Student Sorting and Implications for Grade Inflation

*** PRELIMINARY DRAFT — COMMENTS WELCOME ***

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June 1, 2015

The authors thank Craig Volden for the conversation that inspired this paper and Mark McPeek, Russell Muirhead, and seminar participants at Dartmouth College, the University of California at San Diego, and the University of Virginia for helpful comments.

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Abstract

Contemporary literature on American higher education often draws attention to what is described as rampant grade inflation, and this phenomenon is almost always portrayed as reflecting underlying problems in educational institutions. We offer here, however, an interpretation of grade inflation that turns on the choices that students have over the academic departments in which they study, and we argue that patterns in grades themselves cannot be considered in isolation from the fact that students face incentives to sort themselves strategically across departments. To make this point clear, we present a simple game-theoretic model in which students of varying abilities face a choice between enrolling in a department whose grades are inflated and thus ability-concealing versus enrolling in a department whose grades are ability-revealing. In equilibrium, all grades are high, which on the surface appears troubling. However, what looks like ubiquitous grade inflation is a result of the fact that the ability-revealing department attracts highly talented students seeking to distinguish themselves from students of lesser ability, who avoid said department because enrolling in it is costly. Overall, our formalizations show that student sorting confounds the interpretation of grades and that, in order to understand what a university’s grades truly mean, one must know first which departments in a university are ability-concealing and which, in contrast, are ability-revealing.
Introduction

Grade inflation appears by many measures to be endemic across the American higher education landscape. A 2010 study of grade trends at over 160 American universities, for example, revealed that mean grade point average has risen by nearly a tenth of a point per decade (Rojstaczer and Healy, 2010). Such a rise in average grades has attracted substantial attention in the popular press, and the revelation that the most commonly awarded grade at Harvard University is an “A” unleashed a firestorm of media attention on the university’s grading policies.\(^1\) In 2001, 91 percent of Harvard’s graduating class received honors, rendering this indicator of student ability essentially meaningless in the sense of distinguishing among Harvard undergraduates.\(^2\) At Yale University, approximately 62 percent of grades were “A” and “A-” as of Spring, 2012, whereas in 1963 this proportion was only ten.\(^3\) Empirical research on grading tends to show that the phenomenon of grade inflation is broad-based although Kuh and Hu (1999) argue that grade inflation is disproportionately a feature of research universities and elite liberal arts colleges as opposed to all types of educational institutions.


As we review shortly, scholars have offered a variety of explanations for observed upward trends in grades, and these explanations range from changes in the incentives facing professors to changes in university grading policies that lead to perverse incentives for students. Broadly speaking, some explanations for grade inflation focus more heavily on (and tend to blame) the role of educational institutions and their rules and procedures while others focus on the behaviors of students and the extent to which their choices over departments and courses have downstream effects on measures like average grades. Obviously there are connections between these two ways of thinking—students make decisions in an environment crafted by universities—but it is nonetheless still useful to distinguish between incentives facing faculty members and incentives facing students.

Having noted this dichotomy, we offer an interpretation of grade inflation that turns on student choice over departments. In particular, we present below a simple formal model that allows students to choose whether they want to study in—i.e., major in—a department whose grades are inflated versus a departments whose grades are accurate. We explain shortly what we mean by accurate, but for the moment it suffices to note that inflated grades are *ability-concealing* and that so-called real grades can be thought of as *ability-revealing*. Thus, in our model each student faces a choice between majoring in a department whose classes lead to an ability-concealing transcript and, in contrast, in a department whose classes generate a transcript that meaningfully reveals said student’s ability. The students in the model differ in the extent to which they want their grades to reveal their underlying abilities, and this leads students to sort themselves strategically. The act of sorting, as the model shows, confounds the interpretation of grades, and interpreted liberally the model shows that trends in grades—upward or downward—can be induced simply by changes in student sorting abilities or opportunities. It follows from the model that our understanding of grades and how we interpret trends in them must be informed by a simultaneous focus on student sorting across departments and the effects that sorting has on grade distributions.
In the next section we briefly discuss the literature on grade inflation, and we then present our model and explain its various components. The basic model has two types of equilibria, and we show that, in equilibrium, strategic sorting by students leads to high grades. We then present an extension of the model which generalizes our initial, and somewhat coarse, characterization of student ability and also allows for what we call an education bonus. In the extension we observe high grades in equilibrium, and most notably the extension yields a somewhat counter-intuitive result, namely, that the more difficult a university’s ability-revealing department, the higher are average grades and the more it appears that grade-inflation is ubiquitous. This extension has policy implications for efforts to rein in what are considered inflated grades, and we discuss such implications and others in the conclusion.

**Grade inflation**

Concerns over grade inflation are not new, and see Ekstrom and Villegas (1994) for a discussion of grade trends in American colleges and universities through 1990. Although grade inflation is generally viewed in a negative light, not all scholars share this perspective. For example, Millman et al. (1983) argue in a slightly dated study that grade inflation through 1983 had not eliminated the signaling value of grades. Pattison, Grodsky and Muller (2013) makes similar claims with more contemporary data.

One of the commonly cited reasons for an increase in grades across American colleges and universities has been the increasing weight placed on instructor evaluations in hiring and tenure decisions (Stratton, Myers and King, 1994; Eiszler, 2002). Students tend to give better evaluations to professors who award them higher grades (Johnson, 2003), and thus an increased reliance on teacher evaluations during evaluation processes can create incentives for high grades. Nelson and Lynch (1984) argue relatedly that the relationship
between evaluations and grades can be exacerbated by stagnating faculty salaries. And, Pressman (2007) notes that pressure for high grades will tend to be stronger for untenured but tenure-track professors and strongest for adjunct faculty, whose employment depends on enrollments.

Perrin (1998) and Kelly (2009) draw attention to the fact that a university professor may in the course of grading compare her students not just to other students at her own university but also to the typical American student. Perrin writes, “[Professors] imagine our students at a mythical Average U., and give the grades they would get there.” If a faculty member believes that her institution’s admissions policies lead to a highly-talented student body, then it follows that said faculty member should in general assign high grades. On this point, see Achen and Courant (2009) and their anecdote of “a [University of Michigan] chemistry professor who had stuck to the standards of his own undergraduate work for decades, but who came to notice that incoming graduate students at Michigan often had better grades than graduates of [his] department with similar knowledge and skill.”

