. Electric field lines can be located by keeping the probes at a constant separation and rotating one probe around the other. When the voltmeter registers a maximum reading, the electric field is changing at its maximum rate. The electric field at that point is parallel to the line joining the two probes. By doing this at a variety of positions, the electric field can be mapped.

In practice, it is not possible to map an electrostatic field in a vacuum or air using a

real voltmeter (in contrast to an ideal voltmeter) because a voltmeter works by drawing a small current. Therefore, we shall instead measure the electric field that is set up in a medium that is slightly conducting. In this lab we will use paper impregnated with carbon (i.e. conducting paper). Since the paper is only slightly conducting, the electric field produced by the charged electrodes is nearly the same as the one that would be produced in air with similar geometry, and yet the paper is sufficiently conducting to supply the small current needed by the voltmeter.

In this lab we will use this method to map the electric fields and equipotential surfaces of three charge configurations: (1) two coaxial metal cylinders, (2) two parallel lines of charge and (3) two parallel plates.

Experimental Purpose

The general purpose of this lab is to use the experimental method for mapping electric fields and equipotential surfaces developed in the introduction to map the electric fields and equipotential surfaces of the following charge configurations:

- a. two parallel lines
- b. two concentric circles
- c. a circle between two parallel lines
- d. a "pear" shape near a straight line
- e. a shape of your choosing with at least one sharp point, and a straight line

and to compare these lines with those predicted by the formulas describing the electric field and potential.

Procedure

1. Draw the patterns listed below on separate sheets of the special black paper with the conductive fluid provided. Read the following instructions carefully before beginning.

You may choose the size of the specified shapes that you draw, but remember that they must be large enough so that you can obtain several potential readings around them using the given probes.

Place a sheet of the special black paper directly on the table to ensure a firm drawing surface and apply a uniform line of fluid from 2 to 4 millimeters wide. Be careful not to touch the ink until it is completely dry. Drying should take about 15 minutes depending on the thickness of the line you have drawn.

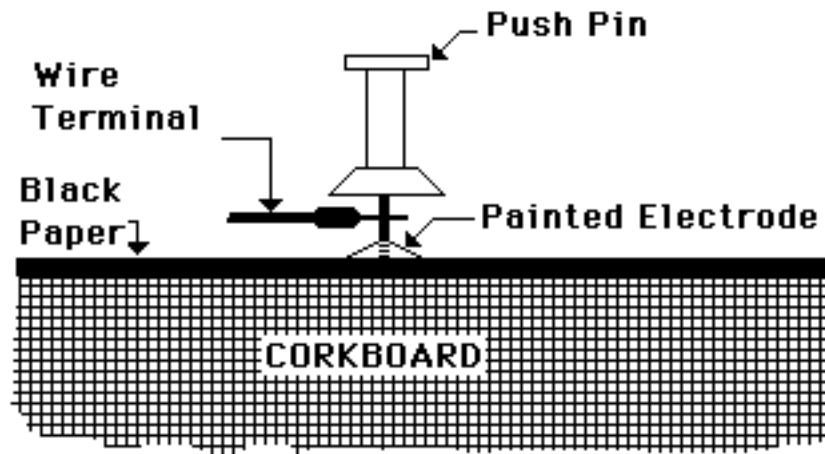
Draw:

- two parallel lines
- two concentric circles
- a circle between two parallel lines
- a "pear" shape near a straight line
- a shape of your choosing with at least one sharp point, and a straight line (you will need to explain the field lines around the shape, so keep it fairly simple).

Note that each sheet contains at least two shapes. These individual shapes will be referred to as "electrodes" below.

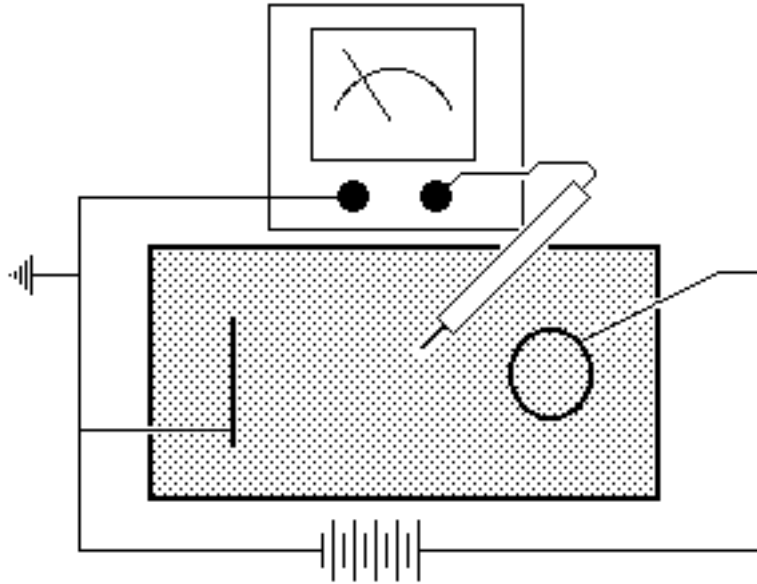
While the conductive fluid is drying, predict what the equipotentials and electric field lines will look like for the patterns you have drawn, and carefully sketch your predictions in your lab notebook. What is the magnitude of the electric field inside a conductor? What is the relationship between the potential and the electric field? What should the potential be inside the conductors?