Another theory of grade inflation posits that trends in high grades have been driven by student behaviors. For example, in some universities students are allowed to take classes without grades appearing on transcripts. Strategically-minded students may seek to take advantage of this practice by ensuring that grades for their most difficult classes are not visible in this way. If students strategically select certain classes to have non-visible grades, then average grade point averages may increase even in the face of fixed grading policies (Birnbaum, 1977). Foreshadowing the model that we present here, Prather, Smith and Kodras (1979) argue that changes in average grades can reflect changes in enrollment patterns; their empirical research finds that “English majors tend to receive relatively higher grades in education courses than in their other courses, while the grades they receive for physical science and foreign language courses are, on the average, lower. Physical science courses generally record lower grades for all majors, while teacher education courses comparatively
record higher grades for all majors” (pp. 21-22). The implication of such a finding is that average grades reflect student selection into coursework of interest.

To some extent students choose the classes that interest them, and Sabot and Wakeman-Linn (1991) show that college students who received relatively high grades in a given course are significantly more likely to enroll in a subsequent course in the department of the former. Bar, Kadiyali and Zussman (2009) analyze data from Cornell University, and find that publicly-available median grades allow students to select into leniently graded classes. Strenta et al. (1994) and Ost (2010) have similar findings, and both note that low grades in science classes are strong predictors of a student’s continued participation in science. There is also evidence that grading policies respond to the perceived value of a major (Freeman, 1999). When a department’s graduates do not perform as well on the job market, they are forced to “buy” students with higher grades.

Sabot and Wakeman-Linn (1991) show as well that introductory course grades received in low-grading departments are better predictors of student performance in future classes on related subjects compared to grades given in what appear to be grade-inflating departments. Similarly, they show that alternate predictors of student ability (e.g., standardized test scores, parental education levels, high school grades, and so forth) are associated with student grades in low-grading departments but not departments that routinely give high grades. Among other things, these findings show that inflated grades effectively mask student abilities and diminish the extent to which grades signaling underlying skills and talent levels.

There is some formal work on grade inflation, but the literature is not extensive. Three examples are Yang and Yip (2003), Chan, Hao and Suen (2007), and Franz (2010). In the former, schools have incentives to give high grades because this helps weaker students obtain jobs; this leads to labor market inefficiencies. In Chan, Hao and Suen employers are faced with a dilemma because they cannot determine whether a set of students with high grades are deserving or whether the university that granted said grades is an easy-grading
institution; this leads to inefficient labor market outcomes. Lastly, Franz (2010) models professor-student interactions with an eye on the costs on faculty that students impose by requesting high grades. In equilibrium, the “nuisance” students in Franz lead professors to inflate grades.

Students and institutions in this limited formal literature are strategic, but the extant models in the literature do not allow students to sort themselves across departments (or other academic units) in the way that we describe here. Insofar as contemporary university students appear to be very attune to grading policies and how they vary by field of study (and even class and professor), our model of sorting fills a gap in the literature.

Model

We now describe a model that sheds light on the dynamics of student sorting across university departments and subsequent patterns in grades. The model is very simple, but it nonetheless illustrates the extent to which student sorting confounds our understanding of grade distributions. The model also helps us understand how situations in which there is no variance in student grades—situation that on the surface appear very problematic—should be interpreted.

The model is set in a single university and includes a group of students and departments. The model’s premise is that the students in the university have already been admitted but must choose a department in which to enroll (or major). As will be clear, a student’s choice between departments is informed by her interest, or lack thereof, in signaling her intrinsic ability level to a labor market that she will enter upon completion of her studies.
Students

There are in the model two types of students, low ability and high ability. A student’s ability is fixed and exogenous, and let the proportion of high ability students in our hypothetical university be \( \pi_H \in (0, 1) \). All students know their own abilities, either low or high, but face the problem of credibly signaling their talents to a labor market. In the sense of Spence (1973), students attend university in order to earn a transcript which can subsequently be used to indicate ability. Like the students in Love and Kotchen (2010), our students value grades because of what they signal to a labor market. 

We assume that being of high ability is valuable to a student insofar as, by assumption, the post-university labor market disproportionately rewards high ability students. In particular, if a student is known to be of high ability, then after her education is complete she earns in the market a wage that we normalize to be one. In contrast, a low ability student known to be of low ability earns in a post-education market a wage normalized to be zero. If after graduation a student’s ability is not known to the labor market, then said student receives a wage proportional to the probability that she is of high ability. For example, if the market believes that a student is of high ability with probability 1/4, then her post-education wage will be \( 1/4 \times 1 + 3/4 \times 0 = 1/4 \). We do not model the post-education labor market explicitly, and implicit in our assumptions about post-education wages is that firms in the labor market are risk neutral and that all students receive jobs after graduating. The assumption that all students are employed post-education is not binding; one could treat what we call a low wage as an unemployment or welfare benefit. Moreover, one could also interpret what we call a post-education wage as a placement in a post-graduate institution like a medical school, law school, and so forth. Viewed from this perspective, our model posits that high ability students will earn slots in better medical students than their low ability counterparts, \textit{ceteris paribus}. Key here is the assumption that being of high ability is more valuable than being of low ability.
Asserting that post-education wages are either zero or one, or perhaps a value between, is of course a simplification. While we could have asserted that low ability students receive a post-education wage of $w$ and high ability students a wage of $\bar{w}$ where $\bar{w} > w$, the conclusions of the model do not depend qualitatively on the magnitudes of these two wages. Rather, only the difference $\bar{w} - w$ is important. Moreover, as long as this difference is positive, then a student values having high ability, ceteris paribus. Hence we normalize wages so that $w = 0$ and $\bar{w} = 1$, and thus the difference between low ability and high ability wages is one.

Similarly, asserting that there are two types of students—low and high ability—is a simplification but one without serious consequences. We could have assumed a priori that student ability exists on a continuum, and indeed we consider such an extension to our model after we present our main results.

Departments

We assume that our hypothetical university contains departments whose grades are either ability-revealing or ability-concealing. These types of departments differ only in the manner in which they assign grades to their students.

An ability-revealing department (shorthand for a department whose grades are ability-revealing) is one that offers courses with regular and discriminating examinations, projects, assignments, and so forth. These examinations, say, allow the department to know whether a given student enrolled in the department is of low ability or is of high ability, and the department indicates this knowledge via grades. In particular, a grade-revealing department assigns an “A” grade to high ability students (because these students did well on the department’s examinations) and a “B” grade to low ability students. Assuming that an ability-revealing department assigns grades of “A” and “B,” as opposed to “A” and “C” or “A” and “D,” is of no consequence. The key here is that an ability-revealing department assigns grades that discriminate between low and high ability students.
The regular and discriminating examinations given in an ability-revealing department require an effort cost for enrolled students who know that these examinations and related assignments will ascertain their underlying abilities. This cost, paid by each such student, is parameterized as $c$, and we assume that $c > 0$.

In contrast, an ability-concealing department is one whose courses do not discriminate between low and high ability students. The courses in an ability-concealing department are by definition not excessively challenging, and the key is that all students enrolled in them receive excellent grades, in particular, marks of “A.” Moreover, the lack of discriminating examinations means that students in an ability-concealing department are not subject to a cost comparable to the effort cost $c$ required of a students who enroll in a grade-revealing department.\footnote{The key here is that an ability-concealing department gives all enrolled students “A” grades. One could posit that the examinations in such a department are in principle ability-revealing but that the faculty in said department simply ignore this when assigning course grades.}

We could have assumed that students enrolled in an ability-concealing departments are forced to pay an effort cost akin to the cost required of students in a grade-revealing department. Had we done this, our model’s equilibria, which follow shortly, would have been a function of the difference between the effort cost required of a student in an ability-revealing department and the cost required of a student in a grade-concealing department. Thus, the assumption that $c > 0$ is akin to assuming that it is more costly for a student to enroll in an ability-revealing department than in an ability-concealing department. This ordering is intuitive given the discriminating examinations associated with the former and the fact that students in the former know that they are being judged.

We also could have assumed that low and high ability students pay different costs for attending an ability-revealing department. That is, we could have posited that high ability students pay $c^H$ when enrolling in an ability-revealing department and low ability students $c^L$. It would probably be natural to assume that $c^L > c^H > 0$, meaning, low ability students have
to work harder in ability-revealing departments than do high ability students. Regardless, our main results do not depend on whether costs for attending an ability-revealing department vary by student type.

Finally, the assumption that ability-concealing departments give all students “A” grades is not strictly necessary. Rather, the key feature of an ability-concealing department is that the grades given by it do not vary by student ability. The grades could always be “A”s, as we assume here, or it could be that they are all “B”s or “C”s. We note that grade inflation is one form of grade compression, and the point here is that students enrolled in an ability-concealing department are assumed to know that the grades they receive do not discriminate between low and high ability students.

**Labor market**

As we noted above, we do not formally model firms in a post-education labor market, nor do we model, say, admissions committees in graduate institutions. However, we assume that the market knows which types of departments are ability-revealing and which are ability-concealing. This does not strike us as a particularly strong assumption although we do recognize that one could argue that firms, graduate schools, and other post-graduate institutions are not informed about which departments in universities give ability-discriminating grades.

**Average grades and grade inflation**

As will be clear shortly, our model generates a distribution of grades across students, and from this distribution we can calculate average grades in equilibrium. Moreover, before we discuss the equilibria of our model we need to define grade inflation. In our two-type model, high ability students cannot receive inflated grades because they are of high ability; only low ability students can have inflated grades. Thus, we say that a low-ability student receives
an inflated grade if said student receives a grade that is equal to or greater than the grade received by a high ability student. Given the model description above and our associated assumptions, if a low ability student receives a grade of “A,” then we say that this student’s grade is inflated. Why? A high ability student who attends an ability-revealing department will by definition receive an “A” grade. Thus, a low ability student who also receives an “A” grade has an inflated grade.

As will be clear shortly, it is important to distinguish between the fraction of inflated grades in equilibrium and the fraction of high grades. These two quantities are distinct: a high grade is not necessarily inflated if it is earned by a high ability student. Distinguishing between inflated versus high grades helps us understand the difference between situations where there is serious grade inflation (many students receive grades greater than their abilities) versus situation where there are many high but accurate grades.

**Equilibria**

We assume that a university has two departments, one that is ability-revealing and the other, ability-concealing; the assumption of two departments is not constraining and we comment on it later. The university’s students, $1 - \pi_H$ of whom are of low ability and $\pi_H$ of whom are of high ability, choose simultaneously whether to enroll in one of two departments. Subsequent to this the departments assign grades, and then the students enter a post-education labor market. As we have noted above, the labor market pays each student based on her ability or her expected ability if the market cannot discern based on grades whether the student is of low or high ability.

In our model only the students are strategic actors. Overall, a given student’s utility is equal to her post-education wage minus the effort cost $c$ if the student enrolled in an ability-revealing department.
The (Bayesian) equilibria of the model depend on the fraction $\pi_H$ of high ability students and the cost $c$ of enrolling in an ability-revealing department. As characterized in Lemma 1, the model always has a separating equilibrium and has a pooling equilibrium if $\pi_H \geq 1 - c$.

**Lemma 1.** There is always a separating equilibrium wherein low ability students enroll in an ability-concealing department and high ability students enroll in an ability-revealing department. If $\pi_H \geq 1 - c$, then in addition there is a pooling equilibrium wherein all students enroll in an ability-concealing department. Regardless of the values of $\pi_H$ and $c$, there is never a pooling equilibrium in which all students enroll in an ability-revealing department.

The proof of Lemma 1 is in the appendix, and the lemma characterizes the model’s pooling and separating equilibria. We discuss these equilibria in this order.