2. When the conductive fluid has dried, mount the sheet with the two parallel lines drawn on it on the corkboard by pushing a metal pushpin through each corner of the paper.
3. Connect one electrode (one of the lines you have drawn) to the negative terminal of the power source. To do this, use a pushpin to connect the "eye" connector end of the black, plastic-coated terminal wire to the electrode: push the pin through the metal loop (the "eye" connector), through the dry conductive fluid, and firmly into the corkboard. See the figure below.

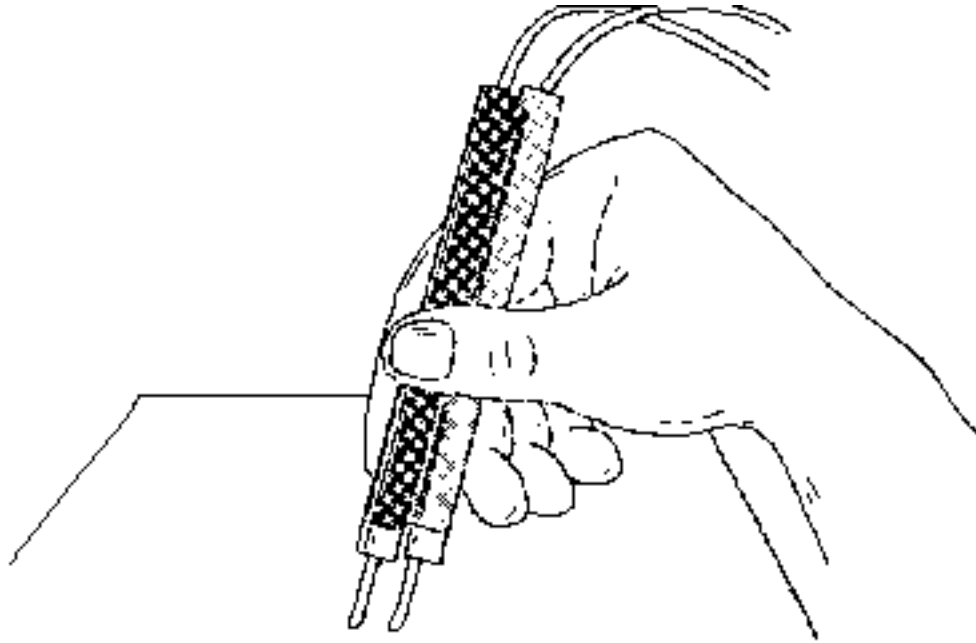


Insert the other end of the wire (it should be a banana plug) into the negative terminal of the power source.

4. Connect the negative terminal of the power supply to ground with another black wire with banana plugs on either end.
5. Connect the other electrode (the other shape you have drawn on the paper) to the positive terminal of the power source using the procedure outlined in (3) above, except with a red lead. When you turn on the power source in step (8), it will produce charge on the electrodes.
6. Use a black lead to connect the negative terminal of the power supply to the black or "ground" or "LO" INPUT connector on the multimeter.
7. Press the ON button on the multimeter and press the V button so that the meter will read potential difference. Press the 20 volt button.. Insert the banana plug of the red probe wire into the "HI" INPUT terminal of the multimeter



8. Turn on the power supply and place the probe tip on the black paper. The multimeter serves as a voltmeter to record the difference in potential between the electrode connected to the negative terminal of the power supply and the probe. Avoid touching the grid printed on the paper as the ink may prevent proper connection. Is the potential affected by touching the paper with your hand? Why or why not?
9. Use the probe to find points of equal potential and mark points having the same potential in pencil on the paper. Connect these points with a pencil line to form lines of equipotential. What is the value of the potential along each electrode? If it is not constant, see your TA. Label each equipotential line with the potential along that line.
10. Disconnect the black lead connecting the common terminal of the voltmeter to the negative terminal of the power supply. Insert the banana plug at the end of the black probe wire into the common terminal of the voltmeter. You should now have a red and a black probe connected to the voltmeter. Hold the probes firmly together as shown in the diagram below and measure the potential difference between the two probes.



Slowly rotate the probes while keeping them firmly together and in contact with the paper. Observe the voltmeter. How does the potential change? Assuming that the distance between the tips of the probes is one centimeter, calculate the maximum electric field present. In what direction does this maximum field point? Draw a vector on the black paper showing the direction and magnitude of the electric field. Repeat this process until you have found the electric field at enough points so that you can map the electric field lines. What are the units of electric field? What can you say about its magnitude between two parallel plates?

11. Map the electric field lines and the equipotentials for the remaining figures. Is the electric field uniform between two concentric circles? Were you surprised by any of the results? What is the angle between the equipotentials and electric field lines in general?

Lab Report

Follow the usual lab notebook format. Your lab report should include the answers to all of the questions asked in the introduction or procedure, all raw and derived data, and an estimate of the magnitude and sources of error in any data recorded. When answering any question or when giving any comparison or explanation, always refer to specific data to support your statements.