**Pooling equilibrium.** When $\pi_H \geq 1 - c$, the model has a pooling equilibrium in which all students enroll in an ability-concealing department only. Consider the implication of the aforementioned (weak) inequality. When it holds, then the fraction $\pi_H$ of high ability students is large and indeed almost all students are of high ability. The intuition for this claim is as follows. Recall that $c$ denotes an effort cost, presumably short-term, that students enrolling in an ability-revealing department pay as a consequence of having to endure discriminating examinations, assignments, and so forth. Recall as well that high ability students identified as such earn a normalized wage of one post-graduation. If this wage of one is interpreted as the present value of a stream of wages, then $c$ is presumably much smaller than one and in particular should be understood as being quite close to zero. And, when $c$ is close to zero, then $1 - c$ is close to one. Thus, when $\pi_H \geq 1 - c$ we have a rarefied situation in which there are almost no low ability students at all.

Continuing, when every student enrolls in the ability-concealing department, which is what happens in the pooling equilibrium under consideration, then all students receive the same grades, in particular grades of “A.” An outside observer assessing this situation—in which the fraction of “A” grades is one and there is no variance in awarded grades—might
be inclined to say that this is a situation characterized by rampant grade inflation. Such a characterization would be inaccurate, however. Rather, the fraction of inflated grades in the pooling equilibrium is $1 - \pi_H$, which is very small and actually quite close to zero given our earlier discussion of $\pi_H$ and $c$. Intuitively, what our hypothetical observer is seeing is not a consequence of low ability students flocking to an easy-grading department in order to hide their low ability levels, a pattern of behaviors that would in fact indeed induce high rates of grade inflation. Rather, the situation the observer sees is one in which strong or high ability students flock to such a department.

When $\pi_H \geq 1 - c$, it would be hard to argue that grade inflation as we have defined it is troublesome. After all, when this inequality holds, almost every student is of high ability, and thus the fact that almost every student receives an “A” grade does not connote a serious mismatch between grades and underlying abilities. There is of course some mismatch here, as $1 - \pi_H$ of the student body—the low ability students—receives grades that are inflated. However, $1 - \pi_H$ must be quite small for the pooling equilibrium to exist.

One might want to argue that the presence of all students’ pooling on an ability-concealing department is an observable indication that almost every student is of high ability. Thinking empirically about actual trends in grades, this point reverses the concern that many have articulated about inflated transcripts. To the point, in the pooling equilibrium discussed here, an abundance of students who enroll in an ability-concealing department means that almost every student is highly talented and not, say, that all students lack ability and are choosing an ability-concealing department because they fear being exposed as such by a grade-revealing department. In our pooling equilibrium, high ability students do avoid paying the effort cost $c$, which could be criticized on normative grounds, i.e., perhaps the students are lazy. However, these students are not enrolling in an ability-concealing department to hide their abilities, and this is the key point here. The small percentage of low ability students who pool with the high ability students do so in order to hide their (low ability)
statuses; this group does end up with inflated grades, but the group is nonetheless very small.

In the pooling equilibrium here, the ability-revealing department has no students in it, i.e., no enrolled majors at all. Presumably this is not ideal for the department, and indeed one might conjecture that such a department would anticipate a lack of students and change its grading policy prior to student enrollment decisions. Department grading policies are probably sufficiently sticky so that changing grading norms is not a simple process, and from this perspective treating department grading policies as exogenous seems natural. Nonetheless, we are exploring the matter of strategic department grading policies in other research.

Separating equilibrium. Our model always has a separating equilibrium in which low ability students attend the ability-concealing department and high ability students, the ability-revealing department. In this equilibrium, whose existence is not a function of the relationship between $\pi_H$ and $c$, departments do not contain mixtures of low and high ability students. Rather, in the equilibrium all low ability students attend one department and all high ability students, the other.

This feature of the separating equilibrium has one rather notable consequence: grades appear inflated in both departments in our hypothetical university. To be precise, in the separating equilibrium all high ability students receive top grades—because they are in fact of high ability and are enrolled in an ability-revealing department—and low ability students receive top grades, too—because they enroll in an ability-concealing department which provides everyone with high grades. Thus, our model’s separating equilibrium, which exists for all values of $\pi_H$ and $c$, features a distribution of grades that looks on the surface to be highly inflated.

In fact, the distribution of grades in the separating equilibrium has literally zero variance because in it every student receives an “A.” These numerous “A” grades, however, reflect
fundamentally different dynamics. “A” grades received by high-ability students are accurate evidence of excellent students being willing to subject themselves to an ability-revealing process; thus, these “A” grades do not reflect grade inflation. In contrast, however, “A” marks received by low-ability students are evidence of low-ability students avoiding an ability-revealing process; “A” grades received by these students do reflect inflation, and thus the fraction of inflated grades in the separating equilibrium is $1 - \pi_H$.

The model’s separating equilibrium is more compelling than the previously-discussed pooling equilibrium because the latter only exists when the fraction of high ability students is very large. With this in mind, we argue that our model shows that student sorting by itself is sufficient to lead to a situation in which all students receive identical grades, all of which are “A” marks; this situation looks like one in which grade inflation is a serious problem but, at least for high ability students, it is not.

We earlier mentioned that our assumption about the existence of only two departments in a university is not binding. If there were more than two departments in our hypothetical university—some ability-concealing and others ability-revealing—the separating equilibrium we have described here would continue to exist as long as the ability-revealing department or departments imposed effort costs beyond those imposed by the ability-concealing departments. The key to the equilibrium is not the number of departments per se; rather, the key is the fact that ability-revealing departments impose more of a cost on enrolled students than do ability-concealing departments.

Another notable feature of the separating equilibrium is that it requires only that $\pi_H$ be neither zero nor one. If $\pi_H$ were one, then all students would be of high ability and the only equilibrium that would exist in the model would be one in which students pooled on the ability-concealing department. Observationally speaking, grades would appear to be inflated in this scenario, but in reality they would not be because all students would be of high ability. If on the other hand $\pi_H$ were zero, then all students would again pool on the
ability-concealing department. This would yield a situation with rampant grade inflation, one wherein all students of low ability are labeled by an ability-concealing department as high ability.

**Extension: continuous student ability and an education bonus**

One might argue that our characterization of student ability as binary—either low or high—is rather coarse and that this may be responsible for the result, above, that, when students separate, there is no variance in student grades. With this in mind we now offer an extension of our model that allows us to explore the consequences of allowing student ability to exist on a continuum. Along with this change we also include in the extension an education bonus that a student receives if she enrolls in an ability-revealing department. As shown below, the extension of our model does lead to variance in student grades; however, as will be clear shortly it does not change our fundamental results about grade inflation and the effects of student sorting on the distribution of grades.

Let student ability be denoted $\theta$. We assume that $\theta \sim U(0, 1)$, but this assumption is not qualitatively necessary for the results that follow. We make the uniformity assumption because it allows for a closed-form equilibrium characterization.

When student ability exists on a continuum we can no longer speak simply of “low” and “high” ability students. In addition, with continuous student ability we need a more refined characterization of grades and of post-education wages. We continue to assume that an ability-revealing department is one that assigns grades based on underlying student abilities and in particular suppose that a student’s grade in such a department perfectly reveals her ability level. With a continuous distribution of grades this is obviously a bit of an abstraction insofar as a finite number of class letter grades—“A,” “A-,” “B+,” and so forth—cannot map one-to-one to a continuous range of student abilities. However, one
can imagine that an ability-revealing department issues class grades, returns discriminating assignments, generates ability-revealing letters of recommendation, and so forth, in such a way that a student who enrolls in such a department has an overall record from her educational experience that perfectly reveals her ability level $\theta$. Thus, in the extension of the model a student with ability $\theta$ who attends an ability-revealing department receives a grade of $\theta$.

With respect to ability-concealing departments, we continue to assume that such a department awards very high grades to all of its students. In particular, in the model extension we assume that every student in it receives a grade of one. This is parallel to our earlier assumption that ability-concealing departments award grades of “A” to their students.

In terms of wages, suppose that a student of ability $\theta$ whose ability is known to the labor market receives a base wage of $\theta$ in the post-education market. If in addition this student attended an ability-revealing department, then she receives a wage boost of $e \geq 0$. Such an education bonus is intended to capture the fact that studying in an ability-revealing department and bearing the requisite effort cost can lead to increased knowledge and, accordingly, higher wages.

As was the case in our initial model formulation, a student whose wage is not known receives a base wage in the labor market corresponding to expected ability level where this expectation is taken given equilibrium student behavior. Such a student cannot receive the education bonus $e$ because the only way to receive such a bonus is to enroll in an ability-revealing department, an action that would signal the student’s ability.

Considering both effort cost, education, and the effect of ability on what we are calling base wages, if a student with ability $\theta$ enrolls in an ability-revealing department, her net utility is $\theta - c + e$. Similarly, if students with $\theta \in \Theta \subset (0, 1)$ enroll in an ability-concealing department, then each student with $\theta \in \Theta$ receives net utility of $\int_{\Theta} \theta d\theta$.

The equilibrium of the model extension depends on a cutpoint that we call $\bar{\theta}$, and this
cutpoint is characterized by student indifference between enrolling in an ability-concealing and an ability-revealing department. The equilibrium is described in Lemma 2, whose proof is in the appendix.

**Lemma 2.** Let $\bar{\theta} = 2 (c - e)$.

If $\bar{\theta} \in (0, 1)$, then the equilibrium of the model is semi-pooling. Namely, students with ability $\theta < \bar{\theta}$ enroll in an ability-concealing department and students with $\theta > \bar{\theta}$ enroll in the ability-revealing department. Students with $\theta = \bar{\theta}$ are indifferent between the two departments, and we ignore these students since they are of zero measure.

If $\bar{\theta} > 1$, then the equilibrium of the model is pooling wherein all students enroll in an ability-concealing department. If $\bar{\theta} < 0$, then the equilibrium of the model is pooling wherein all students enroll in an ability-revealing department.

The knife-edge condition $\bar{\theta} = 1$ (equivalent to $c - e = 1/2$) is consistent with either semi-pooling as described above or students pooling on the ability-concealing department; without loss of generality, we assume that $\bar{\theta} = 1$ leads to the latter type of pooling. Similarly, the knife-edge condition $\bar{\theta} = 0$ (equivalent to $c = e$) is consistent with either semi-pooling or student pooling on the ability-revealing department; without loss of generality, we assume that $\bar{\theta} = 0$ leads to the latter type of pooling.

Lemma 2 shows that the indifference cutpoint $\bar{\theta}$, which determines whether the extended model’s equilibrium is semi-pooling ($\bar{\theta} \in (0, 1)$) or is pooling ($\bar{\theta} \leq 0$ or $\bar{\theta} \geq 1$), depends on the relationship between the effort cost $c$ required of students enrolled in an ability-revealing department and the education bonus $e$. We have earlier argued that $c$ should be thought of as small compared to one because this parameter represents a short-term effort cost as opposed to a discounted stream of wages. Henceforth we assume that $c - e < 1/2$, and we thus focus our attention on two possibilities for our cutpoint, either $\bar{\theta} \leq 0$ or $\bar{\theta} \in (0, 1)$.

\footnote{Put another way, if the effort cost $c$ is close to zero and hence small, then $c - e$ must be small as well, even if $e$ is negligible.}
Suppose first that $\bar{\theta} \leq 0$. Because $\bar{\theta} = 2 (c - e)$, it follows that the education bonus $e$ must be greater than or equal to the cost $c$ that students must pay when taking classes in an ability-revealing department. When $e \geq c$, all students attend the ability-revealing department and, accordingly, all students’ grades are accurate measures of their abilities. There is no grade inflation when $\bar{\theta} \leq 0$ because, in this situation, there are no students in the ability-concealing department.

Suppose one were to argue on normative grounds that student pooling on the ability-revealing department is a good thing, i.e., that society benefits when $\bar{\theta} \leq 0$. One might then ask, how might students be induced to pool in this way? Based on Lemma 2, the answer here is simple: our hypothetical university should increase its education bonus $e$ until the value of education overwhelms the cost of attending an ability-revealing department. As soon as this happens, then all students become willing to attend said department, an outcome that, as we noted, has no grade inflation at all. It is also true that making the effort cost $c$ small will have the same effect.

Now consider the case $c > e$ with the previously-noted proviso that the effort cost $c$ remains small. Intuitively, the inequality $c > e$ implies that education is valuable to those students who enroll in an ability-revealing department—but not overly so. If one thinks of education like Spence (1973), solely in the sense of signaling, then $e = 0$, in which case the restriction $c > e$ certainly holds.

When $c > e$ the model’s equilibrium is semi-pooling, and the first feature to notice about the equilibrium is that the fraction of students who enroll in the ability-revealing department is $1 - \bar{\theta} = 1 - 2 (c - e)$. This expression is decreasing in the cost term $c$, which is intuitive: the more effort required in the ability-revealing department, the fewer the number of students who are willing to enroll in it. It is also increasing in $e$: the greater the value of education for those students who work hard, the greater the number of students who are willing to enroll in an ability-revealing department.
In the semi-pooling equilibrium, the average student grade is

\[
\int_0^\theta d\theta + \int_\theta^1 \theta d\theta = 2 (c - e) + \frac{1 - 4 (c - e)^2}{2}
\]

Differentiating this expression with respect to \( c \) yields \( 2 - 4 (c - e) \), which is positive as long as \( c - e < 1/2 \) (which noted holds by assumption, as we noted earlier). Thus, the more difficult the ability-revealing department becomes, the higher are average grades in equilibrium, *ceteris paribus*. On the other hand, the opposite result hold with respect to the education bonus \( e \): the greater this bonus, the lower are average grades in equilibrium, *ceteris paribus*.

The result about the effect of the effort cost \( c \) on average grades is particularly notable, and there are two reasons that increases in \( c \) lead to higher grades on average. First, when the difficulty parameter \( c \) increases, the indifference cutpoint \( \bar{\theta} \) increases as well. When \( c \) is large, that is, only high ability students remain in the ability-revealing department; when their abilities are revealed, they are accurately revealed to be high. This is not grade inflation; to the contrary, it is accurate reporting. Second, and on the other hand, an increase in \( c \) causes some students to select out of the ability-revealing department and into the ability-concealing department; these students receive high grades on account of enrolling in the letter, indeed grades higher than they should based on their underlying abilities. This is a form of grade inflation, and the fraction of students with inflated grades is \( 2 (c - e) \).

Overall, the point here is that the increase in grades due to increasing effort cost \( c \) has two components which manifest themselves similarly but have different underlying causes, only one of which can be considered problematic.

When the education bonus \( e \) increases, then two the effects described above move in the opposite direction. In particular, an increase in \( e \) leads students who previously would have enrolled in the ability-concealing department to enroll in the ability-revealing department
instead. This has the feature of both lowering average grades and decreasing the number of students whose grades are inflated.

One can imagine a dean or other administrative figure in a university arguing in the face of high grades that departments need to make their classes more difficult so as to drive down ostensibly inflated grades. In the framework of our extended model, this would be equivalent to making $c$ greater. Positing that increasing difficulty will lower average grades is, however, a partial equilibrium assertion. Since students sort themselves conditional on department difficulty, making an ability-revealing department more difficult will drive students away from it and thus have the opposite effect of what the dean or other figure intended.

Such an argument holds conditional on the existence in our hypothetical university of an ability-concealing department, and this highlights the possibility that there is a collective action problem in university grading, one that the dean could overcome if she could simultaneously convince an ability-revealing department to be more difficult while convincing (or compelling?) an ability-concealing department to become ability-revealing. We will return to this point later. At the moment, though, it suffices to note that, when students have an ability-concealing department as an option, the more difficult the ability-revealing department, the higher are average grades and thus the more grades look like they are inflated.

One seems a similar point when examining the variance in grades in the extended model’s semi-pooling equilibrium. This variance is

$$\int_0^\theta d\theta + \int_\theta^1 \theta^2 d\theta - \left(2(c-e) + \frac{1 - 4(c-e)^2}{2}\right)^2.$$  

Algebra shows that this variance approaches $1/12$, the variance of the uniform distribution on the unit interval, as $c$ approaches $e$. This is appropriate because, as $c$ becomes smaller, more students enroll in the ability-revealing department and this leads to a grade distribution that gets close to the true ability distribution.
More importantly, the derivative of the above variance is negative for relevant parameter values. In other words, the more difficult the ability-revealing department becomes, the less variability there is in student grades. This is because increasing difficulty leads to an increasing number of students in an ability-concealing department and accordingly less grade variability. If, say, our aforementioned university dean or administrator were to argue that his or her institution should seek extensive variability in grades—because, say, variability in grades makes it easier to distinguish low and high ability students—the implication of our extended model is that the dean should insist that the ability-revealing department be as easy as possible on its students.

Given the definition of $\bar{\theta}$, another option open for a dean who wanted to increase grade variance would be to encourage the ability-revealing department to increase its education bonus. This would have the effect of inducing more students to enroll in said department, and this would lead to increased variance in grades and less grade inflation. This option may be more palatable than decreasing the effort cost $c$.

We motivated the extension of our model with the recognition that the coarse way in which we modeled student ability—low versus high—diminishes our ability to ascertain whether in equilibrium there is variance in student grades. Our extension shows that this concern was indeed valid. Namely, as long as the effort cost $c$ associated with an ability-revealing department is not excessive compared to the education bonus ($c - e < 1/2$, which seems very intuitive), then there is indeed variance in student grades. This variance is conditional on effort cost $c$ and the education bonus $e$, and above we have explained how changing these two parameters changes average grades and grade variance.

Our final point about the model extension concerns the possibility of including a cost term for enrolling in an ability-concealing department. Were we to have done this, then we would have seen that what we call $c$ in the model extension proxies for the difference in cost associated with an ability-revealing department and an ability-concealing department.
Having said this, there are two ways to interpret the effects of an increase in the effort cost $c$. In particular, $c$ in the extended model can increase because the ability-revealing department becomes more difficult; and, it can increase because the ability-concealing departments becomes easier. These dual perspectives on $c$ do not change any of our earlier interpretations or derivatives, but they do imply, for example, that increasing the cost required for a student to enroll in an ability-revealing department is equivalent to decreasing the cost associated with an ability-concealing department.

**Discussion**

In this paper we have offered a game-theoretic analysis of student choice and grading, an analysis motivated by empirical studies documenting upward trends in grades in American higher education institutions. We have considered here what one might make of these trends and whether or not they are a harbinger of fundamental problems with the operation of colleges and universities in the United States.

The basic model of grading we have offered is by design very simple, and the extension to a more nuanced characterization of student ability is not appreciably more complicated. In spite of this simplicity, our formalizations shed some light on a key feature of student grading processes in American universities, a feature that often is neglected discussions of grading trends in American higher education. Namely, the models here focus attention on the role in grading of student choice over departments and the effects on grade distributions of student sorting. The students in our model are both effort-averse and forward-thinking, and this induces a dynamic in which the best students seek to distinguish themselves from their lower-ability counterparts and are willing to undertake costly behaviors so that their true abilities are revealed to a post-education labor market. The end result of this is that average grades are high but not because of, say, lax standards or enrollment pressures. Rather, grades are
high because good students appropriately earn them from an ability-revealing department and lesser students garner them, so to speak, from an ability-concealing department.

To be clear, we are not arguing that our model should be thought of as a (or “the”) comprehensive explanation of grade inflation in American higher education institutions. Our primary objective has not in fact been to offer a complete theory of grading in educational institutions but rather to encourage scholars interested in grading to consider assiduously the consequences of sorting and strategic behavior on grade distributions. Presumably there are a variety of explanations for the types of grade inflation that empirically-driven scholars of American education have identified, and our models should remind those considering these explanations to be mindful of how sorting can manifest itself.

What does our model say about contemporary trends in grades across American universities? Perhaps the most obvious implication is as follows: within-institution trends in grades are hard, if not outright impossible, to interpret in isolation from trends in the extent to which students sort themselves strategically into departments. Put another way, student sorting confounds grading, and therefore analyses of grade trends that are executed independently of sorting dynamics can be very misleading. In our basic model where parameters are reasonable (i.e., the cost of attending an ability-revealing department is not too high compared to long-term wage streams), literally all students receive high grades and there is correspondingly no variance in the distribution of grades. Viewed from the lens of grades only, this situation looks problematic and in need of remedy; it is problematic, however, only for low-ability students as the high grades received by top students correctly reflect these students’ high ability levels. Challenged to defend its plethora of high grades, an ability-revealing department in this situation might respond, “All of our students are excellent!” Due to student sorting driven by high ability students seeking to distinguish themselves from low-ability students, this claim would be essentially accurate.

Simply put, our models show that the grading practices of individual departments cannot
be assessed simply by observing whether they assign many “A” marks or, say, “C” marks. Suppose that a university dean were to compare the grades across two departments in her jurisdiction, and suppose that she were to notice that both consistently give many (or perhaps exclusively) “A” grades. Should the dean insist that these two departments raise their grading standards or, say, ramp up the effort levels required for classes in said departments? Not necessarily. Of the two departments, if one is ability-concealing, then only top students choose the other, thus inducing this department to give a plethora of high grades that are accurate. This department’s becoming even more difficult will not resolve the student selection incentives that we have explored here.

It is clear from empirical literature on grading that, as of this paper’s writing, grades given in American universities are higher than they were ten years ago. Our models suggest that one source for this trend may be an increasing ability of students to sort themselves based on intrinsic ability. Another possible explanation for the observed trends in grades is the emergence of one or two ability-concealing departments in a university. That is, suppose that many years ago all departments in a hypothetical university were ability-revealing and entailed effort costs. Were this the case, then we would expect these departments to have issued both low and high grades. Indeed, and as this example shows, if all departments in a university were ability-revealing, then students would not be able to avoid having transcripts reveal their abilities.

Continuing with this example, suppose that an exogenous shock—the Vietnam War, as some have conjectured⁶—led one department in a university, or perhaps a small number of departments, to adopt grade-inflating practices and simultaneously reduce the effort needed to enroll in said department or departments. As soon as this were to have happened, we would be in a situation where the university had a combination of both ability-concealing

departments and ability-revealing departments. In the presence of both types of departments, our model suggests that forward-looking students of high ability will seek to separate themselves from lower-ability students, the latter of whom will choose ability-concealing department and the former, ability-revealing departments. The result of this will be that all students earn high grades. In this example, the culprit for high average grades overall is the presence in a university of a small number (or even a non-small number) of ability-concealing departments. Indeed, one could argue that ability-revealing departments are somewhat at the mercy of ability-concealing departments: once some of the latter exist, the former will enroll only good students. This leads to a flattening of the grade distributions produced by grade-revealing departments.

This point highlights a collective action problem associated with grading. A department that by itself wanted to address institution-wide grade inflation can be stymied by the ability-concealing behaviors of other departments. If an ability-revealing department were to make its classes increasingly challenging in an attempt to mitigate inflation, then it would make the overall grade inflation problem worse (by driving students to ability-concealing departments) and in so doing decrease its own enrollments. To the extent that low enrollments are problematic for departments who might want to use enrollment figures to argue for faculty positions, no department has an incentive on its own to increase the cost associated with its classes. This sort of collective action dilemma means that university administrators should not assume that individual departments will ever be able to coordinate themselves and form a solution to what administrators might consider a grade inflation problem.

We end with two general comments. First, the model adduced here shows how student sorting at one level of the higher education landscape—students within universities—leads to distributions of grades that seem extensively inflated but, for high-ability students at least, are nonetheless accurate. Of course there are additional levels of sorting that we have not discussed, and these include sorting across universities and sorting within departments.
Although we have not modeled choice of university, one could envision a more general model of education wherein students choose universities and then choose departments within these universities. If, say, some universities are known for being ability-concealing and requiring little effort, then students of higher ability will select against these schools and instead pursue education in costly, ability-revealing institutions. This may be very valuable for these latter institutions, since they will be populated by high ability students, but they will face grading dilemmas since the variance of talent in these schools will be low. Given the recent increase in competition for admission into elite colleges and universities, it is conceivable that across-institution sorting may be a notable factor in explaining nationwide increases in average grades. Not all schools may be subject to sorting in the same way; the University of Texas, for example, is subject to a state law that mandates admissions policies for freshman from Texas high schools. This law may lead to a larger variance in underlying student ability levels than would be expected otherwise.7

Student sorting is also possible at the course level within departments. If, say, a department offers a small number of ability-concealing courses that always give high grades, then only top students will select into costly, ability-revealing courses. Or, say, if a department offers several sections of one course, and these sections vary in the extent to which they are ability-revealing, then we should expect sorting to affect the grade distributions from alternative sections.

Second, while this paper draws its conclusions exclusively from equilibria of game-theoretic models, these conclusions suggest empirical strategies for those interested in understanding the dynamics of grade inflation. Namely, suppose that a researcher at a university wanted to know about the extent to which her institution’s departments were ability-

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7See “Report to the Governor, the Lieutenant Governor, and the Speaker of the House of Representatives on the Implementation of SB 175, 81st Legislature For the period ending Fall 2014,” The University of Texas at Austin, available at https://www.utexas.edu/student/admissions/research/SB_175_Report_for_2014.pdf (last accessed May 12, 2015).
concealing or ability-revealing. Knowledge of this type could in principle inform university policy decisions insofar as regulating ability-concealing departments by compelling them to issue ability-revealing grades could “solve” the problem of grade inflation to the extent that it is considered a problem. In contrast, regulating ability-revealing departments is not necessary if one’s goal is simply to generate a distribution of grades that has significant variance.

How would an interested party determine which departments in a university are ability-concealing? As is clear from our theoretical results, looking at which departments have high grades is not sufficient and can actually be misleading. Rather, ability-concealing departments can presumably be identified because their students vary in ability yet receive similar grades. Presumably the administration in most if not all educational institutions knows, say, standardized test scores of all its enrolled students. Departments whose enrolled majors, say, have high variance in test scores yet have received disproportionately high grades may be ability-concealing.

To be precise, suppose that of two departments in a university, one had high variance in student test scores and low variance in grades, and suppose that the second had low variance in test scores and high variance in grades. This would presumably indicate differences in grading practices in a way that falls roughly along the ability-concealing versus ability-revealing dichotomy described here. These variances are still subject to a student sorting confound, but this is generically true unless a university were to compel all of its students to enroll in common course. Absent such a common course, combining variances in measures of ability like standardized tests with variances in grade distributions might yield a plausible picture of which departments in a university are ability-concealing and which are ability-revealing. In the long run this will aid the general understanding of grading dynamics and how educators and researchers should interpret trends—both upward and downward—in grades.
Proof of Lemma 1.

Proof. Suppose that $\pi_H \geq 1 - c$. If all students enroll in a grade-inflating department, then each receives $\pi_H$ as a post-education wage. This situation obtains because the labor market cannot distinguish the students based on their transcripts, all of which contain identical marks (all “A” grades, as the ability-concealing department by construction issues only “A” marks) and all of which were issued by one department. A low ability student who deviates to a grade-revealing department will pay a cost of $c$ and, by virtue of enrolling in an ability-revealing department will be identified as having low ability. If deviating this student will thus receive utility of $-c$, which is strictly less than $\pi_H$. In contrast, a high ability student who deviates to the ability-revealing department will receive a wage of one, which is better than receiving $\pi_H$, but will also have to pay a cost of $c$. If $\pi_H \geq 1 - c$, then this deviation is not optimal.

Now let $\pi_H$ and $c$ be given such that $\pi_H \in (0,1)$ and $c \in (0,1)$, and consider the existence of a separating equilibrium as posited in the lemma. In such an equilibrium, low ability students receive utility of zero. If a low ability student were to deviate to the ability-revealing department, she would continue to receive a wage of zero and in additional would pay the effort cost $c$. Since $c > 0$, deviating in this way is not optimal. Similarly, with separation a high ability student receives a wage of one and an overall utility of $1 - c$. Deviating to an ability-concealing department would lead to a utility of zero because the student would be considered low ability. Since $0 < 1 - c$, deviating is not optimal.

To show that there is no pooling equilibrium in which all students enroll in a grade-revealing department, suppose that such an equilibrium were to exist. A low-ability student would earn utility of $-c < 0$ in such an equilibrium. Were such a student to deviate to an ability-concealing department, the student would not pay the effort cost $c$ and would receive
a wage between zero and one, inclusive. This would lead to positive utility greater than $-c$, thus contradicting the conjectured equilibrium.\footnote{We need not specify the off-equilibrium path belief of the labor market that a deviating student in an ability-concealing department is of high ability conditional on all students pooling on enrolling in an ability-revealing department. This is because any such conditional belief will contradict the existence of the conjectured pooling equilibrium.}

As an aside, this proof does not depend on the value of $c$ being identical for both low and high ability students. This point supports the claim made earlier in the body of the paper that, as long as enrolling in an ability-revealing department imposes an effort cost, the results of the model do not depend on whether $c$ varies by student ability.

Proof of Lemma 2.

Proof. If in equilibrium a student with ability $\theta_1$ prefers an ability-revealing department, then it is straightforward to show that a student with ability $\theta_2 > \theta_1$ does as well. This follows from the fact that a student’s utility for an ability-revealing department is increasing in underlying student ability. Similarly, if in equilibrium a student with ability $\theta_1$ prefers an ability-concealing department, then it is straightforward to show that a student with ability $\theta_2 < \theta_1$ does as well. Thus, if in equilibrium students separate in their choices between departments, they must do so where a cutpoint, which we denote $\bar{\theta}$, partitions students based on ability.

We now derive the value of $\bar{\theta}$. Let a student with ability $\theta = \bar{\theta}$ be indifferent between enrolling in an ability-concealing department and enrolling in an ability-revealing department. The former nets the student $\bar{\theta}/2$ and the latter, $\bar{\theta} - c + e$. Equating these two utilities yields

$$\bar{\theta} = 2(2 - c - e).$$

If $c - e \geq 1/2$, then $\bar{\theta} \geq 1$ and all students pool on the grade-revealing department. Similarly, if $c \leq e$, then $\bar{\theta} \leq 0$ and all students pool on the ability-revealing department. If $\bar{\theta} \in (0, 1)$, the equilibrium is semi-pooling.
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URL: http://econ.ohio-state.edu/hyang/grade-inflation.pdf